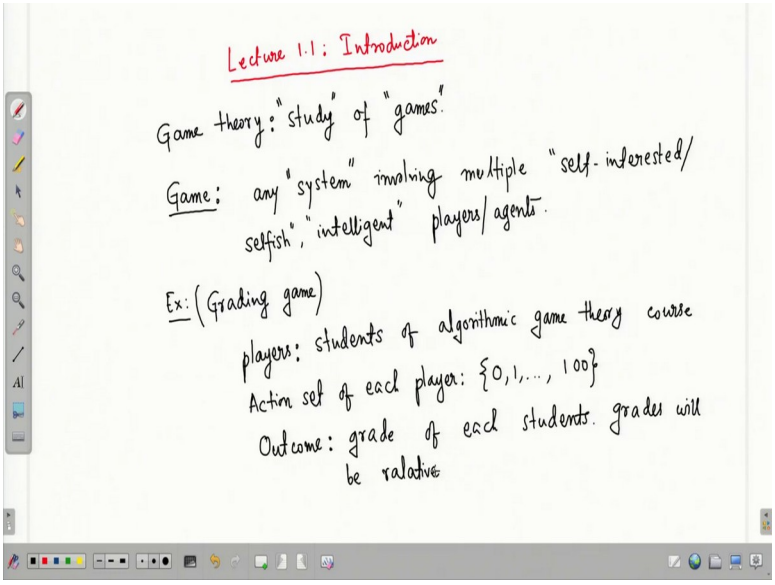
Algorithmic Game Theory
Prof. Palash Dey
Department of Computer Science and Engineering
Indian Institute of Technology, Kharagpur

Lecture - 01
Introduction

So, today is the first lecture of our course. We will introduce the various concepts of Game Theory. So, what is game theory?

(Refer Slide Time: 00:32)



This is study of games. Now, two things require explanation; what do we mean by study and what do we mean by game. So, let us first answer this; what do you mean by game. So, what is game? This is intuitively speaking any system involving multiple self interested self interested also called selfish self interested intelligent players also called agents. These terms are used interchangeably. So, before defining it formally let us do a example.

So, example of a game; the most important example from the students is what is called what I call a grading game. So, what is a grading game? So, who are the players? The players are students. Now, why we do not say game is a set of intelligent agents? It is not only a set of intelligent agents, it is a system. Now, what do you mean by system?

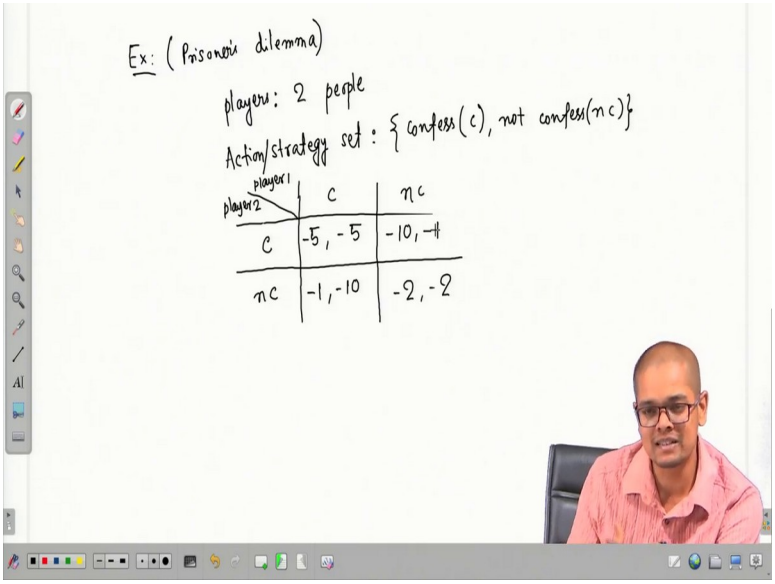
That means, there are some more things some more components involved which makes it a system. So, what is it? So, each player has a action set. So, action set of each player. Suppose the players are students of algorithmic game theory course. And the action set also what are the possible actions.

The action sets is how much marks that they get in the exam out of 100. So, action set is this numbers 0, 1 up to 100. How much they can score what could be their aggregate score. In this that is action set and what is the outcome. So, this is what we mean by system; that means, each player can play one of the possible actions and based on the action played by each player there is a outcome; outcome is grade of each students.

See students are not interested at the end of the day not in what marks they get, it is what grade they gets. And how the grades will be decided? So, grades will be relative; that means what? That means, each say the top 10 percent get the highest grade, the next 20 percent may get the second highest grade and so on.

So, you see that the outcome of the system depends on not only it depends on the entire action profile of the players, it depends on all the actions played by all the players, it not only depends on the action of each player its. So, it depends on all the players and that is that that sort of ah justifies my use of the word system this way these intelligent players are connected the outcome affects all of them and the outcome is a function of action of all the players ok. So, this is one example.

(Refer Slide Time: 06:50)



So, let us continue with another example. This is a famous example of prisoners dilemma. So, what is the setting? Suppose a crime has happened and two people have committed the crime together. So, what are the players? Players are 2 person 2 people who have committed the crime together; one crime committed jointly by together for maybe say thief theft or something like that stealing.

Then after the crime has happened police have get hold of them, but police does not have enough evidence to put together a strong case against them. So, what police does is police interrogates them separately and asks each of them to confess. So, each player has two options. So, action set; these also called strategy set action or strategy set.

Action set or strategy set of each player it is they have two options either confess that they have committed the crime let us denote this action by c and are not confess nc. So, both the players have two options. And what is the outcome? So, here is the rule. So, suppose this way this side is player 1 this column and row belongs to player 2. Now column player 1 has two options either confess or not confess. Player 2 also have two options either confess or not confess.

So, if both players confess then the police has a strong case and in that case the judge has no doubt that the commit the crime is committed by both the players and in that case both of them gets a 5 years of jail term. On the other hand if both of them does not confess then the police does not have a strong case and the judge is not 100 percent convinced that they committed they had that they have committed this crime and in that case both of them gets small jail term, the judge is not fully convinced.

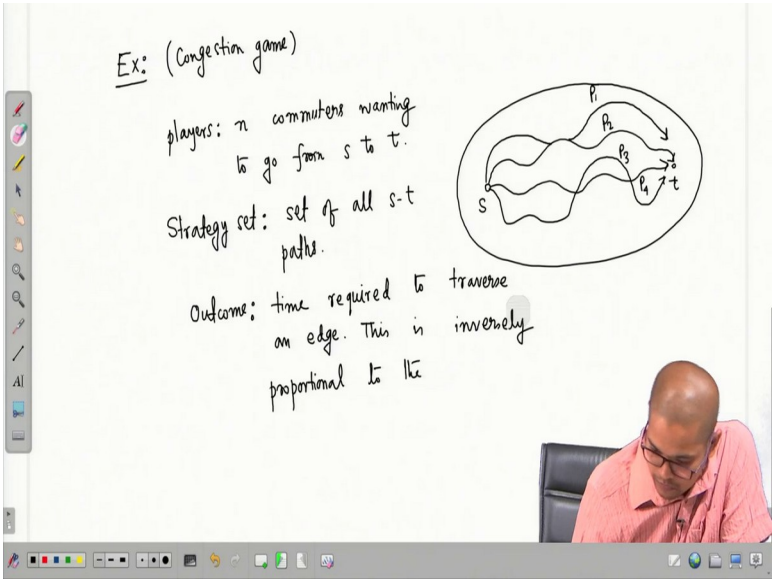
But if say column player confesses and the row player does not confesses then what happens? Then column player gets a benefit of cooperating with the police and gets a jail term of minus 1. The first number denotes the jail term of the first player and the number immediately after comma denotes the jail term of the second player. Now, what happens to the second player? Second player has now is has first of all committed the crime and now he is not cooperating with the police also. So, that is why the second player gets a jail term of 10 years.

Symmetric situation happens when player 1 does not confess and player 1, player 2 confess this. Here also what is the system outcome? The system outcome is the jail terms awarded to each of the players and you see that the system the outcome depends on the

action profile the tuple of actions played by all the players. It does not depend as solely on the action of one player, it depends on the action played by both the players. This is the salient feature of game theory ok.

So, this is another cornerstone example of game theory prisoner's dilemma. And so, here also you see that you know the players are selfish, intelligent, rational they wants to minimize their jail term and what should they do, that is what we mean by studying this game and that is what game theory is all about.

(Refer Slide Time: 12:35)



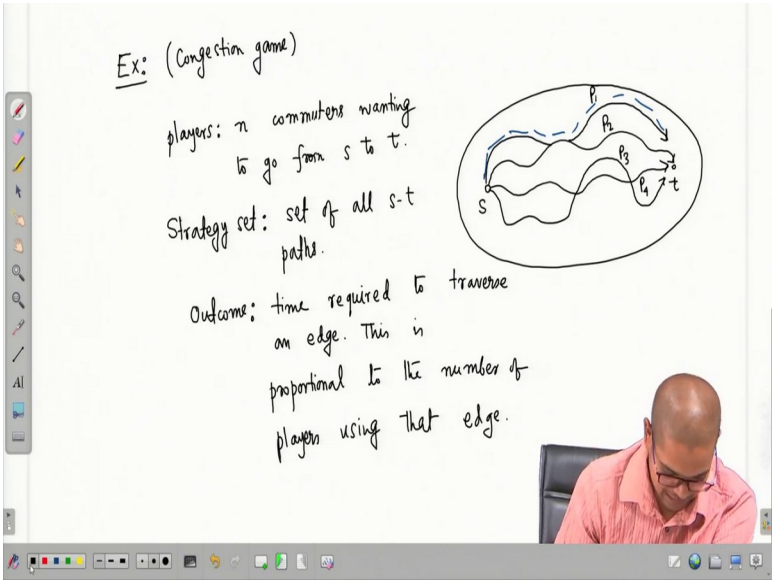
So, let us see another example before formally defining what we what we mean by game, how we formally denote it and so on. Another example is congestion game. So, think of a network say a road network and there is a source node and there is a destination nodes and there are various available paths from source to destination anyone who wants to and these paths can intersect also these paths can overlap also. The point is that there are various possible paths and ok.

So, let us see. So, what are the what the players; say n commuters. Suppose there are n number of commuters wanting to go from s to t ok. Now what do they want? They want to reach the goal for the reach t from s as early as possible. So, what is their action set or strategy set? The strategy set is the set of all s to t paths. Each player is free to take any path of his or her choice from s to t.

Now, what is the outcome? So, system outcome: Define outcome. We need to define how much time it takes to reach from s to t in one path. Suppose there are these are four paths; P_1 $,P_2,P_3,P_4$. Now to define how much time it takes to take each path we will define what is called time required to traverse an edge.

So, path is nothing but a sequence of edges and if we define what is the time required to traverse an edge, the time required to traverse the path will be simply the sum of all the time required to traverse each of the edges. So, what is the time? This time required and this is and this is this time this is inversely proportional to the or not inversely proportional let me use the term proportional.

(Refer Slide Time: 17:02)



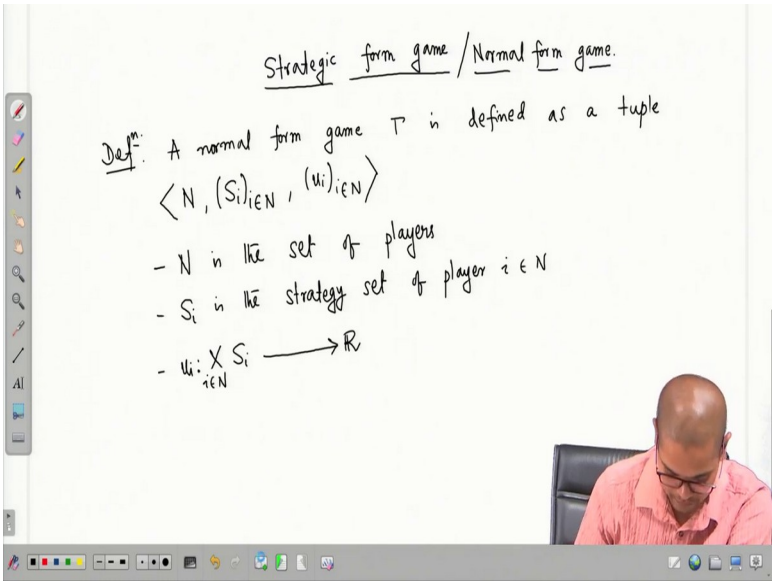
This time required this time is proportional to the number of players using that edge ok So, what do you mean by that? That means, take any path say suppose P_1 and it is a collection of edges and for each edge you see how many of other players are also using this edge or how many in total number of players are using this edge.

The idea is that the more the number of players use an edge, the edge becomes so much congested the traffic increases and the time required to traverse that edge also increases. So, here also you see that the system outcome is a function of the action profile the strategy profile of all the players.

So, in that sense each player is independent they are free to choose the path that they want to take, but the outcome of the system is dependent or is a function of the or if of all the actions taken by individual players that way the all the players are connected it is one system.

So, I hope you have got some idea about what is game, what are its components namely players. Each player has a strategy set and there is a system outcome and each outcome from each outcomes each player derives certain utilities ok. So, with that thing let me now define formally what is game or how do we formally define an abstract game.

(Refer Slide Time: 19:14)

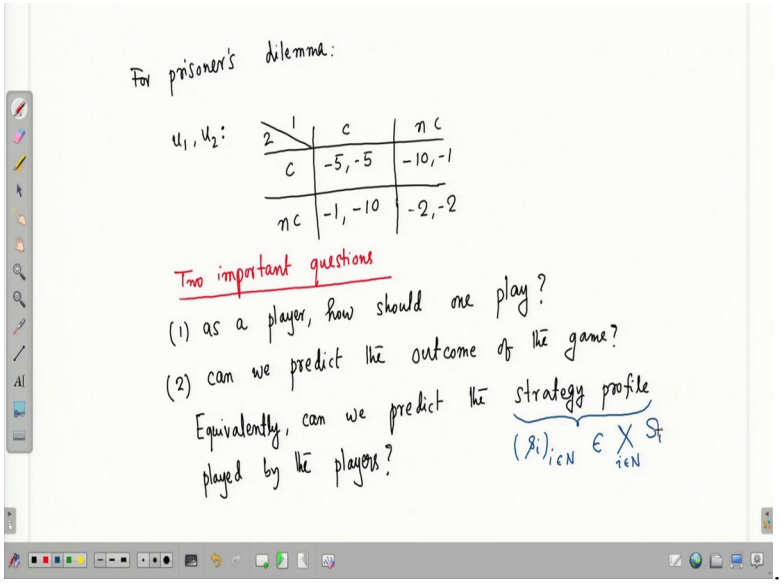


So, that is called strategic form game; strategic form game. This also goes by the name of normal form game. So, definition: So, a normal form game gamma is defined as a tuple. What are its components? N, the set of players, strategy set of all the players. In the in our example still now the strategy set is same for all the players. It can; it need not be same, the strategy set of each player could be different and utility function of the players of each of the players.

Utility function is the what is the utility derived from the outcome from each of the outcome for each player. So, let us write it formally. So, N is the set of players S_i is the strategy set of player $i \in N$ and utility function is a function from output to real number from each output what is the utility derived by player i from that output.

Now in output, the output is a function of the strategy profile or the tuple of strategies that is called strategy profile. So, in turn utility of player i is a function from Cartesian product of i equal to $i \in N S_i$ to real numbers ok. So, as an example we have already seen the prisoner's dilemma, let us recall.

(Refer Slide Time: 22:50)



So, for prisoner's dilemma the matrix that you have written are basically the utility functions. So, u_1 and u_2 these are the utility functions. So, again so, let us write. Here was player 1, here is player 2. Each of them has options of confess or not, confess not confess; if both of them confesses then both of them get derives a utility of minus 5.

If both of them does not confess both of them gets a derive of minus 2. If player 1 confess, it gets a utility of minus 1 and in that case if player 2 does not confess, player 2 gets a utility of minus 10. The symmetric case happens when player 1 does not confess and player 1 player 2 confesses ok.

So, we have a game and we have we now know how to formally define it. Now what? So, what are the important questions? So, there are two important questions of game theory and we will answer these questions throughout the course. So, till now we have modeled a situation in the form of a game and. Now, we will ask two important questions and the answer will be what is game theory. So, two important questions: The first question is from player's perspective, as a player how should one play?

That means, you see that this situation is bit complicated from the players perspective also because the outcome and hence the utility derived does not depend solely on the action of one player, it depends on the action of all the players. So, in such a complicated scenario what action is best for me to play?

So, that is one important question. The second important question is - can we predict system outcome the outcome of the game? Equivalently can we predict the strategy profile played by the players? So, what is strategy profile? It is a tuple of strategies. So, let me write this way: $(s_i)_{i \in N}$, this is a element of this set S_i . So, we will continue from here and we will see that how to approach this question. So, we will continue in the next class ok.

Thank you.