Algorithmic Game Theory

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Lecture - 41
Implementation of Social Choice Functions

Welcome. So, in the last class we have started mechanism design and we have defined what is a mechanism indirect mechanism and direct mechanism. So, today we will start seeing the connection of Bayesian game in with the mechanism.

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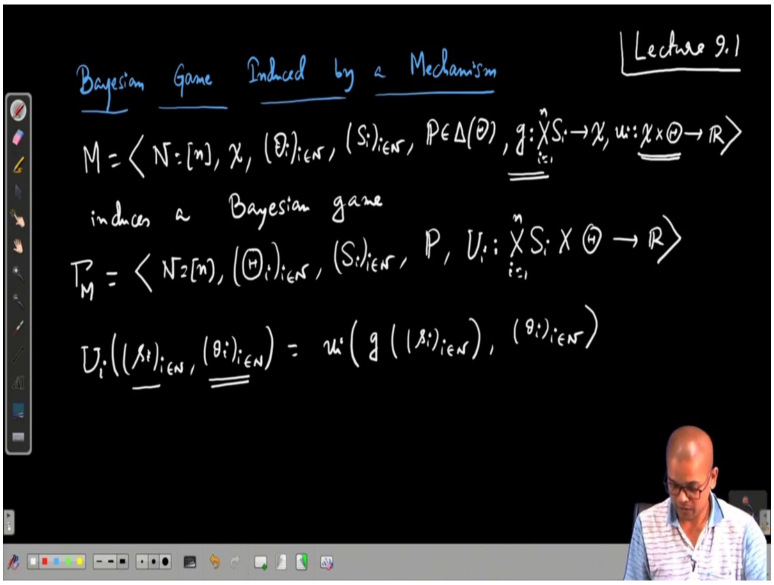

So, first topic is Bayesian game induced by a mechanism each mechanism induces a Bayesian game on the set of players Bayesian game induced by a mechanism. So, suppose I am given a mechanism M N set of players say we have small n number of players set of outcomes X, $(\Theta_i)_{i\in N}$ then $(S_i)_{i\in N}$. Then a prior distribution P in which is a probability distribution over set of all type profiles g from strategy profiles two outcomes and utility function from outcomes and type profiles to real number. So, this mechanism induces Bayesian game.

What is the Bayesian game? Let us denote it by Γ_M which is the set of players remains same then for Bayesian game there is no set of outcomes, outcomes are implicit $(\Theta_i)_{i\in N}$ typeset remain the same strategy set remains the same for each player. Then prior distribution also remains the same there is no g and there is utility let us denote it by capital U which is from strategy profiles I equal to one to nth time theta type profiles to real number.

So, only thing that I need to define is the utility function and it is defined as $U_i((s_i)_{i\in N},(\theta_i)_{i\in N})$ small u_i. This a small u_i is a function of outcomes and type profiles type profiles I have here and outcome is a function of s_i that is connected using this function g so $u_i(g((s_i)_{i\in N}),(\theta_i)_{i\in N})$ ok.

Now once we have connected mechanism with Bayesian game in all those concepts of game theory like Nash equilibrium and all those things all those things becomes relevant here now.

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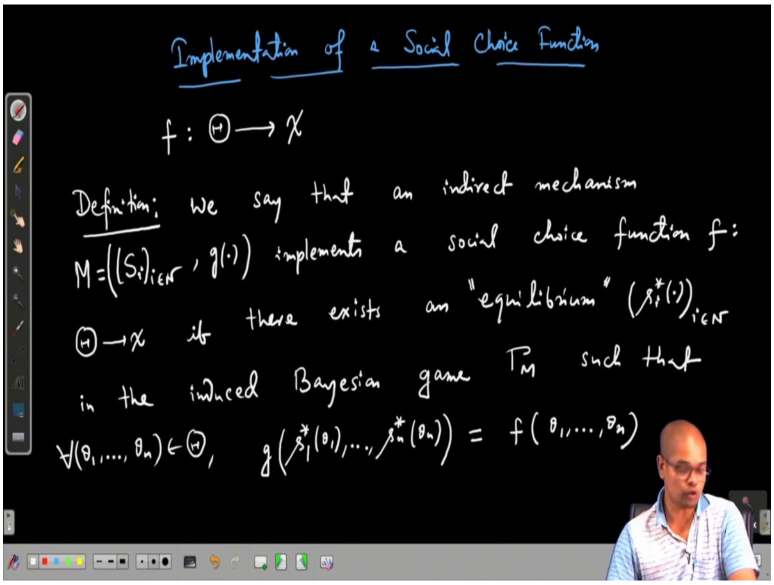

Now, we formally define what do we mean by implementation of a social choice function implementation of social choice function. So, suppose as usual I have a social choice function f from type profiles two outcomes. And I want to implement it that is mean what why what; that means, that I want to induce a game such that from the ways players play participate in the game assuming players are rational intelligent and selfish and they participate and from their action we are able to compute this function f.

So, what do you mean by that definition? What do you mean by implementation? We say that an indirect mechanism M recall if everything is clear from the context we drop those unnecessary components from the tuple M and we focus on only what mechanism designer need to needs to design; that means, the strategy set S_i and g. We say an indirect mechanism implements a social choice function f from type profiles to outcomes if there exists an equilibrium.

Now, how does an equilibrium looks like in a Bayesian game it looks like a function from type sets to strategy type sets to two strategy sets. So, s_i^* it is a function each strategy in a Bayesian game is a function from type sets to the strategy sets $(s_i^*)_{i\in N}$ ok. If there exist an equilibrium, in the induced Bayesian game, Γ_M such that for all $(\theta_1,\dots,\theta_n)$ for all type profile.

Look at what the players are playing with in this particular type profile. What strategies they are playing $s_1^*(\theta_1)$ is the strategy played by player 1 and in this type of type θ_1. So, this $(s_1^*(\theta_1),\dots,s_n^*(\theta_n))$ this is the strategy profile played by the players in the Bayesian game in that when the type profile is sitting on to the time.

And in this strategy profile if you apply the function g that is the outcome this should be same as $f(\theta_1,\dots,\theta_n)$. That means, for each type profile you look at what strategy profile players are playing and you apply the function g that then the outcome that we get is should be equal to the desired outcome of f under the same type profile.

If this holds for all type profile then we have we are able to implement the social choice function f using this indirect mechanism M. Now, here one thing is not completely specified equilibrium what do we mean by equilibrium there are so many concepts of equilibrium.

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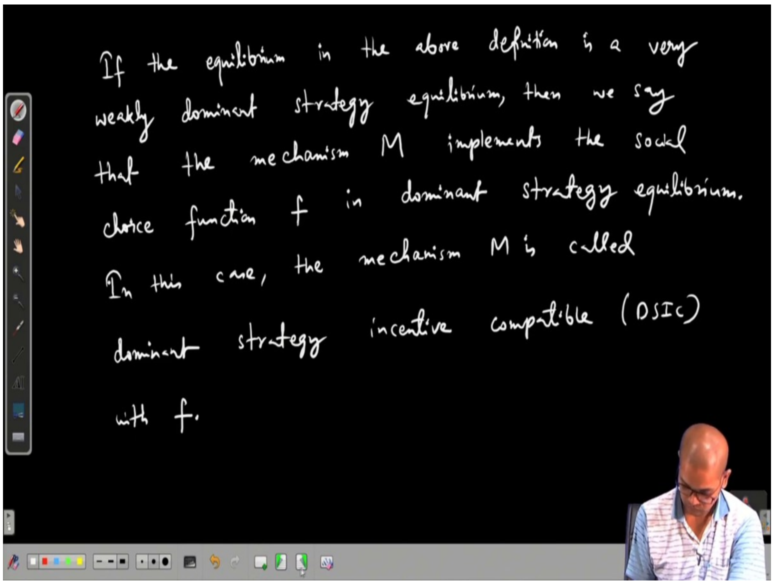

So, if the equilibrium in the above definition is a very weakly dominant strategy equilibrium. Then we say that the mechanism M implements the social choice function f in dominant strategy equilibrium. In this case, f is called not f the mechanism M is called dominant strategy incentive compatible incentive compatible DSIC with f ok.

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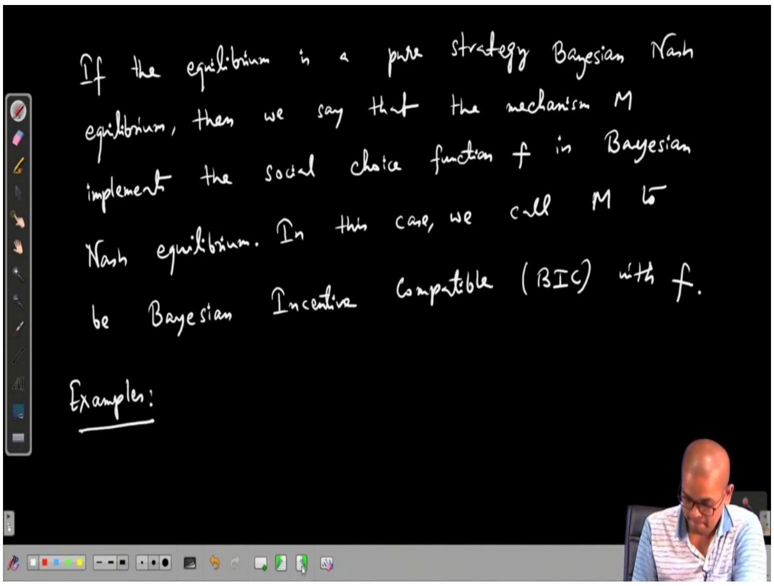

So, let us see or otherwise on the other hand if the equilibrium is a pure strategy Bayesian Nash equilibrium. Then we say that the mechanism M implements the social choice function f in Bayesian Nash equilibrium. In this case, we say we call f we call M to be Bayesian incentive compatible Bayesian incentive compatible BIC with f. So, let us see some examples of social choice functions. So, examples of social choice functions.

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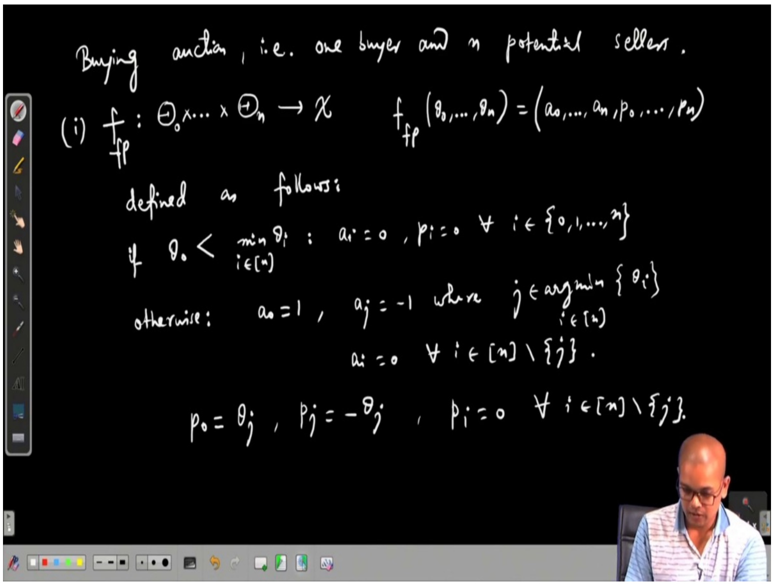

So, again consider a buying auction that is one buyer and n potential seller ok and suppose two so one first social choice function is the first price auction and the second social choice function or for example, will be the second price auction. So, first let us see the first price auction what is the corresponding social choice function from say.

So, it is a function from type profiles to outcomes where $f_p(\theta_0, \ldots, \theta_n)$ is the outcome has two parts allocation which seller is eventually going to sell the item and payment ok and how does this how this is defined as follows how.

So, think of θ_0 the type of the buyer is the maximum amount that he or she the buyer is willing to pay, θ_0 is the type which is the maximum amount that the buyer is willing to pay. If the maximum amount is less than the minimum of $\theta_1, \ldots, \theta_n$, so think of theta I from for from $\theta_1, \ldots, \theta_n$ this is the amount minimum amount for which the seller is willing to sell.

So, if the buyers maximum amount to buy is less than the smallest amount by the sellers coated then they are there cannot be any buying or selling can happen. In that case all the $a_i = 0$ and all the $p_i = 0$ for all $i \in \{0, 1, \ldots, n\}$. Otherwise buyer buys the item $a_0 = 1$ and suppose j is the winner j is the highest bidder. So, $a_j = 1$ where j is $argmin_{i \in N} \theta_i$ and breaking the ties arbitrarily may be like geographically if there are more than one minimum.

So, $a_j = -1$ if j is going to gives the item minus 1 and $a_i = 0$ for all $i \in N \setminus \{j\}$ ok. And what is the payment? p_0 in the first price auction the bidders are the by are the sellers and it gets this much it is amount. So, p_0 is θ_j that much amount the buyer pays and $p_j = -\theta_j$. This is the amount that seller receives and for every other seller $p_i = 0$ for all $i \in N \setminus \{j\}$.

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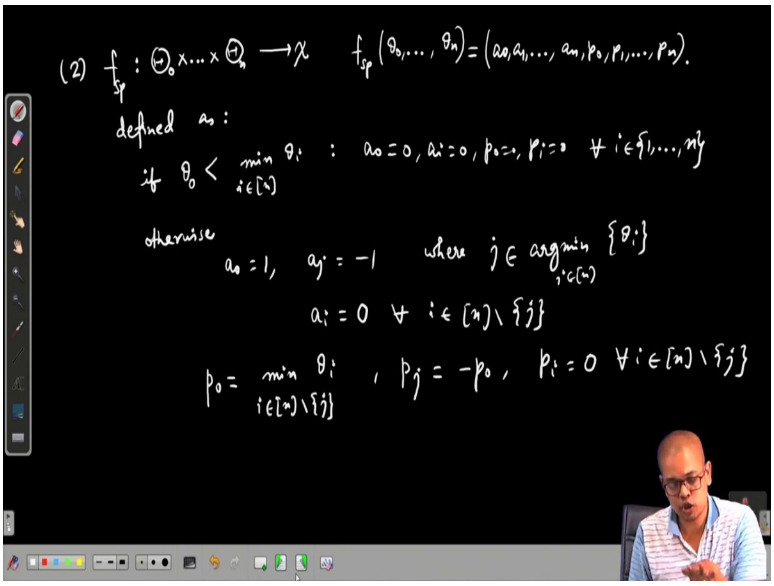

So, this is how the first price auction looks like looks. See how the second price auction look like means what is the corresponding social choice function, $(\theta_0, \ldots, \theta_n) \rightarrow X$ as usual $f_{sp}(\theta_0, \ldots, \theta_n)$. The outcome has two parts one is allocation and payment defined as if as usual. If buyers valuation θ_0 is less than sellers valuation then no trade can happen $a_0 = 0, a_i = 0, p_0 = 0, p_i = 0$ for all $i \in \{1, \ldots, n\}$.

Otherwise sales happens and the buyer gets the item $a_0 = 1$ and who is the seller is the highest bidder. So, $a_j = -1$ who gives the item and where j is as usual $argmin_{i \in N} \theta_i$ whoever beats the smallest that seller sells the item and breaking that is arbitrarily and all other sellers lose for all $i \in N \setminus \{j\}$ok. And what is the payment that is where the most crucial thing.

So, p_0 is the second highest bid. So, it is the minimum of θ_i where i where is this $i \in N \setminus \{j\}$. So, this is p_0 and this is the amount of money that seller i buyer i pays and this money is received by the say by the winner the seller $p_j = -p_0$ and other players they do not pay or get anything $p_i = 0$ for all $i \in N \setminus \{j\}$. So, this is another social choice function which is second price auction.

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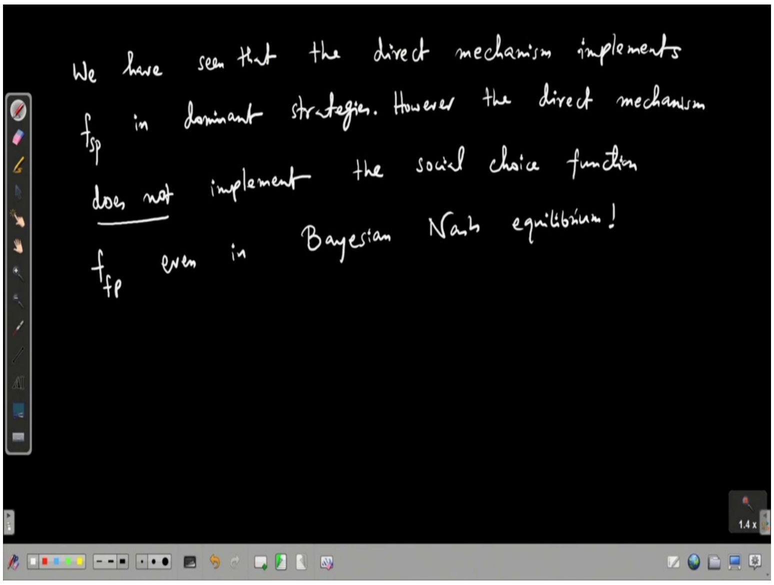

Now, what we have seen? We have seen that the direct mechanism implements f_{sp} the social choice function for the second price auction the direct mechanism implements the social choice function f_{sp} in dominant strategies. Recall we have proved this in the beginning of our courses course that you know simply tell simply ask the players to reveal their true type and declare that this is how I am going to decide the winner, this will be the allocation rule, this will be the payment rule.

Then bidding their true valuation is a weakly dominant strategy equilibrium it is not even a very weakly dominant strategy equilibrium it is even stronger, it is weakly dominated dominant strategy equilibrium. On the other hand it is clear that you know for the first price auction at least the direct mechanism does not work and not only that we have seen that in the first price auction it is one can easily see that you know revealing the true type bidding the true type is not at all up even a pure strategy Nash equilibrium.

Indeed if there are n players bidding $\frac{n-1}{n}$ of true type because is a pure strategy Bayesian Nash equilibrium. So; however, let me write; however, the direct mechanism does not implement the social choice function which is for first price auction even in Bayesian Nash equilibrium. So, the immediate question is that ok may be the direct mechanism does not implement the first price auction in the Bayesian Nash equilibrium.

But can there exists exist some other more clever indirect way of inducing a game and indirect mechanism which can implement the social choice function for the first price auction and the answer is no. We will see in the next class that there does not exist even an indirect mechanism; however, complex; however, sophisticated it is which can implement the first price auction in Bayesian Nash equilibrium. We will start the next class from this point ok.

Thank you.