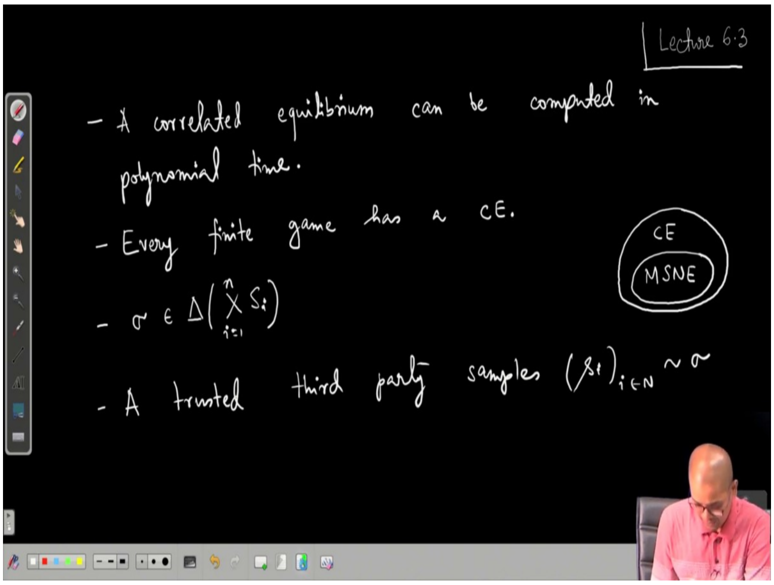
Algorithmic Game Theory
Prof. Palash Dey
Department of Computer Science and Engineering
Indian Institute of Technology, Kharagpur

Lecture - 28
Coarse Correlated Equilibrium

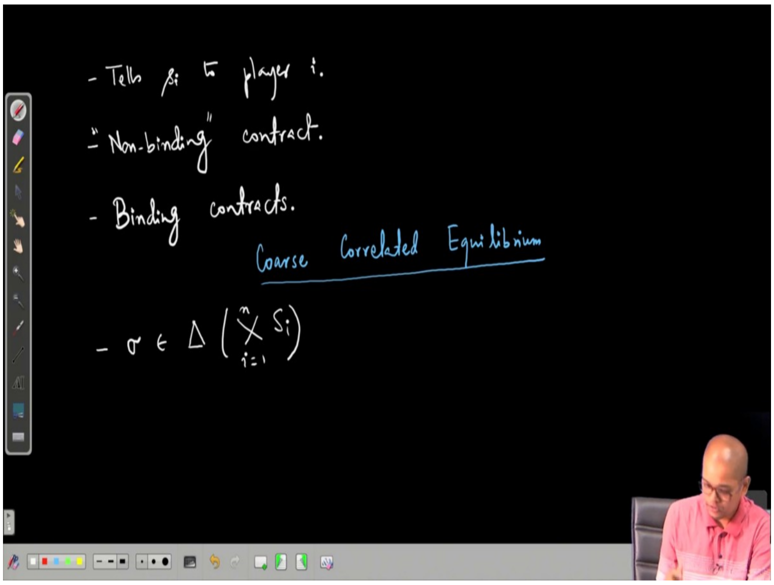
(Refer Slide Time: 00:36)



Welcome in the last lecture we have studied Correlate Equilibrium, so and we observe that a correlated equilibrium can be computed in polynomial time and we also observe that every finite game has a correlated equilibrium CE in short. So, this sort of resolves our problems of NASH equilibrium being hard to hard to compute in particular it is PPAD hard.

And we have found something which is correlated equilibrium and the second treatment follows from the fact that every mixed strategy NASH equilibrium is also a correlated equilibrium. And we give this perspective that you know how we can implement correlated equilibrium it is a joint probability distribution over the strategy profiles and a trusted 3rd party samples a strategy profile samples a strategy profile from this distribution. And every player knows this distribution sigma, but you know every player has the flexibility of not following it.

(Refer Slide Time: 03:34)



So, what the trusted third party does? Is that it first samples a strategy profile and tells Si to player i and sigma is a correlated equilibrium, if players each player after knowing Si and of course it knows sigma there is no incentive for player for any player to deviate from the suggested strategy from the trusted third party.

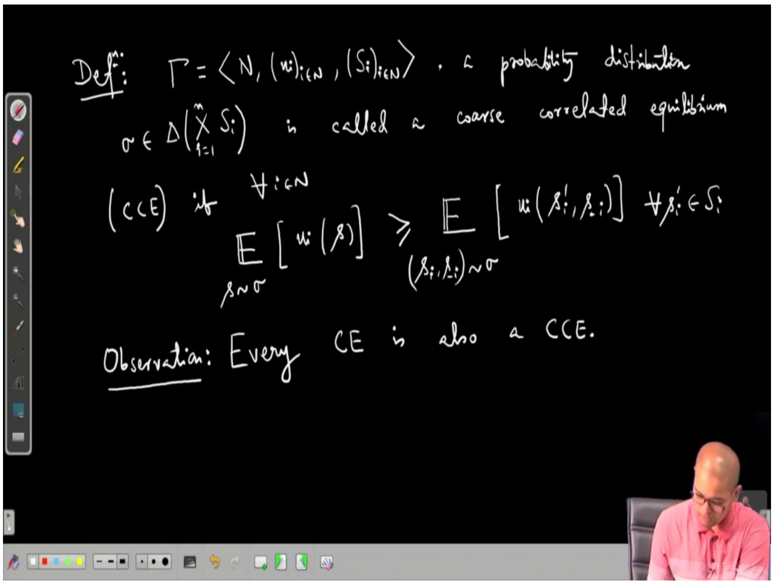
So, this type of settings are sometimes called Non-binding contract, it is like all the players have come together and decided who will be the trusted third party and it have decided that you know sigma is a good probability distribution over strategy profiles to follow that is good for everyone. But still they have a freedom of not following it just before actually playing the action. So, that is why that is what makes this setting a non-binding contact it is like a contract among the players, but it is not binding players are free to not follow it.

And the strategy profile a contract we will be called correlated equilibrium, if there is no incentive for any player to deviate from the distribution unilaterally to be to breach the contract unilaterally. So, because there is a binding contract this this correlated equilibrium corresponds with binding non-binding contracts you know we know we know that there is there are binding contracts. A contract among the players which they have to follow if they do not follow then the then there will be consequences, there will be penalties or something like that.

So, there is a equilibrium notion which is called Coarse Correlated Equilibrium. Which sort of captures this essence and you know before signing the contracts the players have all the liberty in the world to view the contract they know a contract is a probability distribution sigma over the strategy profiles. But after they sign the contract after they agree that they will play according to sigma and a trusted third party samples a strategy profile and tells each player what is his or her strategy is the player players must follow it.

So, the question is when does a player would like to sign the, this contract and that is exactly the notion of coarse correlated equilibrium. It is like at the time of signing the contract players have no incentive of deviating from itself.

(Refer Slide Time: 07:51)



So, definition as usual suppose I am given a game gamma in normal form a probability distribution over the set of all strategy profiles is called a coarse correlated equilibrium CCE in short.

If there is no unilateral deviation which benefits any player, so if expected utility if for all player $i \in N$; if you look at it is expected utility $u_i(s)$ where s is sampled from σ this should be greater than equal to $u_i(s_i^{'}, s_{-i})$. Where, (s_i, s_{-i}) is sampled from σ this should hold for all $s_i^{'} \in S_i$. So, it is not beneficial for any player i to not follow this strategy this probability distribution σ unilaterally; that means, that assuming all other players are following it and play some other strategy say $s_i^{¿}$ ok.

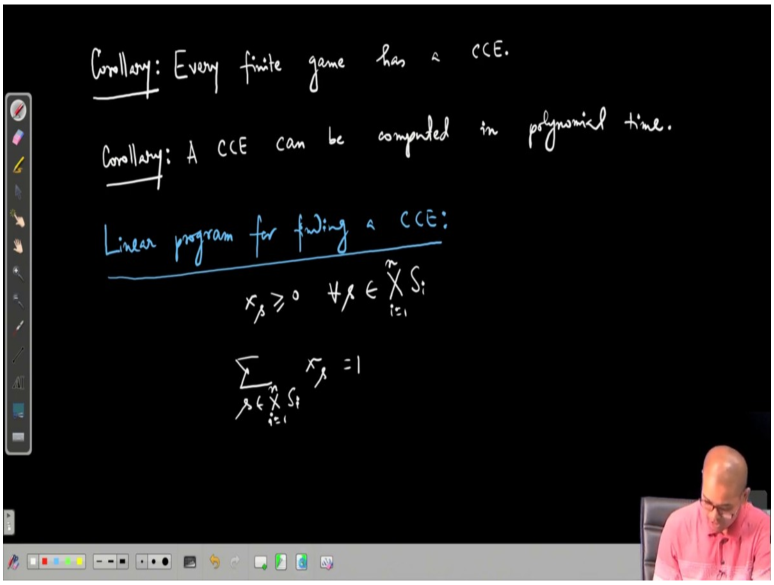
So, as the name suggests it let me put as an observation every correlated equilibrium is also a coarse correlated equilibrium. I will let you proof formally, but intuitively it should be very clear, it is like for a probability distribution to be correlated equilibrium players does not have any incentive to deviate from the to deviate from the probability distribution sigma at the time of signing the contract. On the other hand a correlated equilibrium players does not have any incentive to deviate from following the probability distribution sigma at the time of the contract and also at the time of the playing.

So, it is like correlated equilibrium gives more freedom for the players and a for a probability distribution to be correlated equilibrium, it must safeguard against it must safeguard against unilateral deviation at the sign of at the time of signing the contract and also at the time of playing an action. On the other hand the coarse correlated equilibrium only safeguards again against the, against unilateral deviation at the time of signing the contract.

So, for a coarse correlated equilibrium it can happen that if some player gets to know what exactly the strategy they need to play according to a strategy profile sampled from sigma, they are at that time it may be beneficial for them to deviate. So, that is why to implement coarse correlated equilibrium we need binding contracts, players should not deviate at the time of playing if they have all the freedom till the till they sign the contract sigma. But once they sign the contract they must follow it at the time of playing.

And the notion of coarse correlated equilibrium itself does not safeguard against it and so it must be some external constraint like they must follow it otherwise there will be some penalty or something like that ok. So, every correlated equilibrium is also a coarse correlated equilibrium.

(Refer Slide Time: 13:55)

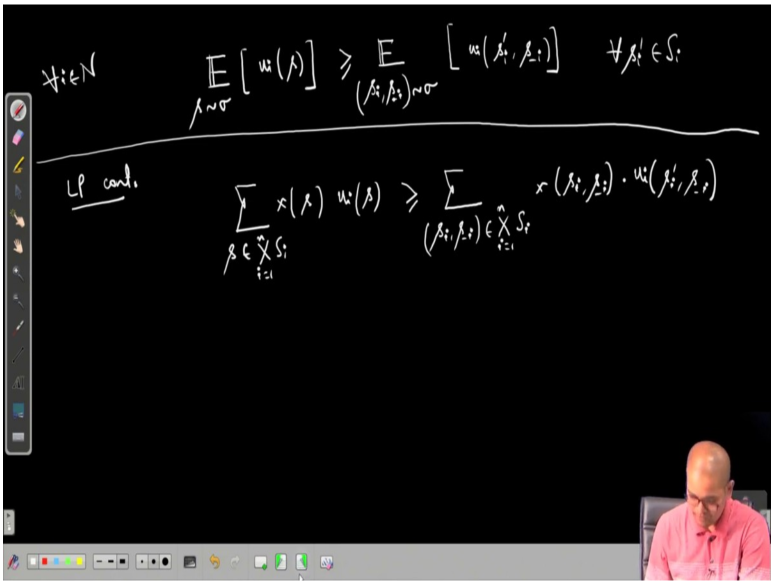


So, there is a corollary from that because every finite game has a correlated equilibrium it follows that every finite game has a coarse correlated equilibrium.

Not only that because correlated equilibrium can be computed in polynomial time, it follows immediately that a coarse correlated equilibrium can also be computed in a polynomial time. But you know we can write a direct LP for solving the coarse correlated equilibrium or finding a coarse correlated equilibrium. So, let us write that the LP will be much simpler. So, linear program for finding a coarse correlated equilibrium.

As usual we have a variable x_s for each strategy profile s which will indicate the probability the probability mass that we are putting on this strategy profile s, this is for all s in the strategy profiles. So, because there are probabilities it should be greater than equal to 0 and this should sum up to 1 and what is the condition that for every player there is no unilateral deviation unilateral beneficial division. So, let us write that condition.

(Refer Slide Time: 16:46)



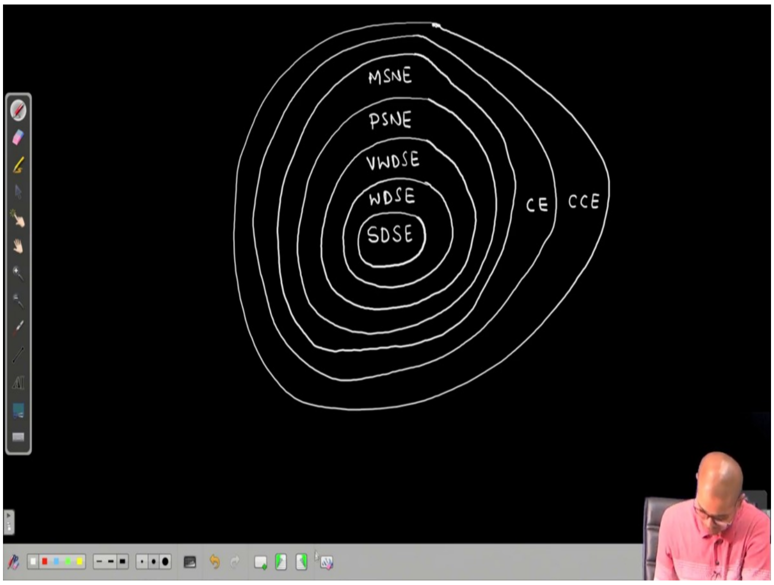
So, let us recall what was the condition expectation expected utility of player i by following the strategy profile sampled from σ, this should be greater than equal to you sample a strategy profile (s_i, s_{-i}) from σ.

If instead of following s_i just play $(s_i^{'}, s_{-i})$ this should hold for all player $i \in N$ and this is for all $s_i^{'} \in S_i$. So, I just unravel this expectations and we will get the linear inequality. So, LP continues so what is expected utility of player i when following σ.

So, sigma so this is s in the set of all strategy profiles, this particular strategy profile is sampled with probability $\sigma(s)$ and in that case the utility of player i is $u_i(s)$. This should be greater than equal to this particular strategy profile is sampled with probability $\sigma(s_i, s_{-i})$.

And then this is but player i is playing $(s_i^{'}, s_{-i})$. Now, these are the exact translations of the inequality above. Now we are doing LP so σ's are not known. So, what I do is that I replace this sigma's with the corresponding variables $x(s)$ ok. So, again we can see that it directly follows from the fact that the linear programs can be solved in polynomial time. So, what are the equilibriums we had? So that is a very big picture so let us again have the big picture.

(Refer Slide Time: 20:08)



We have at the coarse strongly dominant strategy equilibrium strongly dominant strategy equilibrium ,very few game has it and it is most powerful as a predictive tool. If it has if some game has a strongly dominant strategy equilibrium we are convinced that players will follow that that strategy. Of course, assuming that standard game theoretic assumptions. So for example, prisoner's dilemma game has strongly dominant strategy equilibrium.

Next we have Weakly Dominant Strategy Equilibrium, here there are games like say second price auction which has weakly dominant strategy equilibrium, but not strongly dominant strategy equilibrium and this is a weaker notion because every strongly dominant strategy equilibrium is also a weakly dominant strategy equilibrium and there but not the vice versa. So, it we lose predictive power, but we gain towards universality we have more games having this equilibrium concept than the games which has SDSE strongly dominant strategy equilibrium.

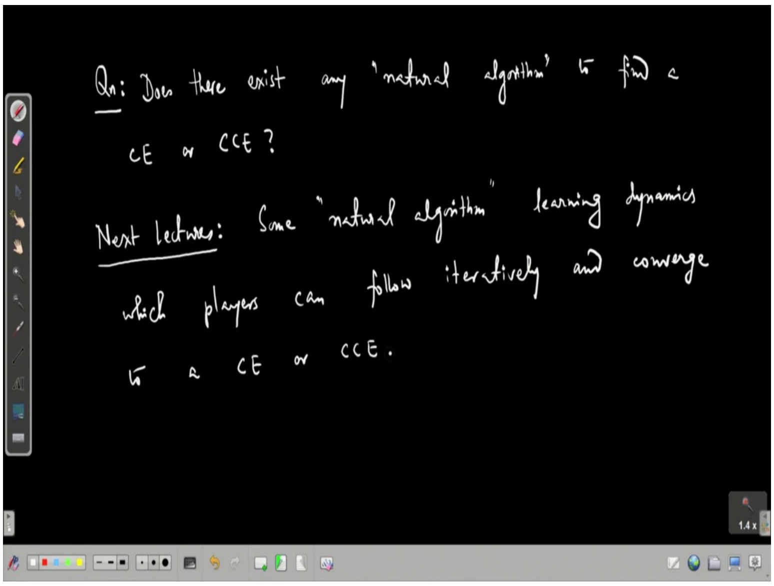
Next we have VWDSE Very Weakly Dominant Strategy Equilibrium, again we go towards universality at the cost of predictability or reliability of this prediction. Then after VWDSE we have PSNE Pure Strategy NASH Equilibrium. Again there are games like say coordination games battle of sixes they have pure strategy NASH equilibrium, but no very weakly dominant strategy equilibrium.

Next we move more towards universality at the cost of reliability and we get MSNE Mixed Strategy NASH Equilibrium and we achieve full universality, because of NASH theorem that every finite game has a mixed strategy NASH equilibrium. Then we ask ok so good that every finite game has a mixed strategy NASH equilibrium. But how about the computational task, how difficult it is? Is it computationally easy because players in the real world although in game theory players have infinite computational power.

So, game theoretically it is not a problem, but you know in real world no player has an infinite computational power. So, whatever model or whatever theory they need to follow they must first compute a equilibrium and that is where we come across the notion of correlated equilibrium. So, these are generalities in different direction at MSNE we already have universality, but with coordinated equilibrium we achieve computational efficiency ironically computational efficiency can also be abbreviated as CE ok.

And then we thought of viewed this correlated equilibrium as non-binding contracts and we then define what is called coarse correlated equilibrium which models binding contracts. So, many equilibrium notions we have, but still we are not satisfied and a real researcher study should not be satisfied and always ask ok.

(Refer Slide Time: 24:54)



So, what is the question that we ask? So, question good that correlated equilibrium and coarse correlated equilibrium can be computed in polynomial time. But you know look at the proofs how are they computed they are computed ok. What is the proof? Because we are able to write down a linear program for correlated equilibrium and coarse correlated equilibrium.

Now, in practical real world scenario where players are often human beings we do not act we do not act or find our best action to perform at any particular situation by solving a linear program, we do not work like that. So, so just because it is polynomiality in solvable whether players will follow it, because at the end what is the goal? The goal of equilibrium concept is to predict how the system will behave. Now we had game theoretic assumptions to make our study going.

But now once the field has matured we will critically relax though we want to critically relax those assumptions and the first relaxation that happened is from MSNE to correlated equilibrium. We said that although we had assumed that players have infinite computational power in real life there is nothing has an infinite computational power.

So, whatever we talk whatever notions whatever theories we develop they must be computationally tractable, they must be efficiently solvable and that is where we got correlated equilibrium and coarse correlated equilibrium.

But now we are not again still satisfied because you know the computational efficiency is fine, but the procedures for discovering correlated equilibrium or coarse correlated equilibrium can they be more natural means people do not solve linear programs to find what they need to do. So, the question is first criticism that or let me put the question does there exist some or not some any natural algorithm to find a correlated equilibrium or coarse correlated equilibrium.

Now, what do we mean by natural algorithm formally then that is a huge topic of discussion, but let us not go there and we just have the intuitive feeling of what is natural it is like in real life we iteratively do something and we learn from mistakes.

And so learning is natural in some sense that we keep doing the same thing every day and we expect to learn from our mistakes. So, that something so this learning phenomena is natural definitely solving a linear program with so many variables and solving it with the computer is not natural.

And so what we will see in next lecture or subsequent lectures is that some natural algorithms just to distinguish from normal algorithms, so we use the term called some learning dynamics which players can follow iteratively. It is like assuming that you know they are playing this game every day and they and they are supposed to learn how they how they will play. Players can follow iteratively and converge to a Correlated Equilibrium or Coarse Correlated Equilibrium. So, we will start this topic in the next class ok.

Thank you.