Algorithmic Game Theory
Prof. Palash Dey
Department of Computer Science and Engineering
Indian Institute of Technology, Kharagpur

Lecture - 39
Extensive Form Game

Welcome. So, in the last lecture, we have studied the notion of Pure Strategic Bayesian Nash Equilibrium and we have seen that for two player first price auctions bidding half of their valuations is a pure strategy Bayesian Nash equilibrium. And, in this class, we will study Extensive Form Game which sort of generalizes the strategic form game which allows sequential moves also not necessarily simultaneous moves which is a which is key to strategic form game.

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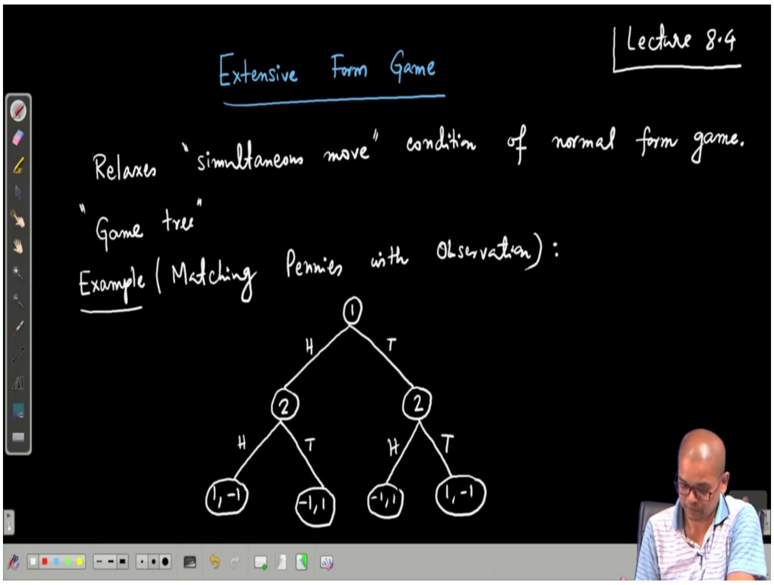

So, today's topic is Extensive Form Game. So, it relaxes simultaneous move condition of normal form game ok. So, let me give understand this with an example and this sequential move is also sometimes represented as what is called game tree. It is a nice pictorial representation of what is happening in a game and let us see this notion of game tree with the example of matching pennies with observation.

So, what is the example let me write example matching pennies with observation. So, first say player 1 moves and it plays either head or tail. Now, after player 1 moves, player 2 sees player 2 can see what player one has played and play accordingly. So, each node is labeled with the name of the player whose turn it is to play.

Now, player 2 also has 2 options in both the cases playing head and playing tails playing head playing tails. This matching pennies with observation whenever some player has to play it knows entire it knows till now all the players who have played what they have played.

So, in that case if for both head and head then player 2 wins player 1 wins and gets a utility of plus 1 and player 2 loses if it is not match then player 2 wins and player 1 loses minus 1, 1, 1 minus 1. So, as you can see that you know with observation this matching pennies game is not so much of an interesting because player 2 can always win because he can see what player 1 has played and just play the opposite. So, player 2 can always win.

So, to make this game interesting, but still extensive form game if you want to want to write the matching pennies game without observation where player 2 is not allowed to see what player 1 has played before playing it is strategy, then what we get is called matching pennies without observation.

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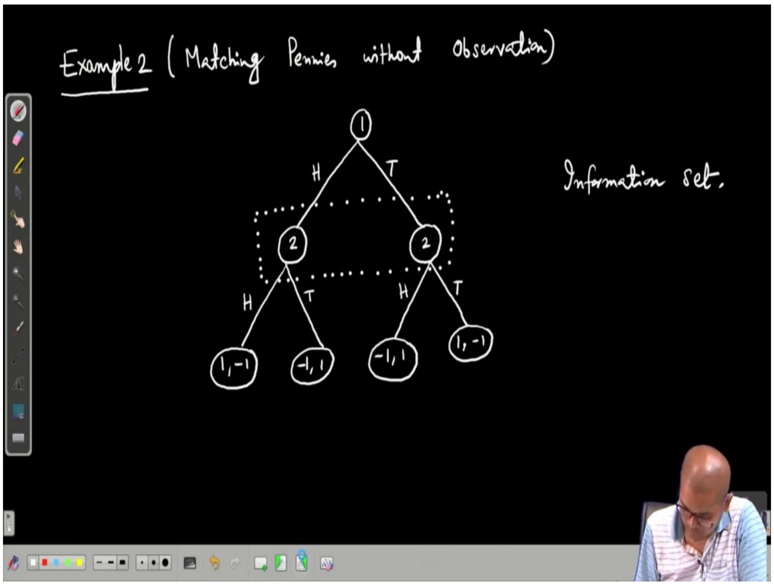

So, example 2, matching pennies without observation. So, here also the game tree remains same. So, suppose player 1 plays first and it can play either head or tail and then player 2 plays, but you know player 2 is not allowed to see what player 1 has played when it is going to play – head, tail; head, tail. The outcome remains same if there is a match then player 1 wins and player 2 loses and if there is a mismatch, then player 1 loses and player 2 wins.

But, how we denote the fact that player 2 is not allowed to see what player 1 has played at till that point and that is where the idea of information set comes handy information set. What is what information set? It is a set of nodes which we denote using say a dotted box and this box is an information set meaning that the player who is playing is not sure which among the nodes in this box it is in at the time of playing at the time of playing certain action, ok.

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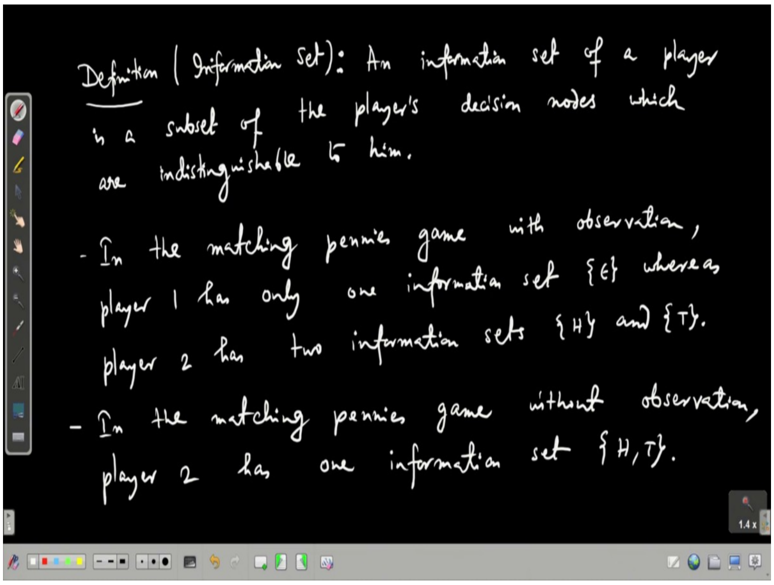

So, what is information set let me define. Definition: information set – an information set of a player is a subset of the players decision nodes of the players decision nodes which are indistinguishable to him interesting. So, you see that in matching pennies without observation player 2 has only one information set although it has two decision nodes.

On the other hand, in matching pennies with observation player 2 has two decision nodes and two information sets, both are singleton ok and player 1 of course, has one decision node and one information set. So, that is how with the concept of set we enable extensive form games to capture the situations of simultaneous move or partial observation and so on and so forth, ok.

Now, you know so, let me write. So, in the matching pennies game matching pennies game with observation player one has only one information set and typically each node is denoted by the path from the root node. So, player 1 decision node is a root node. So, that is denoted as epsilon ok which is like complete history. So, these paths are also called histories in the context of extensive form games.

Whereas, player 2 has two information sets what are they H and T, this is for matching pennies game with observation. For matching pennies game without observation in the matching pennies game without observation player 1s information keeps on continue to have only one information set which is the root node player 2 also has one information set has one information set namely H and T, although two decision nodes, but only one per sets because player 2 does not know at the time of playing the action or strategy played by player 1.

So, now with this example in light we now define formally define what is an extensive form game.

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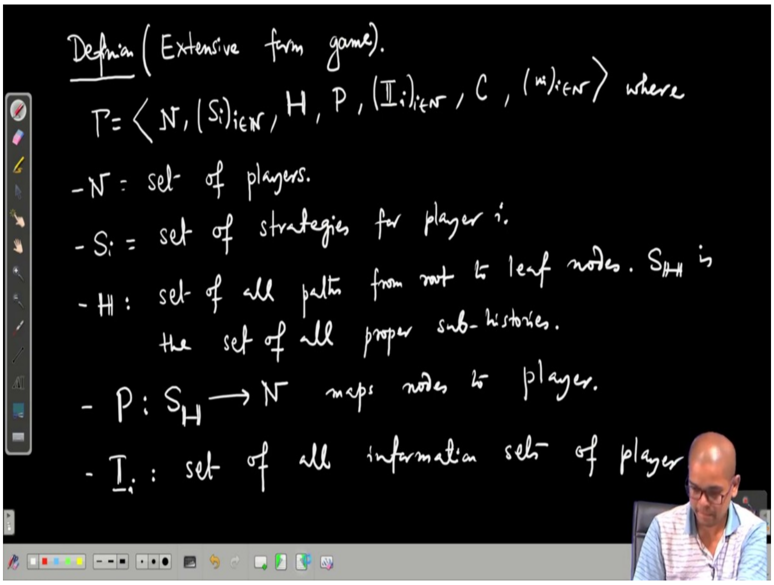

The definition is cumbersome because of so many things. Definition of extensive form game. So, an extensive form game is a tuple in whereas usual we have a set of players then strategy set for each player I in N, then H set of histories it is the set of all sub paths to think of a game tree and the paths from root the paths to all nodes each path is a history.

So, capital H is a set of all histories then prior distribution P no prior distribution not P is not the prior distribution P is a mapping from this sub histories to the player it is like the label it is like at which node which player is going to play. So, we will elaborately write this.

Then we have the set of all information sets ii for all the players N, then we have C will see. What is what is C? It is like for each information set which set of actions are available. So, each player can go in turn and so, depending on the information set various actions are available observed that in each information in an information set at every decision node in one information set the set of all actions available must be same.

Simply because the these states these decision nodes are indistinguishable to the player at the time of play and the utility function i in N, ok. This is an extensive form game where N is we have 10 is the set of players; S_i is set of strategies for player i; H set of all set of all paths from root to leaf nodes S_H is a set of all sub histories sub paths. S_H is the set of all sub set of all proper sub histories which are basically paths from root to internal node.

Then we have a function p which defines from each sub history; that means, in the each node which player is going to play ok. So, its maps nodes to players which player is going to play at that decision node. Then I_i is the set of all information sets of player I that is I_i.

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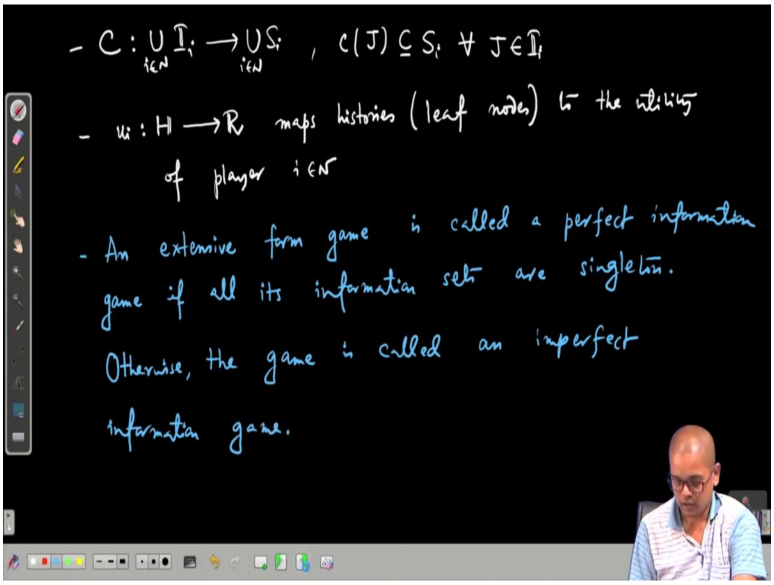

Then we have C it maps in from each information set what are the set of strategies available to a decision node in that information set $\cup_{i\in N} I_i$ to S_i and it ensures that $C(J)$ is less than equal to is a subset of S_i for all $J \in I_i$. That means in if I pick an information set for player I the set of strategies available must be a subset of the set of all strategies available at some point of time to player I.

And, then we have u_i from histories to $\mathbb{R}$ at every leaf node what are the utilities of player I maps histories which are leaf nodes to the utility of player I and then ok. Now, next we define what is called a perfect information game and imperfect information game. A perfect information game is like if everyone can see whatever has happened till now for example matching pennies with observation is a perfect information game on the other hand matching pennies without observation is an imperfect information game.

So, let me write an extensive form game is called a perfect information game if all its information sets are singleton. Each information set contains only one decision node, otherwise if that is not the case that is for example, matching pennies without observation otherwise the game is called an imperfect information game.

So, the last thing that we will see now is that you know again the same question for extensive form game do we need to define the notions of say strongly domain strategy equilibrium, pure strategy Nash equilibrium or correlated equilibrium from scratch or can we borrow the notion or these notions developed for normal form game and the answer is obvious we can borrow.

And, the idea is that is that we can represent the an extensive form game equivalently as a normal form game. This sort of shows why we focus only normal form game although it looks simple simpler than Bayesian game and extensive form game, it is general enough to encode all the complexities involved in more sophisticated games like Bayesian games or extensive form games, ok.

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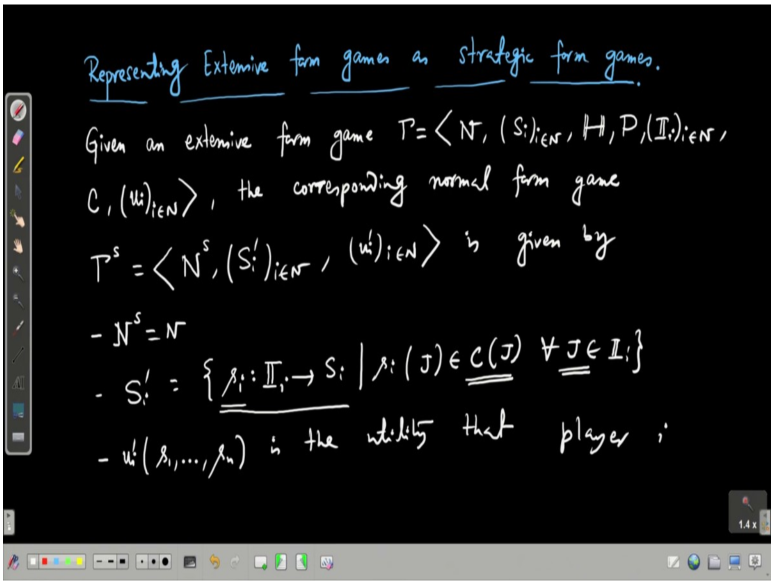

So, let me write representing extensive form games as strategic form games. So, as usual so, given an extensive form game $\Gamma = \langle N, (S_i)_{i \in N}, H, \ldots \rangle$ set of all histories which is basically the set of all paths from root to leaf node. These set of all histories then this mapping P which maps each sub history to the players that which tells at every node it is which player who is going to play.

Then information set of each player $I_i i \in N$, then see this tells that at every internal node at every sub histories what are the set of strategies available to each of the players. And then utility values it maps each leaf node it tells at each leaf node what are the utilities received by player I.

Suppose I am given an extensive form game, the corresponding normal form game gamma s, s for strategic form $\Gamma^s = \langle N^s, (S_i^{'})_{i \in N}, (u_i^{'})_{i \in N} \rangle$ is given by the set of players remains same as it is $N^s = N$ then $S_i^{'}$ let us see what is $S_i^{'}$. It is small s_i from information sets I_i to S_i such that $s_i(J) \in C(J)$ for all $J \in I$. That is the strategy sets and utility $u_i^{'}(s_1, \ldots, s_n)$ is the utility that player i.

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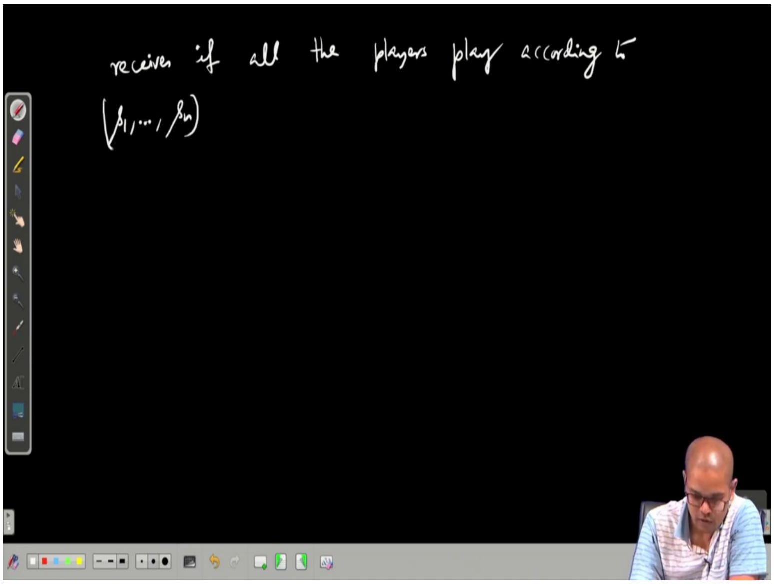

Receives if all the players play according to they are according to this strategy profile $(s_1,\dots,s_n)$. What do I mean? Now let us see let us go back and see you know the set of players is same. So, for matching pennies say without observation we still have in the corresponding normal form game we still have two players, but you know the strategy set is like it is sort of like an elaborate plan from beforehand that is each player is all they have is an information sets and set of all information sets and it is deciding beforehand at every information set what action I am going to play.

So, at the time of playing if I land up in an information set I have already decided which action I play and that is my strategy s_i and that is why strategy s_i is a function from information set to the strategy set S_i and the requirement that I have is that if I look at any particular information set J capital $J \in I_i$ the information sets of player i you know the information set of player i and J is a decision node the set of the strategy I am going to play at that particular decision node must be available must be available that is so, that is why it belongs to $C(J)$.

And, the if all the of the plans are ready then I can simulate this game and follow the path in the game tree and reach a leaf node and get to know what each players utility is and that is what is the utility function $u_i^{'}(s_1,\dots,s_n)$ ok.

So, this concludes the first part of our lecture which is game theory. In the next lecture we will start the second important part which is mechanism design which is also called sometimes called reverse game theory it is like designing a game to obtain desired outcome ok.

So, we will conclude here.