Algorithmic Game Theory
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Lecture - 02
Assumptions of Game Theory

Ok. In the last class we informally defined what is Game Theory with a few examples and then we defined formally what is called a strategic form game or normal form game. And then we finished last class with two important questions of game theory which will drive our study of game theory. So, the first question is from the player's point of view what each player will play and the second question is from overall system point of view that what will be the outcome of the system.

Now, you see that in some sense these questions do not make full sense because you know players are independent, players have all the independence all the choices in the world. They can play any action what do you mean by can you predict which action that the player will play. So, that does not make sense.

So, to put some sense on the question what we will do is that, we will make some assumptions on the behavior of the player. Why players are playing? How they are playing they are playing?

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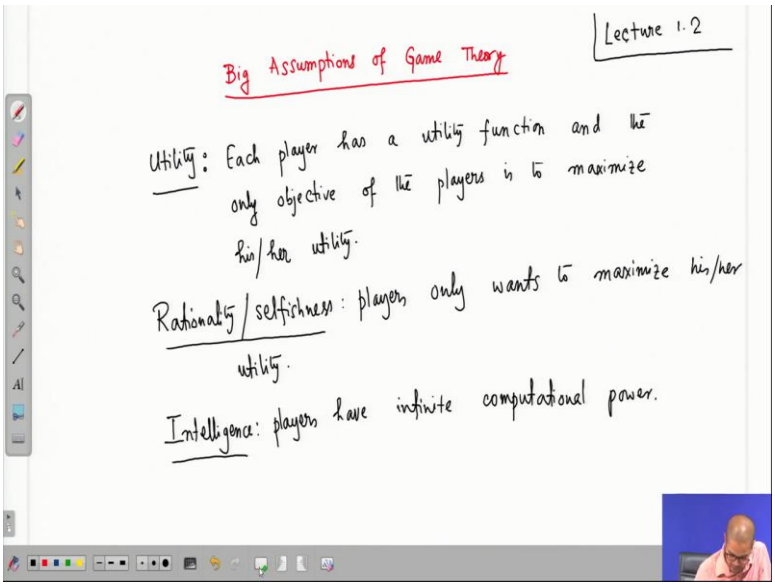

So, we will make some assumptions and these are very important fundamental and sometimes unrealistic assumption you will see.

The first big assumption is utility. So, what is utility? We assume that it its part of the definition of normal form game. So, assume that each player has a utility function and the only objective of the players is to maximize his or her utility. So, if two options among two options if option one gives more utility than option two, player should not hesitate there is no other consideration for players to take into account players will only try to maximize the utility.

So, the assumption of utility is that each player indeed has a utility function and the goal that the assumption that players play to maximize their utility is called rationality. Also it also goes by some other names or rationality or selfishness is that players only want to maximize their utility only wants to maximize his or her utility ok.

Then a third, very important assumption and sometimes unrealistic, we will assume that players have infinite computational power. As you see, infinite if means it does not matter how much computational power players have it cannot be infinite. So, but that is the assumption, that players have infinite computational power.

That means, if player has need needs to do some computation to find out which action which strategy to play from its strategy set, it can it has the available resource to make the computation. That is the assumption of intelligence.

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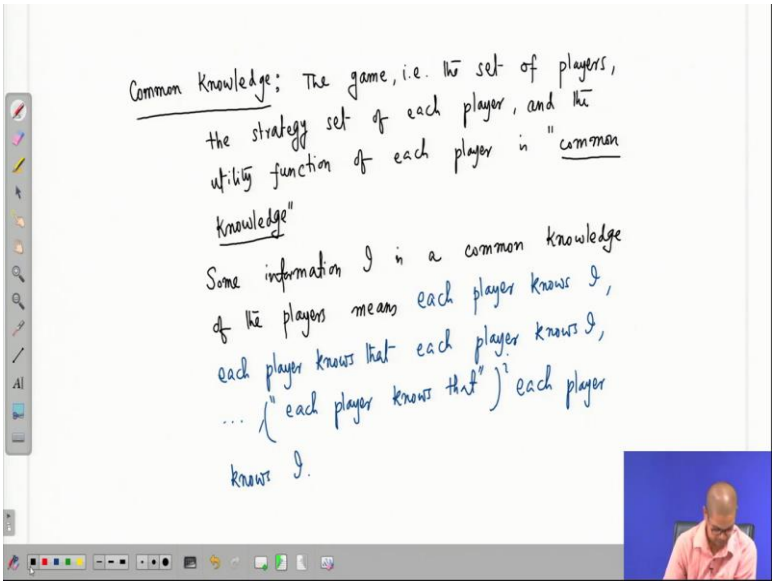

The fourth assumption is what is called common knowledge. So, what is the assumption? The assumption is that the game. What do you mean by game? That is everything in the game for example; that is the set of players, the strategy set of each players and the utility function of each player is common knowledge.

Now what do you mean by common knowledge? So, let me define this term, but it is something more than it is like everyone knows. So, let me define common knowledge, some information say I is a common knowledge of the players means each player knows this that this information is known to all the players each player knows I.

But not only that each player knows that each player knows I. But not like that it is goes on like each player knows that each player knows that each player knows I and so on. That means inductively each player knows that and these repeats say I times. Each player knows that, each player knows that, each player knows that so on knows I knows that each player knows I.

And this is very important without this the game theory will halt and this is also most controversial. So, let us see let us understand this with an example, so with a puzzle say.

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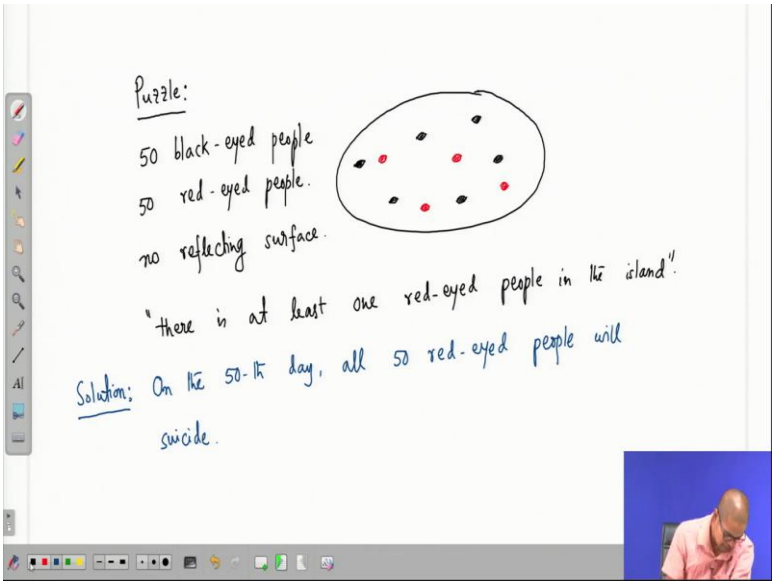

So, what is the puzzle? So, suppose there are there is a island and there are two kinds of people in the island one is black eyed people, one type of people have black eyes and there is other kind of people who are red eyes ok. So, suppose there are 50 black eyed people and 50 red eyed people.

And these red eyed peoples are in some sense cursed it is like if any person knows that, so suppose I am a person in that island and my red colored is my eye colored is red. But if I come to know that my eye colored is red, I will do a suicide. At say 12'o clock in a common place, so that everyone knows.

And there is suppose there is no reflecting surface, no reflecting surface in the island and people are not allowed to talk about the eye colors. No reflecting surface no mirror nothing, no reflecting surface people does not talk about eye color. So, I as a person in that island have no way to know whether my eye color is red or black, but I can see the eye color of all the other people.

Now, one day a visitor comes and visitor does not know that they that your the visitor is not supposed to talk about the eye color and unknowingly the visitor makes a comment this. That, 'there is at least one red eyed people in the island', this is the comment that visitor makes and then visitor leaves, what will happen?

So, people because, so before this comment no one has said no one has any way to know his or her own eye color, they are all living peacefully and no one is suiciding what will happen. Let me give you the answer and you if you listen why this will happen then you have understood common knowledge.

The answer is on the 15th day on the 50th day all 50 red eyed people will suicide, this will happen this is solution. Let me not explain in the solution and I let you think why this will happen and if you are able to successfully get convinced about this answer, then you know what is common knowledge and what why it is different from that everyone knows.

So, some information is known to everyone is not common knowledge. So, for that to be common knowledge everyone should know that everyone knows, everyone should know that everyone should knows that everyone should know, everyone knows that information and so on this should go to till infinity that is common knowledge ok.

So, equipped with these assumptions now players do not behave arbitrarily they behave with a goal. They want to maximize their utility and that is their only goal and we know that players have infinite computational power to compute their best sort of strategy or action.

So, so with this thing we can now proceed to answering these questions, but before that let us see or let us get used to with this game with this normal form game with some more examples. So, let us see couple of more examples or next example we have already seen three examples, namely the grading games the prisoners dilemma and the congestion game.

Let us continue with more examples, so learning with examples is always much easier.

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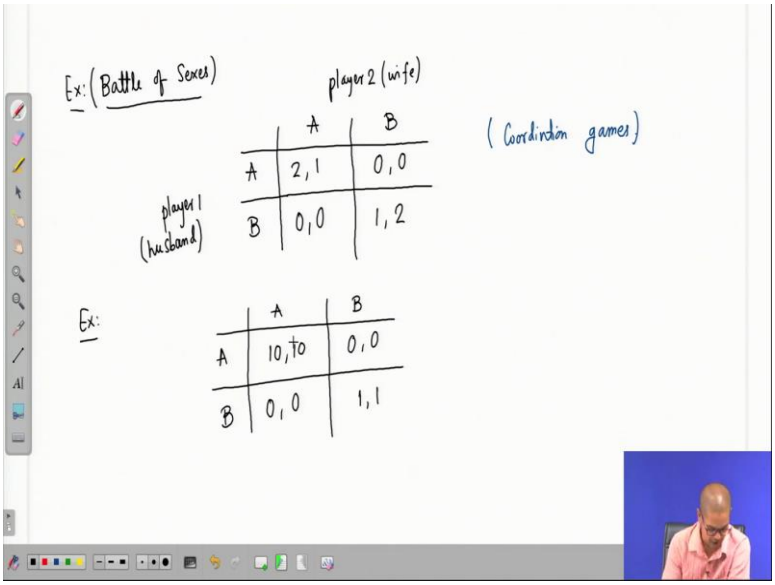

Our next example is battle of sexes. So, it is a game between two players say husband and wife typically row player is called player 1 and column player is called player 2.

Suppose this player 1 is husband and player 2 is wife both the players have two options. Suppose they will they will go to outing today afternoon, today evening and they have two options either they have to they can go to watch a football match that is denoted by A or they can go for shopping.

So, in the morning hustles and bustles they forgot to decide where to go. Either they will go to watch a football match or they will go for shopping and they forgot the mobile also. So, in somebody they do not have any way to communicate and they have to decide independently where to go, to watch a football match or to go for shopping.

Now and so both of them has the option of either go to the football stadium or go for shopping and if it turns out that if they are not together they are not happy they are very unhappy. So, if one player plays goes to the football stadium say and the other player goes to the shopping or vice versa both are unhappy and both derives the utility of 0.

On the other hand, suppose both player goes to football stadium then both are happy, but suppose the husband enjoys the football match more. So, in that case player 1 receives the utility of 2 and player 2 also receives the utility of the he is she is together with his with her husband, but less than the husband 1 2 comma 1.

The symmetrically opposite situation happens if both of them goes for shopping place action B. So, this is called battle of sexes and this comes under another broader class of games which are called coordination games. So, here is let us say another kind of coordination games this is also another kind of coordination game.

Suppose there are two companies and they produce complimentary products A B, A B. As usual, the row player is player 1 and column player is player 2. Now again suppose they need to independently decide which product they will build. If they fail to coordinate; that means, if one player plays A other player plays B or vice versa both of them does not get any utility.

On the other hand, suppose they succeed to coordinate suppose both of them plays B then both of them are happy, but and if both of them plays A both of them also happy but this utility can be different. So, suppose the utility in strategy profile B comma B is 1 comma 1 both the players derive a utility of 1 and the utility for A comma A suppose this is 10 comma 10 ok.

So, this sort of this sort of games arise in many situations. For example, students think of you and your friend you have two options each of you have two options, either come to the class or bunk the class and go to go and play tennis, but you have to decide independently.

So, if you end up if you fail to coordinate then you are unhappy you can neither play tennis or neither enjoy class on the other hand if you both come to the class you are happy you are together, but the lecture is so boring. So, you derive a utility of 1 comma 1, on the other hand if both of you go to play football then you are super happy and say table tennis, then you are super happy and derive a utility of 10 comma 10.

So, these are this all these two both battle of sexes and this game comes under the class of games known as coordination game, players need to coordinate to get utility.

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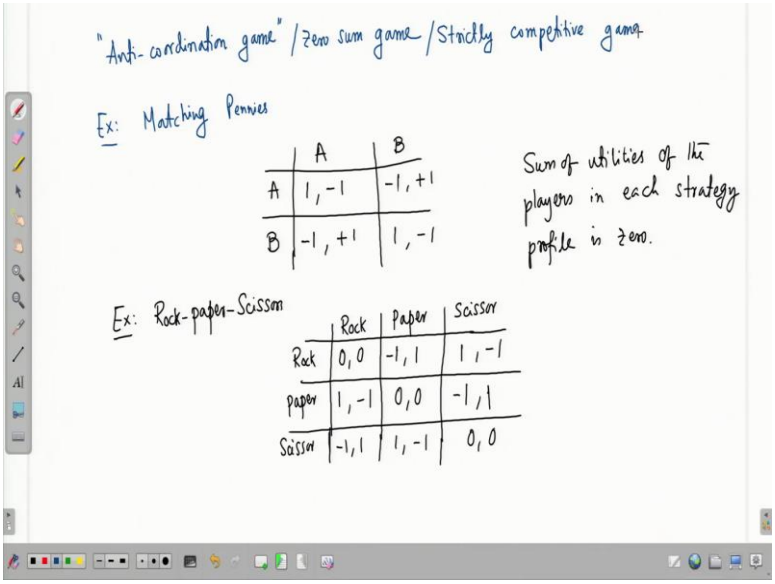

Our next class of examples are what is called anti coordination games, they should not coordinate anti coordination game. So, in this class of games let us see these are also sometimes called zero sum game.

Although zero sum game is a strict subset a special class of anti coordination game, zero sum game ok. Let us see couple of examples. Our first example is matching pennies simple, but very powerful game. So, what is the utility functions? Each of the players have a penny and they will play either head or tail. So, let me denote as A and B following our previous notation think of A as head B as tail.

If both if say both players play A then player 1 gets a utility of 1. So, if their action or strategy matches then player 1 benefits player 1 is trying to get a match and if there is a mismatch player 1 losses. So, player 1 wins this also called win loss game, player 1 wins if there is a there is a match the pennies match; that means, both players play A and losses if pennies does not match.

On the other hand, player 2 gets exactly opposite; that means, player 2 loses if pennies match and wins if pennies does not match ok, so this is matching pennies game. Another famous very important game is what is called rock-paper-scissor, rock-paper-scissor. So, here we have again two players, but each player has three options rock-paper-scissor, rock-paper-scissor ok.

If both players play same strategy then it is like a draw; that means, no player wins or no player loses, you know rock paper scissor game many of them has have played. Between rock and paper, rock loses and paper wins. But between rock and scissor, rock wins and scissor loses and between paper and scissor, paper loses scissor wins.

Symmetric thing happens here, so this is 1 comma minus 1 you can just reflect it minus 1 1 and 1 comma minus 1. So, this is the rock-paper-scissor game and these are also called zero sum game because you know sum of utilities of the players in each strategy profile is zero. That is why the name zero sum game this is also called strictly competitive games.

So, these are the names so or strictly competitive games. So, we will thoroughly investigate this sort of games, strictly competitive game competitive games. The win of one player means the loss of another player ok. So, let us stop today stop here today, so we will continue in the next class.