Algorithmic Game Theory
Prof. Palash Dey
Department of Computer Science and Engineering
Indian Institute of Technology, Kharagpur

Lecture - 26
Approximate MSNE Computation

Welcome. In the last week, we see Complexity Theoretic Machinery for proving hardness for various problems regarding finding MSNE, for a bimatrix game or finding a pure strategy NASH equilibrium for congestion games and so on. So, today, we will see a concrete epsilon Approximate Algorithm for MSNE problem, for epsilon equal to half and bimatrix games.

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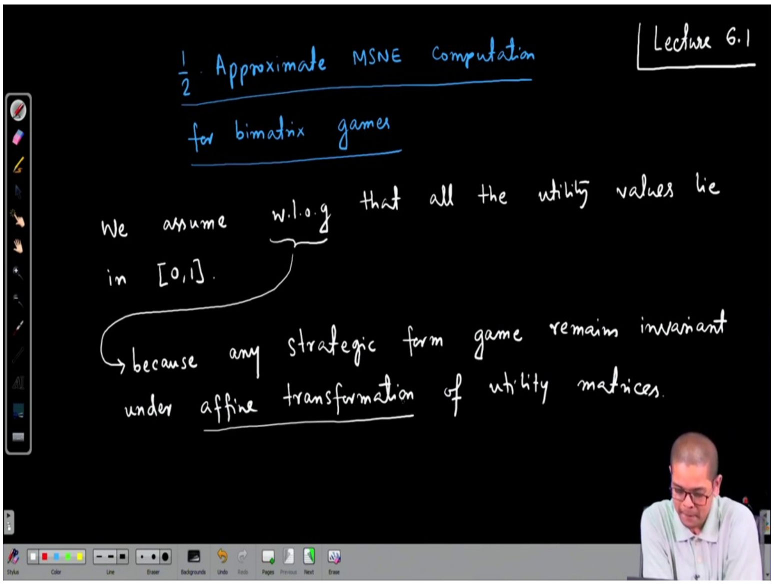

So, today's topic is Half Approximate MSNE Computation for bimatrix games. The very simple yet elegant algorithm for. So, we will assume, we assume without loss of generality that all the utility values lie in $[0,1]$. Why without loss of generality? Because any strategic form game remains invariant under affine transformation of utility matrices. What does this mouthful word affine transformation mean?

It simply means that you can add any constant number to all the utility values of both the players and also you can divide or multiply all the utility values with any positive integer. That is it. That is what this means. And I will let you prove this statement formally. But with this assumption, let me let me go ahead and present the algorithm.

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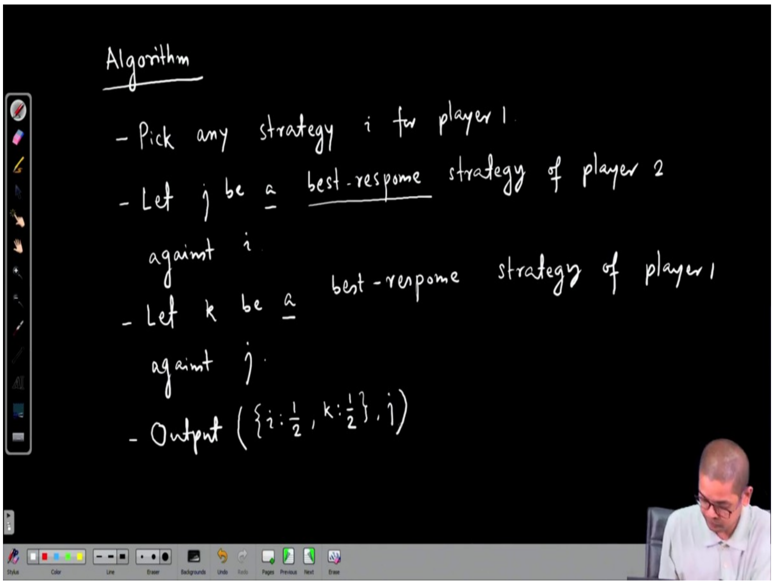

So, algorithm. The first step is pick any strategy i for player 1. Second step, let j be a best-response strategy of player 2 for i or against i. Let k be a best-response strategy of player 1 against j. So, what are we doing? We started with picking any arbitrary strategy for player 1 and asking what is the; what is the best strategy for player 2, if player 1 plays i. Suppose, this is the, that is j, that is what do we mean by best-response strategy. Then we ask, ok, so what is the; what is the best response strategy of player 1 when player 2 plays j.

Why a best response strategy? Because there can exist more than 1 best response strategy. So, and what is the last step? Output, this strategy profile, player i plays i with probability half and k with probability half whereas, player 2 plays j. This strategy profile you output.

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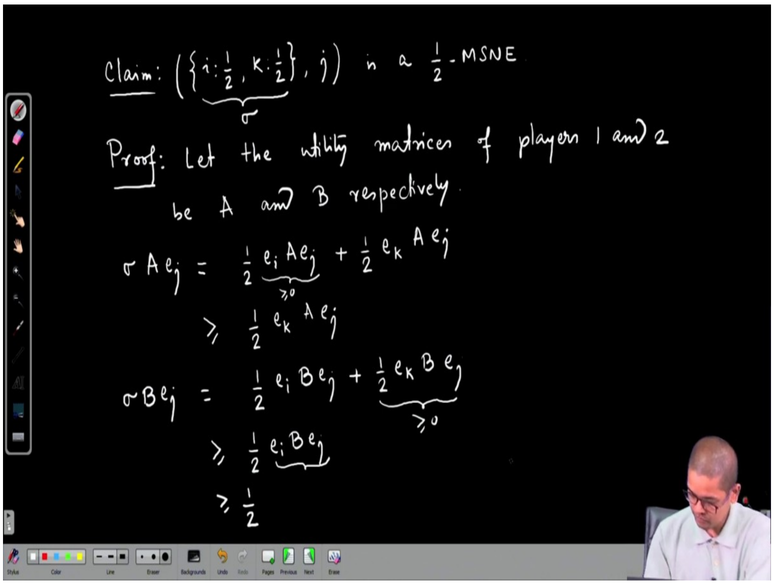

What is the claim? i with probability half, k with probability half, this is what player 1 plays. Player 2 plays j. This is a half MSNE. Proof: so, let the utility matrices of players 1 and 2 be A and B, respectively, ok. What is the utility of player A in this particular strategy profile? So, let us call this particular mix strategy to be σ that is playing i with probability half and playing k with probability half. So, A times σ, this is what is the utility of player A and this is what sorry. So, player A plays σ and player B plays j. So, e_j.

Again, we make standard assumptions that sigma is a row vector and e_j is the column vector. So, that the matrix multiplication makes sense. And e_j is a matrix with one in the jth entry and all other entries are 0. And σ is the vector where ith entry is half, k-th entry is half, all other entries are 0. So, what is this? This is half $e_i A e_j + \frac{1}{2} e_k A e_j$. Now, the first part could be at max 0 or at least 0.

So, this is greater than equal to $\frac{1}{2} e_k A e_j$. And that is what it proves. If you see that player e_k or k is a best response strategy of player 1 against e_j. So, the maximum utility that player 1 can get when player 2 is playing j is by playing k. So, by deviating from σ, it can increase its utility by at most maximum. This is, so this is greater than; this is yeah or this is not required. So, this proves the condition for player 1.

Similarly, for player 2, what is its utility? Player 1 is playing σ, player 2s utility function is B and it is e_j. So, what is it? $\frac{1}{2}e_i B e_j + \frac{1}{2}e_k B e_j$. Again, B; these are utility values are, these are in between 0 and 1, so this part is greater than equal to 0, same was the case here.

And so, we have, this is greater than equal to $\frac{1}{2}e_i$, this greater than equal to $\frac{1}{2}e_i B e_j$. But what is this value? What is $e_i B e_j$? This is the (i, j)-th entry of matrix B and its value can be at max 1. This is greater than equal to half. Now, by deviating, it can get a utility of at most 1, which is within half fraction of what. So, this is at least greater than equal to half. So, its current utility is within half of the maximum utility that it can gain.

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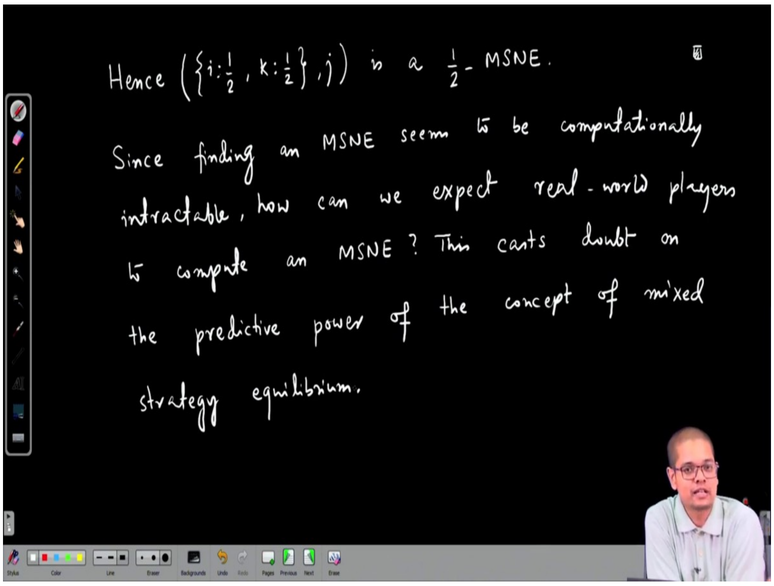

So, this proves that; hence, this strategy profile is a half MSNE, which concludes the proof. Very good. But you know the fundamental result of this thing of our last class is that complexity theoretics study of PSNE for congestion games, for special kind of succinct games and the fundamental question of MSNE, computing MSNE being hard raises doubt in the predictive power of MSNE.

So, since finding an MSNE seems to be, why seems to be? Because it is not a unconditional proof. It just shows that if MSNE computation is polynomial time solvable, then all problems in PPAD will be polynomial times solvable, that is it which is bit unlikely to the research community. It is not as unlikely as P equal to NP, but still it is considered unlikely.

So, that way we can at most, at least, at most try it seems. Since, finding an MSNE seems to be computationally intractable, how can we expect real world players. Real-world players, recall, we had a game theoretic assumption like players are intelligent they can perform any computational task. They have infinite computational power at their disposal.

But strictly speaking, in real world it is not it is not a valid assumption. We are, we have limited computational power and if some problem it is like computers cannot solve, how can we expect to solve, we means real world players; expect real world players to compute an MSNE?

This casts doubt on the predictive power of, on the predictive power of the concept of mixed strategy NASH equilibrium. You see why we are studying equilibrium? Recall, what was the fundamental questions of game theory. Once, we have a game, what are the questions? One of the fundamental question is that is that from system designer or from some player; even for players perspective how will players play, which strategy to play.

And then we started studying of strongly dominant strategy equilibrium, weakly dominant strategy equilibrium and so on and so forth, and we observe that those concepts are not universal. All games does not have that. And we at the end we got the concept of mixed strategy NASH equilibrium which are substantially weaker than strongly dominant strategy equilibrium or weakly dominant strategy equilibrium and so on.

And NASH theorem is a breakthrough in that sense that it shows that all finite strategic form games has a mixed strategy NASH equilibrium. It sort of allows us to predict how players can play and what will be the outcome. But now computer scientist study this problem as a computational problem and discovered that this is a very hard computational problem. And now this sort of shakes the ground of mixed strategy NASH equilibrium. Means, if players cannot compute it, how will they find it later on playing it. So, this motivates; so let us recall what are the equilibrium concepts we had.

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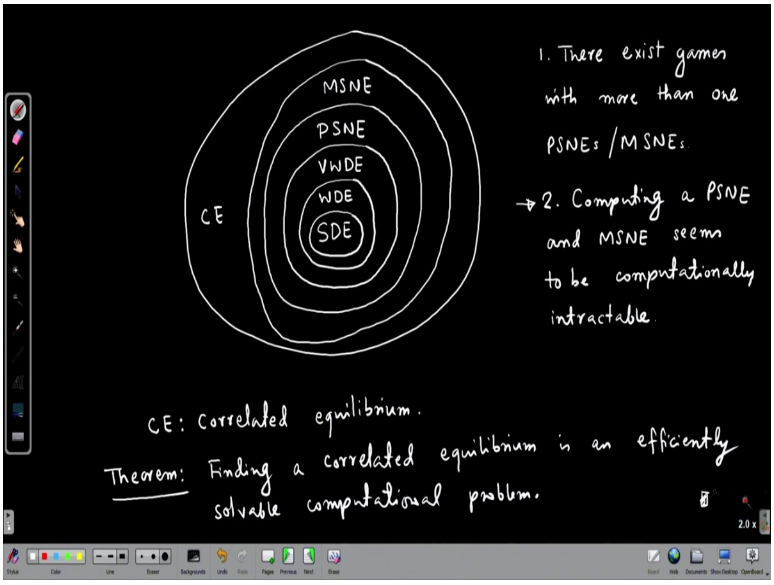

So, let us draw the diagram. We had at the core strongly dominant strategy equilibrium which is which has the highest predictive power, but the problem is universality, most games does not have it. So, we have seen only prisoner's dilemma game which has strongly dominant strategy equilibrium, among the examples that we have seen.

Then, we weakened that, whenever we weaken equilibrium notion its predictive power gets reduced, so weakly dominant strategy equilibrium. And we observe that some games, for example, first price auction, which does not have a strongly sorry; second price auction, which does not have a strongly dominant strategy equilibrium. It has a weakly dominant strategy equilibrium.

Then, we continued, and weakened weakly dominant strategy equilibrium as very weakly dominant strategy equilibrium. But still the problem remains that there are many interesting games. For example, zero-sum games. Most zero-sum games like matching pennies, rock paper scissors, battle of sexes, or say coordination games, or say tragedy of commons. Those games does not have a very weakly dominant strategy equilibrium also.

So, towards that we relaxed our equilibrium notion. Whenever we relax we weaken it, we lose some predictive power, and we get pure strategy NASH equilibrium. Again, most zero-sum games like rock pepper scissor, matching pennies, does not have a does not even have a pure strategy NASH equilibrium.

Then, we move on to mixed strategy NASH equilibrium which is substantially weak than what we had started namely strongly dominant strategy equilibrium. But its appeal, its unique selling point was due to NASH theorem that every finite strategic form game has a mixed strategy NASH equilibrium.

But there are many other criticisms also. The first criticism is that you know this is the criticism for mixed strategy NASH equilibrium, pure strategy NASH equilibrium also, that there exist, there exist games with more than one PSNEs or MSNEs also. I would request you to pause this video and think why this is a criticism? What is wrong if there exist more than one PSNEs or MSNEs? Ok.

So, let me answer. So, you see why we are studying? Always go back to the fundamental question, the motivation, why we are studying equilibrium, we are studying because we want to predict how the players will play. And it also helps the player to reason and help them play, play according to an equilibrium.

Now, if there exist multiple equilibriums, then which one player will follow or how will we predict? Because each player, this is the non-cooperative game and each players play a strategy simultaneously and independently of everyone else. So, ideally, it would it would it would have been excellent if every game has just one equilibrium, unique equilibrium and every game has it, but we do not have that.

Recall, this was not a problem for strongly dominant strategy equilibrium and weakly dominant strategy equilibrium. Any game which has a strongly dominant strategy equilibrium or a weakly dominant strategy equilibrium, it is always unique. So, if it has, that is why it the prediction is much more reliable.

The second objection that we have, that is the contribution of computer science in game theory is which is a very fundamental contribution very fundamental important contribution is that; computing a PSNE and MSNE seems to be, seems because the results are not unconditional results, seems to be computationally intractable.

So, if it is a hard computational problem, how we how can we expect the real world players will be able to play it? Although, there is a game theoretic assumption, but that is the assumption. At the end, this theory will be applied to real world situations. So, what we do is that we weaken the notion of mixed strategy NASH equilibrium further to cater this second issue of computational intractability. And what we get is what is called correlated equilibrium, CE. What is CE? Correlated Equilibrium.

Why we study this? Because, so here is the theorem. So, we will study this in next class. But let me highlight. Finding a correlated equilibrium, finding is an efficiently solvable computational problem.

So, we will stop here. In the next class, we will define what is correlated equilibrium and we prove this theorem, ok.

Thank you.