Algorithmic Game Theory
Prof. Palash Dey
Department of Computer Science and Engineering
Indian Institute of Technology, Kharagpur

Lecture - 37
Bayesian Game

Welcome. In the last two lectures, we have studied price of anarchy, last 2 to 3 lectures, and which sort of quantifies how bad Nash equilibrium outcome can be from the best possible social outcome. And till now in this course, we have been studying the games and we have assumed that the games are given in normal form.

And what is normal form games? It is like players are playing independently and they are playing simultaneously, they are picking their strategies simultaneously, and also it is a one shot game. So, in this lecture, we will start looking at various other kinds of games.

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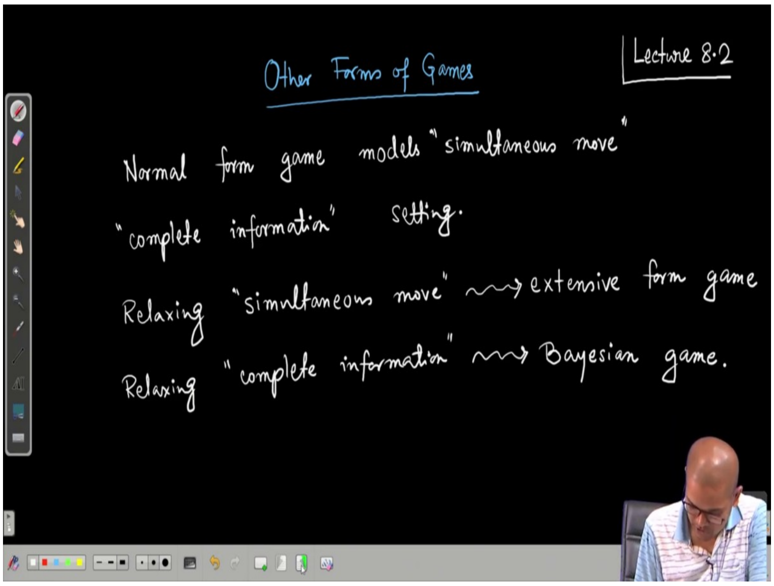

Other forms of games: So, let us recall. Normal form game models simultaneous move; that means, all players are picking their strategies, playing their strategies simultaneously, simultaneous move complete information games. So, what is complete information? That the game is a common knowledge. The number of players, set of strategies, and their utility functions is common knowledge.

There is nothing called there is no notion of private information or any piece of data or information which is private or known only to some player, one player. So, we will relax these two notions one at a time. So, relaxing simultaneous move we get extensive form game, where players are allowed to move sequentially. And relaxing complete information, thereby allowing players to have private information, we get what is called Bayesian Game.

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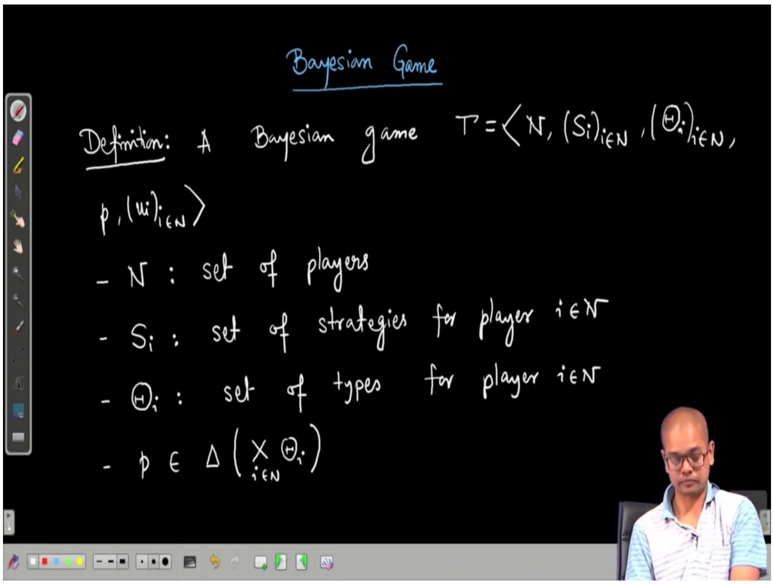

So, in today's lecture, we will study Bayesian games. Definition, what is a Bayesian game? A Bayesian game gamma is denoted by a tuple. The first one is as usual a set of players, then strategy set for each player, then type set for each player, which models private information then a prior distribution small p and the utility functions $(u_i)_{i\in N}$.

So, what are these components? Let us write them one by one. N is set of players, S_i is the set of strategies for player $i\in N$, then theta i is the set of types for player $i\in N$. And then we have a prior probability distribution over the set of type profiles. So, p is a probability distribution over type profiles.

So, although capital theta i is the set of all possible types of player i, by type it just means some information. So, for auction for example, this type of player of a buyer could be it is his or her valuation for the item that could be its type, and this is a private information, but the set is known to all of them.

For example, this set the valuation could be any real number. So, this set real number is known to all the players, it is a common knowledge. But what is the actual type, what is the actual valuation for a buyer for a for that item being auctioned is known only to that particular buyer. Although, there is a common prior over the type profiles which is also common knowledge.

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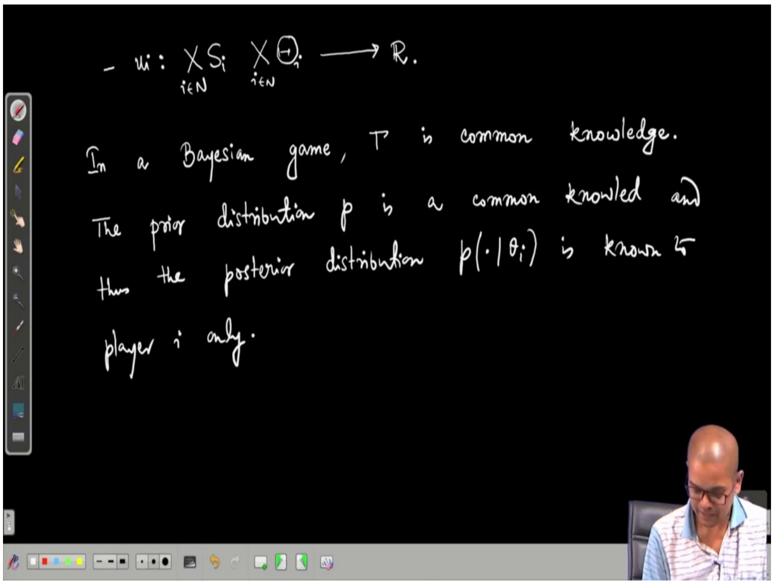

And then, a utility function u i, this is from the outcome which depends on the strategy profile being played times all type profiles to real numbers. So, in a Bayesian game, gamma is common knowledge; that means, set of players, set of strategies for each players, the type set for each players, the prior distribution and the utility functions these are common knowledge.

Now, observe that the prior distribution p is a common knowledge and the posterior distribution, and thus the posterior distribution given θ_i is known to player i, known to player i only because player i knows its current type its true type θ_i.

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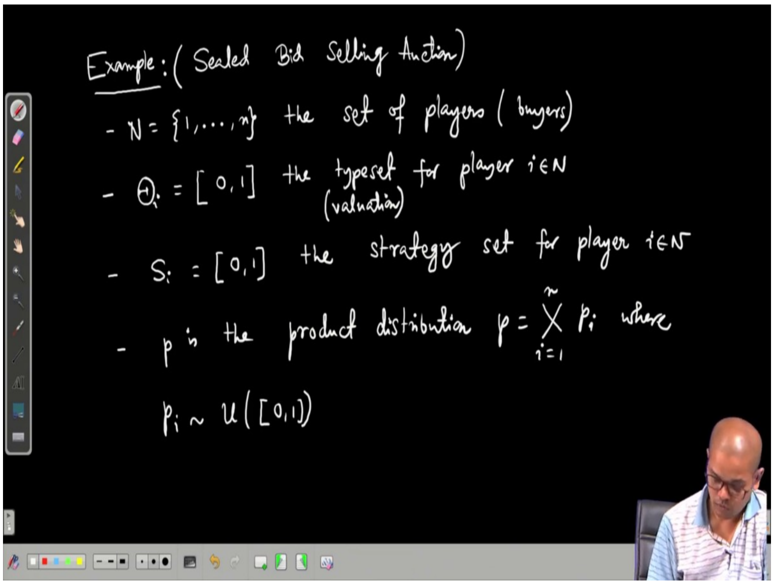

So, let us see an example so, our canonical example of sealed bid selling auction. We will see how this particular situation can be modelled by a suitable Bayesian game. So, we have a set $N = \{1, \ldots, n\}$, the set of players, they are the buyers, prospective buyers.

Then, the types set of each buyer θ_i is some real number between 0 and 1 which type set belongs to the set 0 and 1 which represents the valuation for the item. This is the type set for player i and type set is same as valuation here. And, but what is the particular value, what is the actual value is known only to player i

But they set between 0 to 1 that the value is some real number in between 0 and 1, this particular information is common knowledge. Then, s_i is also in between 0 and 1 which is the strategy set, set of valid bids, strategy set for player i. Then, the prior distribution p is the product distribution $p = \times_{i=1}^{n} p_i$, where p_i is distributed uniformly in 0 to 1, ok.

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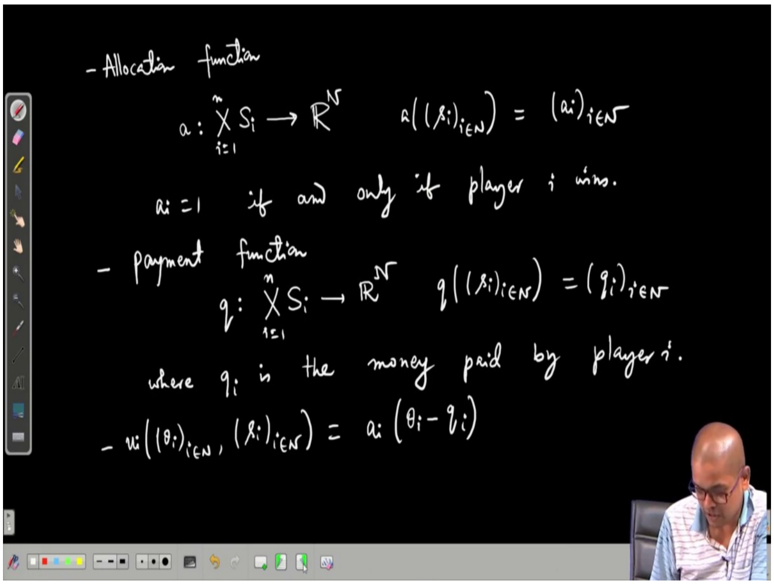

Then, as usual we have an allocation function which will decide who is going to buy the item from the bids. Allocation function which is a, which is a function from strategy profiles to $\mathbb{R}^n$ and defined as $a((s_i)_{i\in N})=(a_i)_{i\in N}$, and it is such that $a_i=1$ if and only if player i wins. That is the ith buyer is going to buy the item. He has the highest bid.

Then, payment function, which is q because p is already taken for prior distribution; it is also a function from strategy profiles to $\mathbb{R}^n$, $q((s_i)_{i\in N})=(q_i)_{i\in N}$, where q_i is the money paid by player i, ok. And then we have utility function, $u_i((\theta_i)_{i\in N},(s_i)_{i\in N})=\theta_i-q_i$ is the payment. So, θ_i-q_i; if player i wins this; that means, times a_i, and if it does not win then its valuation is then its utility is 0, ok.

So, very good. So, this Bayesian game nicely captures or nicely models; if some player has some information which is private to him or her and which is often the case in auctions and many other applications. But the problem we have that, we have developed the entire machinery of Nash equilibrium and the and so many things correlated equilibrium and we have spent so much time studying normal form games. Do we need to develop those notions for the Bayesian game again from the scratch?

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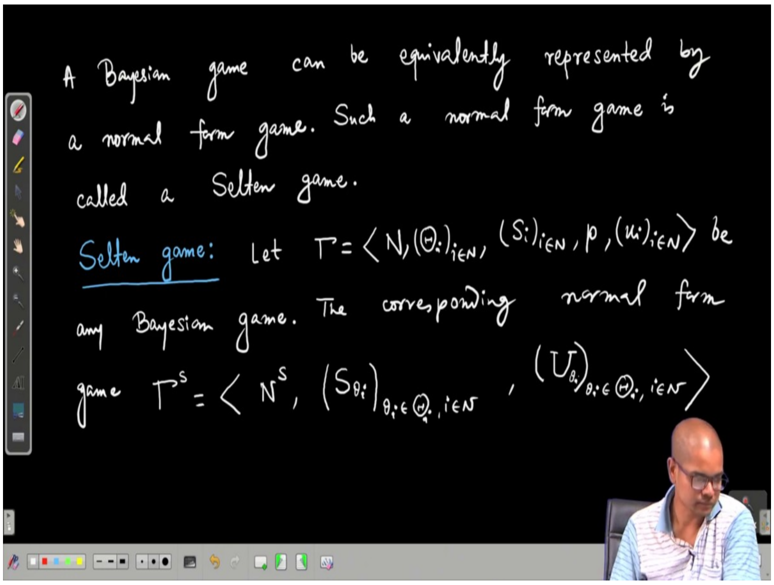

And the good news is that Bayesian game can be equivalently represented by normal form game. That is the beauty of normal form game. Although, Bayesian game is more general than the normal form game, a Bayesian game can be equivalently represented by a normal form game. Such a normal form game is called a Selten game, is called a Selten game; named after the inventor of this game.

So, what is the Selten game? So, let me write Selten game. So, let gamma be a Bayesian game; that means, $\langle N,(\Theta_i)_{i\in N},(S_i)_{i\in N},p,(u_i)_{i\in N}\rangle$, be any Bayesian game. The corresponding normal form game, the corresponding normal form game, called the corresponding Selten game is Γ_s is N^s. Then, you know the set of players is the set of all type profiles for each, set all types, for each type we have a player

So, we will see. So, $N=\cup_{i\in N}\Theta_i$ and then; so, for each such player; that means, for each type I need to define strategy set small $\theta_i\in\Theta_i, i\in N$. And then their utilities let me define it using write it using capital U, small u is the utility function for the Bayesian game, capital U is the utility function for the Selten game; $U((\theta_i)_{i\in N})$, $\theta_i\in\Theta_i, i\in N$, ok.

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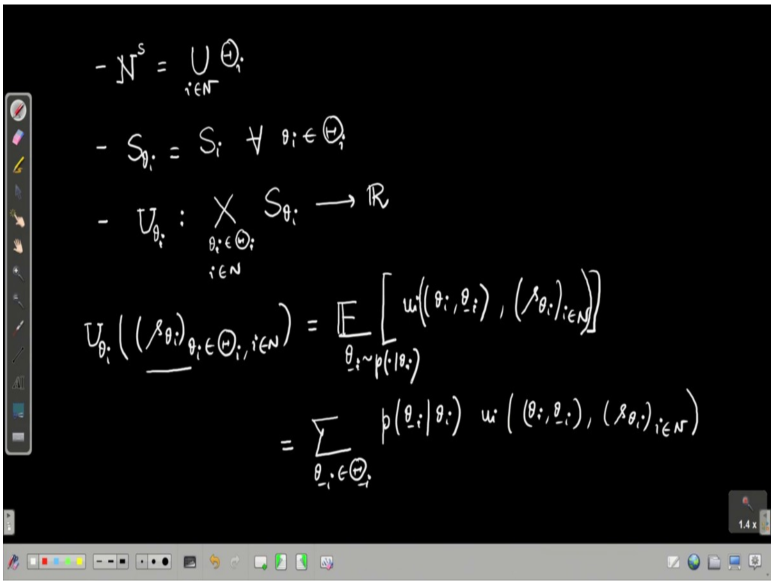

So, what are they? As I said that we have a player for each strategy set. N^s is equal to for each type; $\cup_{i\in N}\Theta_i$. Then, for a particular player i look at its type set Θ_i. And for each type of that type set we have a player and their strategy set is exactly the strategy set of that player. That means, $s(\theta_i)=s_{\theta_i}$ for all $\theta_i\in\Theta_i$.

And what is the utility function? $U((\theta_i)_{i\in N})$ is a function from the strategy profile and we have a strategy set for each player for each player, and strategy for each player and the players are the types; so, $\times_{\theta_i\in\Theta_i, i\in N}\ S_{\theta_i}\rightarrow\mathbb{R}$.

And what is the particular utility value? $U_{\theta_i}((s_{\theta_i})_{\theta_i\in\Theta_i, i\in N})$. This is the expected utility of, expectation with respect to the types of other players, expected utility of player i over the types of other players $u_i(\theta_i,\theta_{-i},(s_{\theta_i})_{i\in N})$.

So, when you want to define the utility of player θ_i, you find what is the expected utility of player i in the Bayesian game when its type was θ_i and yeah and that is it. And so, how will you calculate? You sample θ_{-i} from the posterior distribution of player i when is type is θ_i. And for each sample θ_{-i}, you go to the corresponding players in the Selten game and see what strategy are they playing.

And the utility for that strategy profile will be the utility of player i. Small u_i is a function of type profiles and strategy profiles. So, if I expand this it is summation $\theta_{-i}\in\Theta_{-i}$. What is the probability that this particular probabilities type profile is chosen? This is $p(\theta_{-i}|\theta_i)$, then $u_i(\theta_i, \theta_{-i}, (s_{\theta_i})_{i \in N})$, ok.

So, just expanded the utility so, that is how we view Bayesian game as a corresponding normal form game which is a Selten game. And what we do is, again recall that for each type we introduce a player in the strategic form game and its strategy set is the same as the strategy set of player i in the Selten game in the Bayesian game.

And what is the utility of the player, utility function of the player corresponding to θ_i in Selten game? It is the expected utility of player i in the Bayesian game when its type is θ_i. That is exactly the case. So, in the next class we will define what is the what is mean by pure strategy Nash equilibrium for this Bayesian game and other equilibrium concepts. And we will prove, we will find an equilibrium for the fast price auction, ok.

Thank you.