Computer Vision
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Lecture – 07
Projective Transformation

In this lecture, we will discuss about Transformation in Projective Space.

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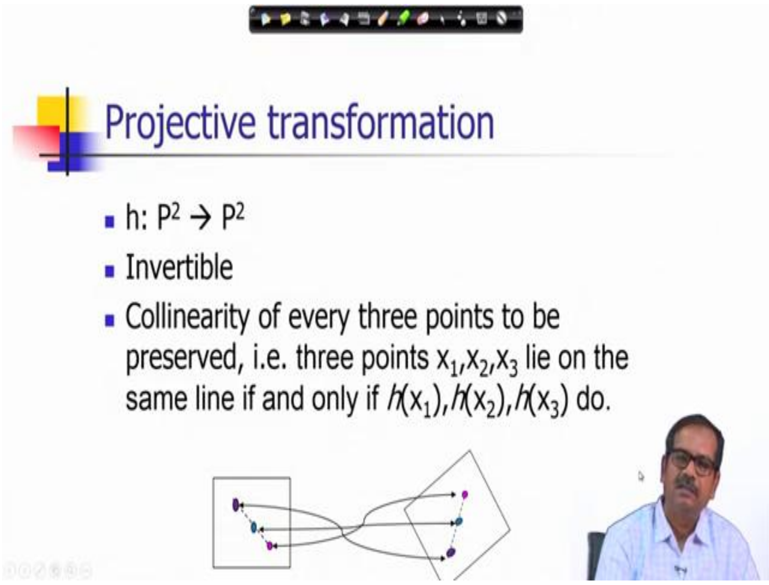

So, let us define what is meant by a Projective Transformation. Firstly, this property says that a projective transformation is the transformation of a point in a projective space to a point in another projective space. This transformation should be invertible and it should preserve the collinearity of every 3 point. For example, you have three points say x_1, x_2, x_3 which lie on the same line, then if you transform those points they should also lie on the same line; on a line. And that property has to be satisfied for any pair of three points, and then any triple of three points, and then this transformation is called projective transformation.

There is a figurative explanation that we can provide here. You consider this particular configuration. Here we have shown there are three points, they are lying on a particular line and they are shown by different colors. You consider a transformation, again their color shows the corresponding transformation in another space and they are mapping, they are shown by these arrows, they are one to one mapping, they are invertible. And also since those lines were collinear they should be also collinear in the transform space. When these properties are satisfied for every configuration of points then this is a projective transformation.

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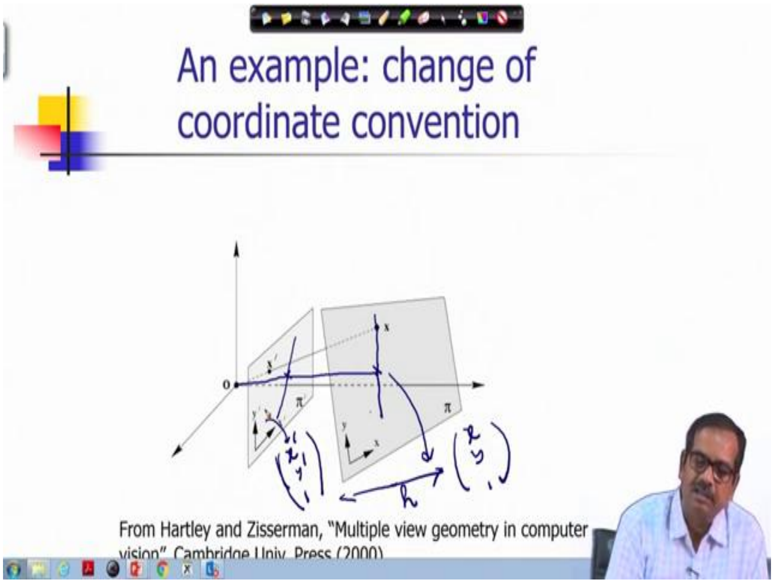

There are various examples of this transformation. For example, change of coordinate convention. You consider this particular case where a point explained in a plane which is shown here as π' plane and that is mapped with a point x in another plane. So, these two points, every point, for every point in this plane there is a corresponding point in this π' plane and they are related by this geometric rule. If I draw a ray connecting that point to the center of the coordinate which is a center of projections in this case then you will find, I mean you will get the corresponding transformed point.

So, you can see that with this particular configuration every point in this space is mapped to another point in this space which is also a two dimensional projective space. There could be coordinate convention. So, coordinate axis could be in different orientations. They need not be parallel to the same coordinate axis of the implicit three-dimensional space, representing this particular phenomena of projection. They could have different coordinate definitions in their particular plane. And so, if I use a representation of this point as say $\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix}$ again a canonical representation and these representation as $\begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$, so we can see that they are related we can observe that they are related by a transformation this is called a projective transformation. We will see that what is the form of this transformation subsequently.

So, this is one example because if I consider any line, so this line will be also projected as a line which means all the points lying on that line after transformation they are also lying on a straight line. So, they satisfy all the three conditions of the projective transformation that is why it is a case of projective transformation.

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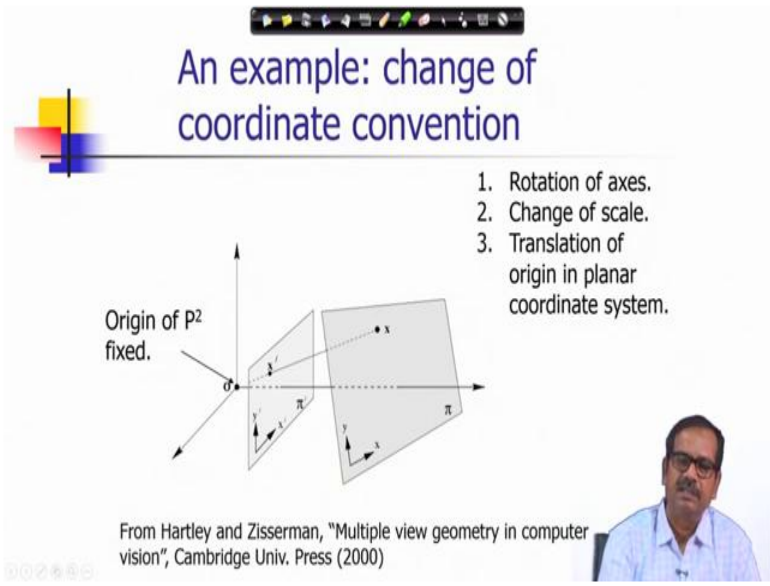

So, there could be several other examples of this projective transformation like rotation of axes, change of scale, translation of origin in planar coordinate system. So, every operation will give you a different set of coordinates for the transform points, but geometrically you can see that they preserve the properties of collinearity, the properties of invertibility of the transformation one to one mapping of the transformation and finally, those are points in 2D projection projective space.

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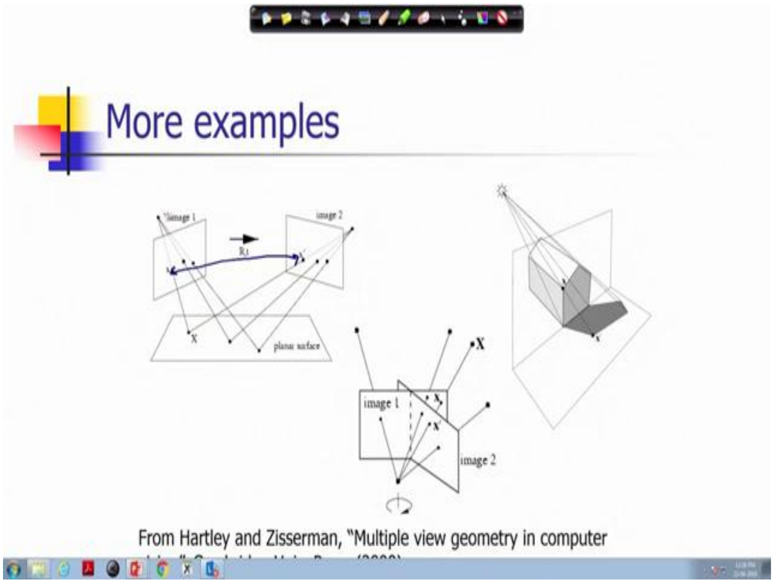

There could be more examples. Like this is another example, where you are rotating your plane of projections about an axis and you consider any two plane of two planes which are related by this rotation and consider a ray passing through the center of, a center origin of this particular plane and the intersection points in this plane they are giving you the corresponding transfer point.

Similarly, some other examples are also given here. In this case, this point x and this point x' they are related by this rule that they are formed, they have two different center of projections, but they are formed by the rays connecting the same point on a particular planar surface. So, here also any straight line, any line in this planar surface they will projected as lying here, which means all collinear points will be also mapped to another set of collinear points in the transform space satisfying the conditions of projective transformation. In this case, also you can see the shadow formation that is also a case of projective transformation because the same set of properties those are true here.

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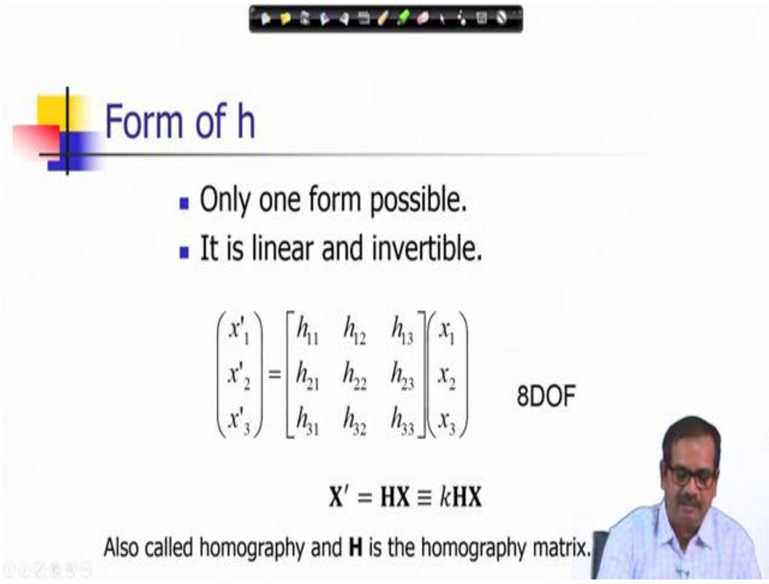

So, let us discuss what would be the form of this transformation and interestingly only one form is possible and that to a very simple linear from, that is what we will be finding out here. And it is as I mentioned, it is linear, it would be invertible that is from the property of the projective transformation and the form is given here, in the linear form as shown below

$$\begin{pmatrix} x_1^{'} \\ x_2^{'} \\ x_3^{'} \end{pmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

. You can see it is a 3×3 matrix which should be an invertible matrix and it maps uniquely a point $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ to another set of another point which is represented by a 3 vector $\begin{pmatrix} x_1^{'} \\ x_2^{'} \\ x_3^{'} \end{pmatrix}$.

In short, we represent this transformation matrix by the symbol H mostly in our lecture. We will see that this representation will be there which is representing a 3×3 matrix. These are the column vectors by bold letter that we are representing here so, the transformation matrix H that is equivalent to kH which is a scalar value. The same relationship can be denoted by this multiplication of a scalar value because a point X' and kX' it is the same point in the projective space. So, I can also multiply by any scalar constant, the transformation matrix I can multiply it with any scalar constant that would give me also the same transformation matrix, which means the transformation matrix itself is an element of the projective space.

So, this particular matrix has 8 degree of freedom due to this fact, because as I mentioned one of them can be treated as a scale and you can express all other elements in proportion to this scale. So, there are 9 elements effectively in the 3×3 matrix, but out of which one of them will denote scale factor, so there are 8 independent parameters or the degree of freedom of this matrix is 8. This matrix is also called homography and this transformation is also called homography, and this matrix is called homography matrix.

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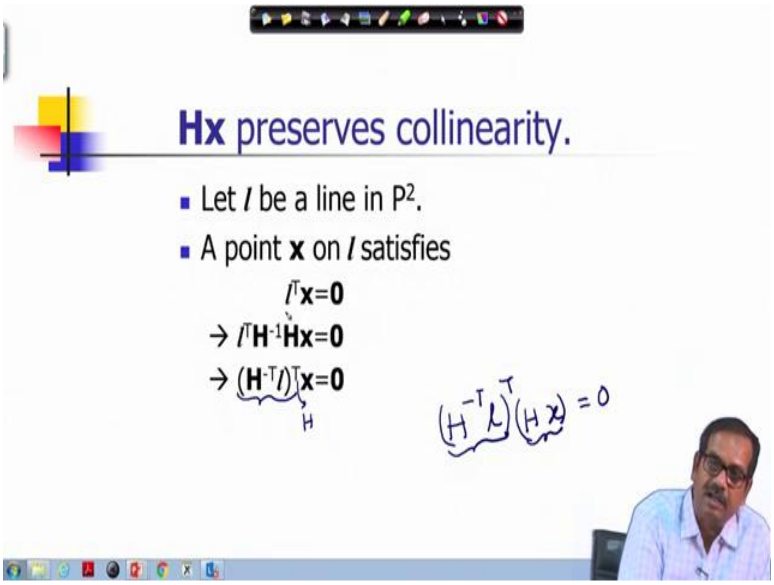

So, Hx preserves collinearity, that property we can very easily verify. Let us consider a line in this space two dimensional projective space. A point x on this line it satisfies this point contentment relationship which is given by this $l^T x = 0$. And these itself I can write it as $l^T H^{-1} H x = 0$. You can see that, that is a trick we are using here. We can consider this($H^{-1} H$) is as an identity matrix.

So, this identity matrix can be replaced by this $H^{-1}H$, and then what we can do that by using the matrix transformational rule $l^T H^{-1}$ can be written as $(H^{-T}l)^T$. So, this quantity itself can be considered as a new line. It is $(H^{-T}l)^T Hx = 0$. So, that is what I was referring at. These itself represents as if a line; on this line this transformed points are lying. So, this line in the transformed space where the all the points of on line l they are lying now.

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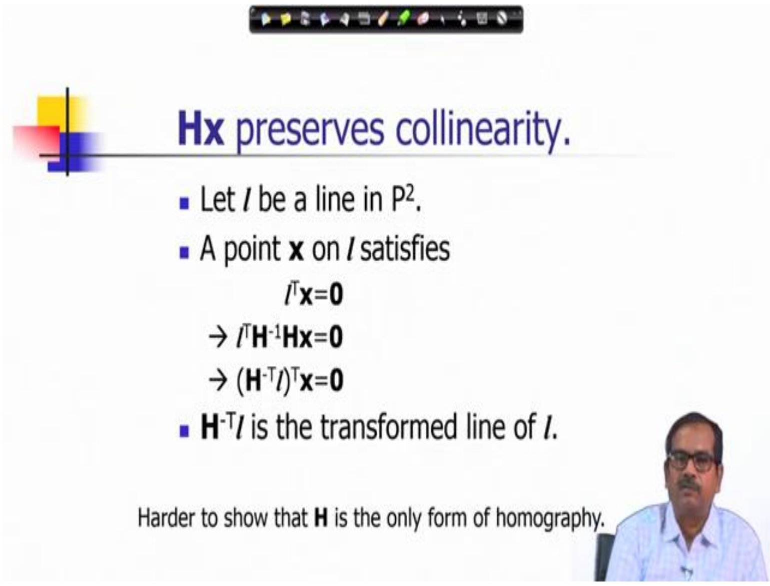

So, this is a summary. So, $(H^{-T}l)$ is a transformed line of l . And so, that shows how Hx preserves collinearity, but it is difficult to show that H is the only form of homography and this is not in the scope of this particular course.

So, we will accept this fact that H is the only form of homography and this has certain advantages in computing H if we accept these particular fact. This is also true, as I mentioned that you can prove it, but that proof is not discussed here because it requires a complex arguments.

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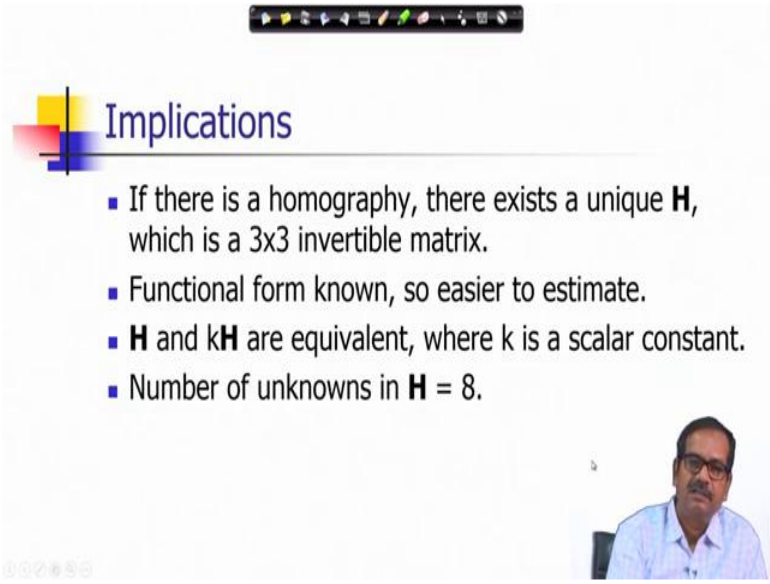

So, the implications of this fact is that, if there is a homography, there exists a unique H and it is a 3×3 invertible matrix. Since, its functional form is known, because we know that is a matrix those are the elements and we can expand the relationships in an algebraic form, so it is easier to estimate. As I mentioned also that H and any scalar multiplication of H which is say kH, they are equivalent. Number of unknowns in H is 8.

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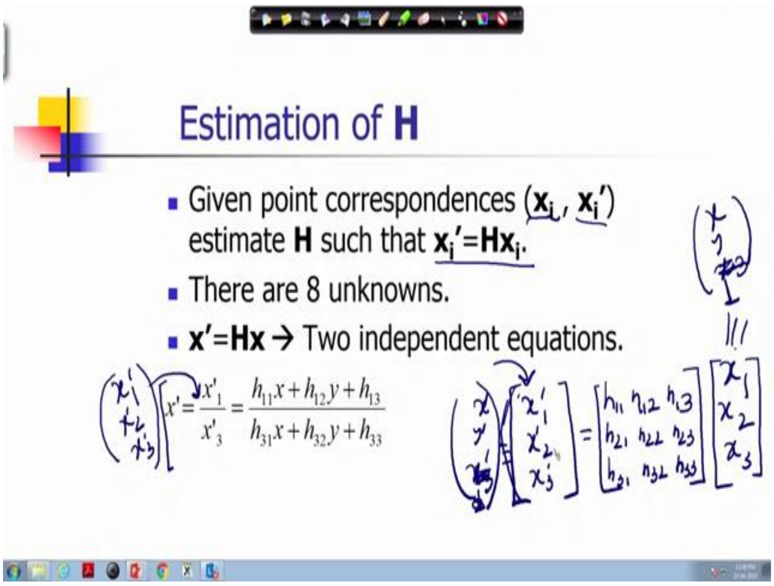

So, now we will be considering how this homography matrix could be computed. Consider, we have a set of point correspondences, and a typical point correspondence can be specified in this form that you have a point x_i in the original space and its corresponding transformed point x_i prime in the transform space, and their relationship is given by this fact $x_i^{'} = Hx_i$ or $x_i^{'} = kHx_i$ you should note that in the projective transformation relationship.

Now, we need to estimate H that is the computational problem. As I mentioned there are 8 unknowns, since there are 8 unknowns and given a particular point correspondence you can get only two independent equations. Let us see how we can get these two independent equations. Let us consider this particular fact. You can see that in the two-dimensional real coordinate space can be expressed as a ratio of $\frac{x_1^{'}}{x_3^{'}}$. This is a division by the scale factor. x'

So, x' is a representation in the homogeneous coordinate of x' is given here as $\begin{pmatrix} x_1^{'} \\ x_2^{'} \\ x_3^{'} \end{pmatrix}$ and this from this representation a x' is given by this form. And since you have the corresponding matrix multiplication. So, if I write the matrix multiplication relationship that means, $\begin{pmatrix} x_1^{'} \\ x_2^{'} \\ x_3^{'} \end{pmatrix}$ this is the transformed coordinate space which is multiplied by the homography matrix whose elements are given by $\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$ and then you have the original space points are represented in the original space in this problem. So, you can see that $x_1^{'} = h_{11}x + h_{12}y + h_{13}$ and which is given here in this numerator.

So, in this particular representation it is considered as $\begin{pmatrix} x^{'} \\ y' \\ 1 \end{pmatrix}$. This representation we are considering this x', y' and this is considered as say x', y' let me take is at x_3 or and this is considered as x, y, and say this is say x_3 and this is say $x_3^{'}$, ok. So, say this is the representation we are using here. So, it is $h_{11}x + h_{12}y + h_{13}$. So, we are taking x_3 as 1 and this is also we are considering this is, this could be x_3', but this is we are taking as 1 because this is a scale factor that would be representing here.

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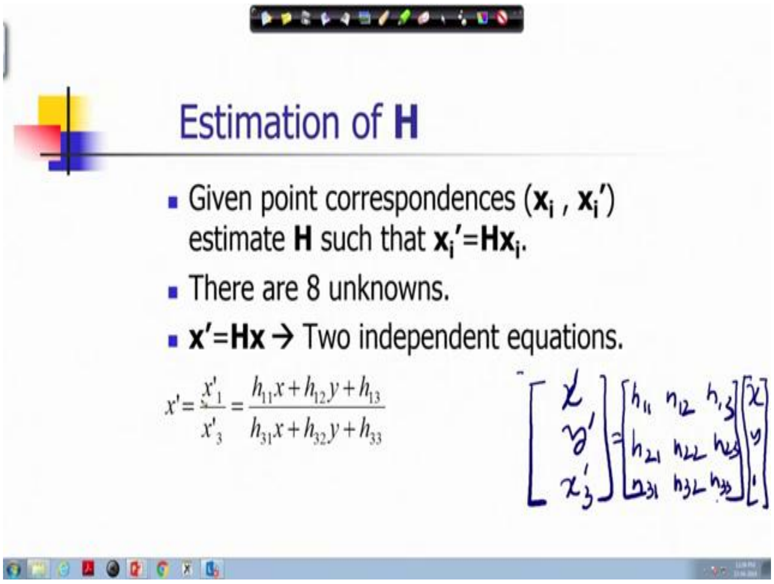

So, let me rub this to make it more concrete. let me write the equation once again. So, this is x', this is y', and this is say x_3', and this is the matrix $\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$ and this is $\begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$ ok. So, from there we can get this equation you can see that numerator is coming from here and the denominator; that means, x_3' is coming from here $h_{31}x + h_{32}y + h_{33}$ and even if this coordinate x' or x_1' is divided by x_3' then you get actually the coordinate in the real space which is observed here.

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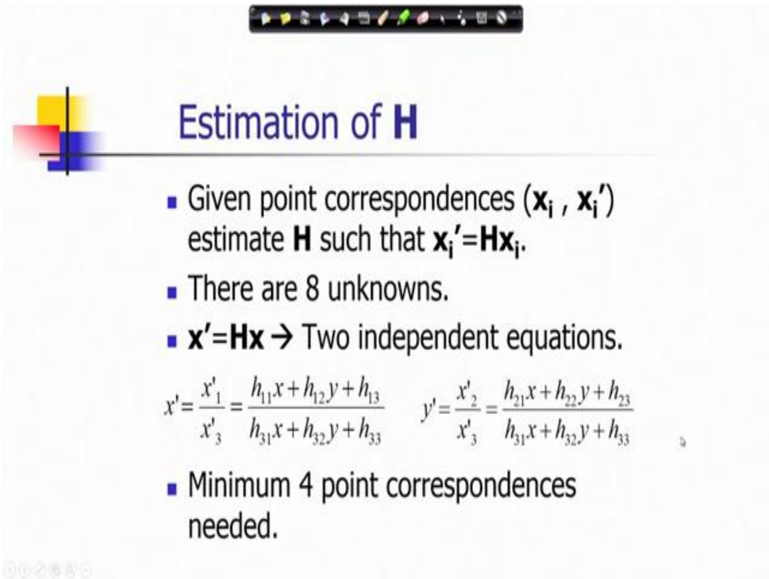

Similarly, we can get also for the y coordinate; that means, y' also can be expressed in this form. So, we get these two equations from this particular relationships. So, there are 8 unknowns. So, you require 4 points, at least 4 points to get 8 equations, but one of the parameters we can set it as equal to some known value and then you can solve this problem. So, minimum 4 point correspondences are needed to solve this problem.

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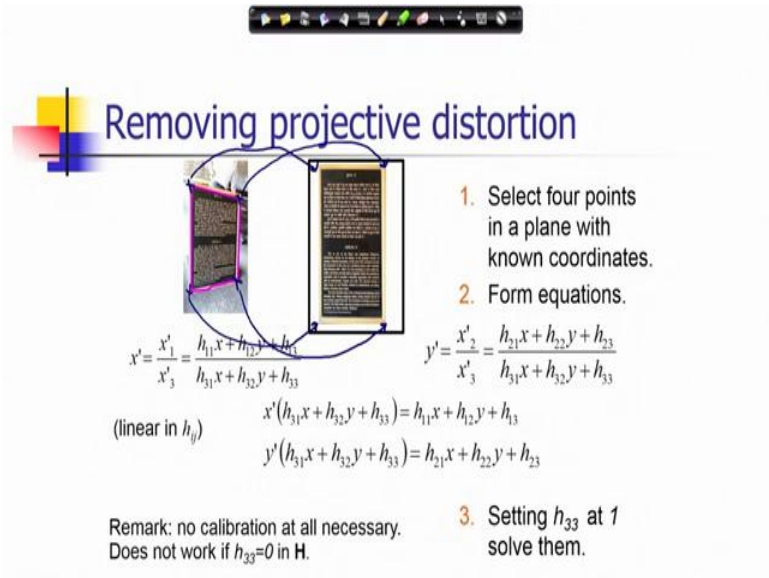

Let us take this particular example consider this image and as you this is the same image what we displayed in a previous lecture and you can see that the horizontal lines they are not remaining horizontal in the projective space because of the projection they look like oblique and meeting at a finite point, that is the vanishing point. If you would like to remove the projective distortion which means let the horizontal line looks like horizontal and vertical lines looks like vertical then we can see how we can apply the concept of homography here.

So, we select 4 points in a plane with known coordinates. So, consider this selection. Here I have shown by drawing a particular contour, but we can select any 4 points, the endpoints of this edges say we have selected this point, this point here, this one and this one. So, these are the points which are selected. And then in the transform space we would like to straighten this lines, so that they look like parallel. So, in the transform space we want this rectangle, this quadrilateral should look like a rectangle; that means, we will be mapping this points to a corner point of this rectangle, the respective corner points of these rectangles. So, this is a kind of mapping you would like to have. And we know the coordinate point. So, we know these coordinates and we can also define the coordinates of this space and we can define the coordinates of these corner points.

So, we know the corresponding pair of corresponding points. So, from there you can form equations because if the coordinates each pair of correspondents will give you two equations, so we will get 8 equations. Like this, this is one example of a pair of equations, similarly we can form another 4 pair of equations and you will get 8 equations and since they are all linear in this with respect to the parameters of transformation matrix; that means, the elements of H matrix. And as I mentioned you can set one of them to a value 1 and in proportion to that value it express others. Because there are 9 elements, but one of them is a scale element, so let us consider say h_{33} is the scale element, we can set this h_{33} at 1 and then you can solve them.

If you do that then you can say that this particular image in the transform space will look like this where the straight lines they still remain parallel. But there is a caution when you apply this method that h_{33} if it is a non-zero value then this method will work, but suppose h_{33} itself is a 0 so then it cannot act like act as a scale factor. And for that you need to choose another element, and we will see actually there is a method which can remove this particular constraint and in generally can be applicable for any kind of homography matrix. And another interesting part here you can see that you do not require any calibration of the cameras here, it is your coordinate definition that is doing the trick to remove this projective distortion.

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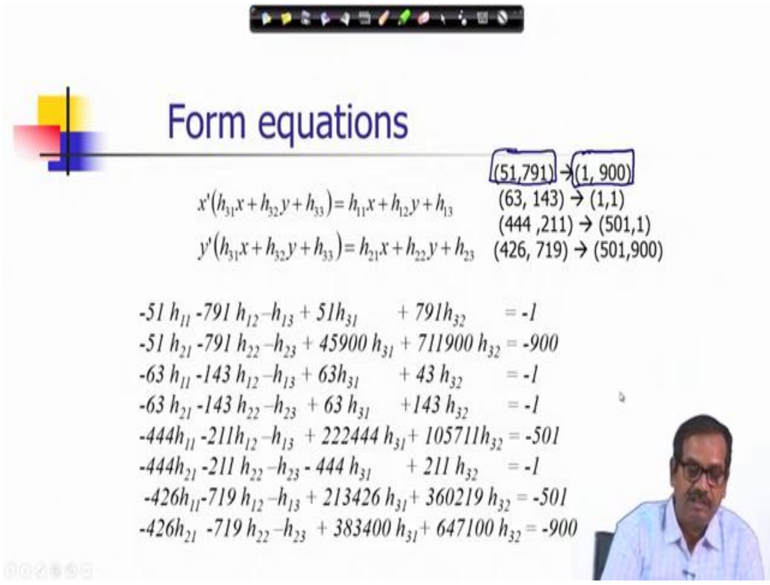

So, this computations let me elaborate a little further. Say take this typical case these are the point correspondences, I have defined these coordinate and these are the points in the in the original image, this is the points in the original image and this is a point in the transforms which is a point of the rectangle. And using this set of point correspondences I can form these 8 equations and then these equation can be conveniently represented in the matrix form in this particular form.

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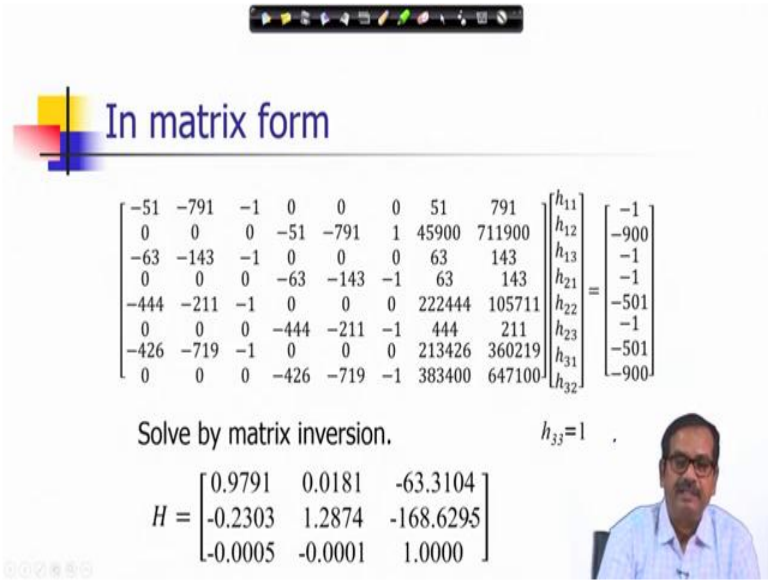

As I set $h_{33} = 1$, I can rearrange the elements in such a way that all the unknown parameters remain in the left hand side and the constant terms they form the column vector in the right hand side. So, the inversion of this matrix and multiplying that inverted matrix with this column vector will give you the solution. And you will get a homography matrix like this.

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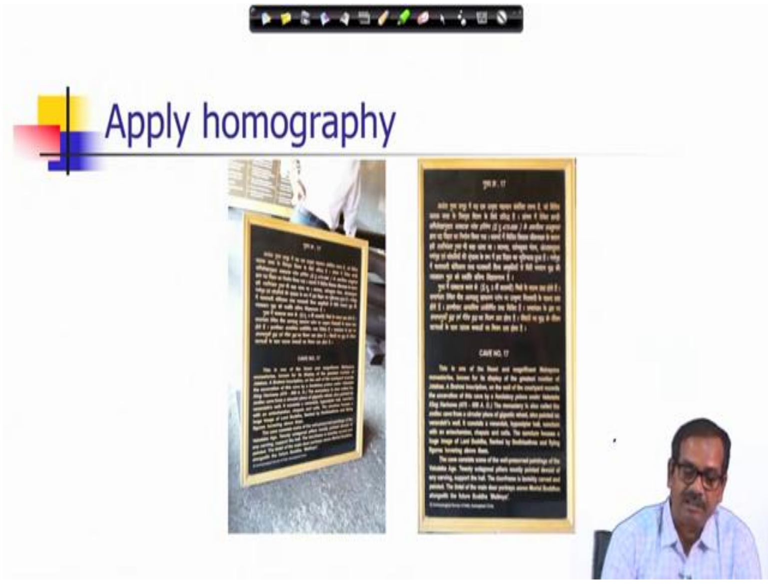

And if I apply the homography then you can see this is the image which I am showing here in an enlarged form. And you can see the utility of this technique, the text is now conveniently being read by applying this homography.

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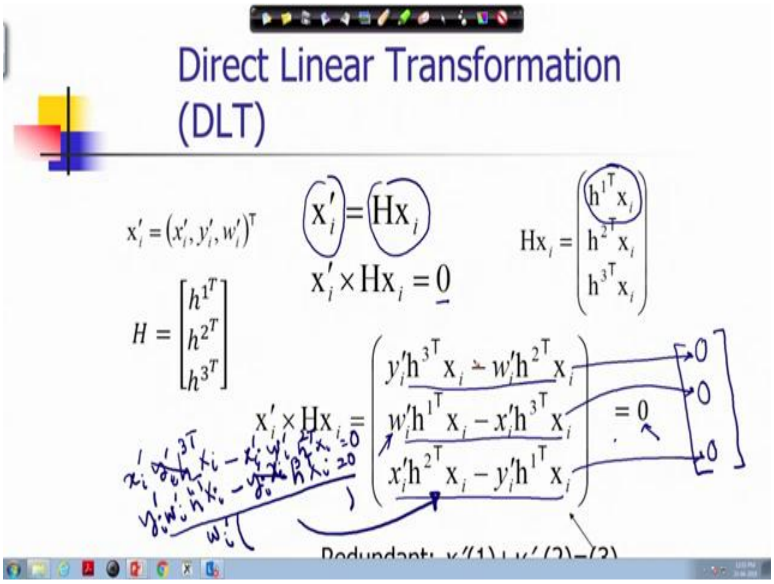

So, the other method by which this homography could be computed is called direct linear transformation method. It is a general method, because yours working with only 4 point correspondences, but you may get more number of observations and you may that would make your method more robust because there could be noisy observations, and if you just use only 4 points to compute homography that noise will heavily affect the quality of your result.

So, let us consider a method where you have more number of observations, more than 4 points. So, in general I would like to represent my transformation coordinate space by this particular representation and consider your homography matrix is represented in this form, a vectorial form where you can you are representing by this particular symbol h^1 it represents a row of H, h^2 represents a row of second row h^3 and transposition of this that represents third row. See this is a matrix representation, all these are row vectors, these are the row vectors.

So, if I multiply this matrix H with the vector x_i, I can also represent in this particular sub matrix form with my multiplication. So, you can see that all row vectors are multiplied by column vectors and each one will give you this particular form. So, you should note that each element is a scalar element. So, this is a 3 vectorial, Hx_i is a 3 vectorial representation that is a transformed point.

$$x_i^{'} \times Hx = \begin{pmatrix} y_i^{'} h^{3T} x_i - w_i^{'} h^{2T} x_i \\ w_i^{'} h^{1T} x_i - x_i^{'} h^{3T} x_i \\ x_i^{'} h^{2T} x_i - y_i^{'} h^{1T} x_i \end{pmatrix} = 0$$

So, since the scale is involved it is difficult to work with this equality sign. You consider $x_i^{'}$ is a vector and Hx_i, is a 3 vectors and they are related with this scale factor. So, they should have the same direction. So, instead of enforcing equality what we can do? We can take the cross product of these two vectors because they have the same directions, so they are parallel vectors. So, their cross product should give you 0 vector. So, this is a 0 vector you should note that this is a 3 vectorial representation this is a 0 vector.

So, if I perform the cross product, the computations can be expanded in this form. So, you can see that these are the 3 components of this cross product. And this should be equal to 0, and this 0 is not a simple 0, its a 0 column vector of dimension 3×1 which means this element should be equated with 0, this second row this is equated with second 0, third row is equated with this 0.

And this can be shown that there is a redundancy in this representation because if I multiply say $x_i^{'}$ prime with this equation, and if I multiply $y_i^{'}$ with this equation. And if I take the addition of this two, so let us see say the first equation if I multiply it would be $x_i^{'} y_i^{'} h^{3T} x_i - x_i^{'} w_i^{'} h^{2T} x_i = 0$ Multiply these second equation with $y_i^{'}$. So, it should be $y_i^{'} w_i^{'} h^{3T} x_i = 0$. If I add them, these two quantity will be cancelled and you can take $w_i^{'}$ as a common and we will find this will give you this particular equation $x_i^{'} h^{2T} x_i - y_i^{'} h^{1T} x_i$. So, that is equal to 0.

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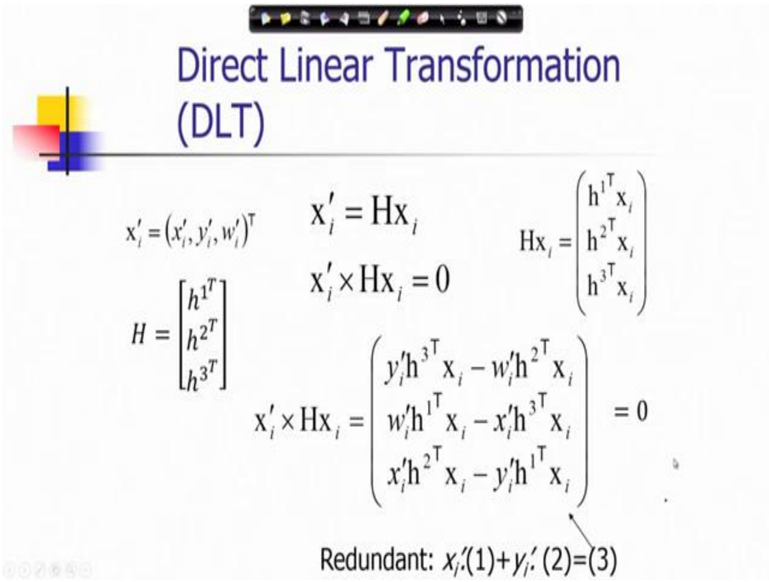

So, this equation is redundant and you get only this two equation. So, that is the summary of this particular exercise. So, this is a redundant equation.

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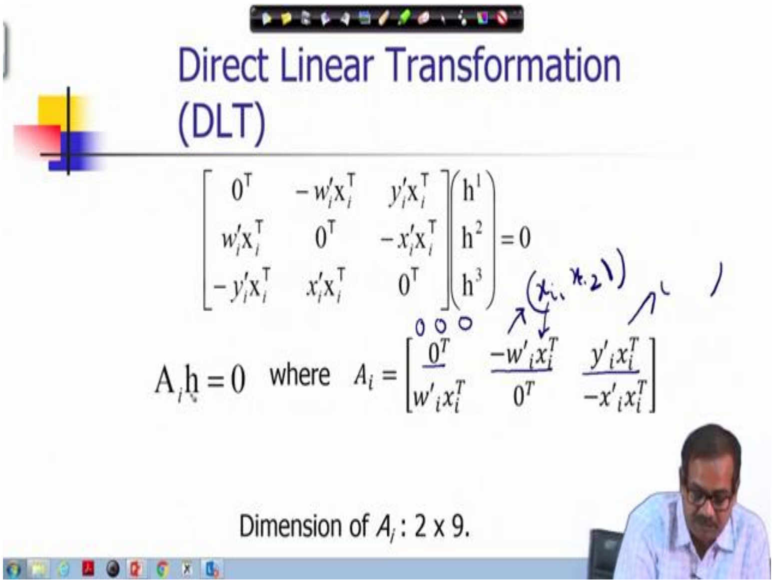

So, in the direct linear transformation we can consider only these two, first two rows of this matrix that is giving you the two equations. So, I can represent this particularform as this equation $A_i h = 0$, where is given by this form $A_i = \begin{bmatrix} 0^T & -w_i^{'} x_i^T & y_i^{'} x_i^T \\ w_i^{'} x_i^T & 0^T & -x_i^{'} x_i^T \end{bmatrix}$. So, you can note that the dimension of A A_i is 2×9, because you know there are two rows and each one is a transpose of 3 vectors. So, 0^T it is a 3 vectors, this is 0 0 0 minus w i prime x i transpose, x i transpose is a point x i, so which is if I write say $(x_{i_1}, x_{i_2}, 1)$ into $w_i^{'}$ This is my original space points. Similarly, $y_i^{'} x_i^T$ is also another point. So, you see that this is also 3 vector. So, if I arrange them there are 9 such columns, there would be 9 such columns. So, it should be 2×9, this matrix is 2×9. So, a single point correspondence will give you; two equations $A_i h = 0$

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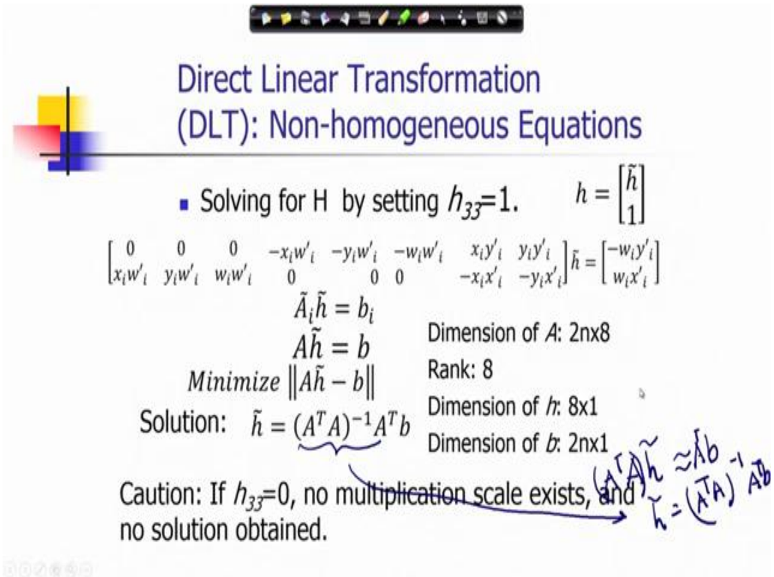

So, this set of equations can be solved by solving a set of non-homogeneous equations and you can see that this is the expanded form what I described earlier. And we can consider that there are more number of points and stack of these two rows for each point correspondence will give you a composite matrix A which is of higher dimension. So, if there are n point correspondences, so dimension of A would be $2n \times 8$ because if we set the parameter h_{33} equals 1, so the representation of h would be $\begin{bmatrix} \tilde{h} \\ 1 \end{bmatrix}$. So, you understand in this representation. So, $\tilde{h}$ consist of the column vector representation of each rows of matrix h. Only the last element that is h_{33} that is set 1 and then rearranging this equation you will get this equation.

So, finally, dimension of $\tilde{A}_i$ is 2×8 and if I get the stack of all endpoints it will be $2n \times 8$. So, since there are more number of equations and only there are 8 unknowns. So, we can use the least square error estimate method. So, all objective is to minimize $A\tilde{h} - b$, where dimension of h is 8×1 and dimension of b is $2n \times 1$.

So, solution of this equation can be obtained in this form. You can see that this is by solving least square, we can multiply the b with this this is called pseudo inverse of the these operation. The whole thing is called pseudo inverse operation. If I multiply with the pseudo inverse matrix then you will get this vector. I can give show you one simple way to obtain this pseudo inverse by using the matrix operations itself. Say, approximately we want $A\tilde{h}$ should be equal to b. So, what we can do. Let us post multiply each side by A^T, ok.

So, now you take the inverse of A transpose and multiply with the resulting operation that would give you the corresponding equations that is what exactly you are getting here. So, if you use these particular solutions, you will get the elements of matrix H in this form and append it with the element h_{33} as 1 then you will get a transformation matrix. But here also the problem is that, that your assumption of h_{33} equals 1 may not hold because if h_{33} is 0, then you simply cannot put it as 1 and you cannot apply this method in that case.

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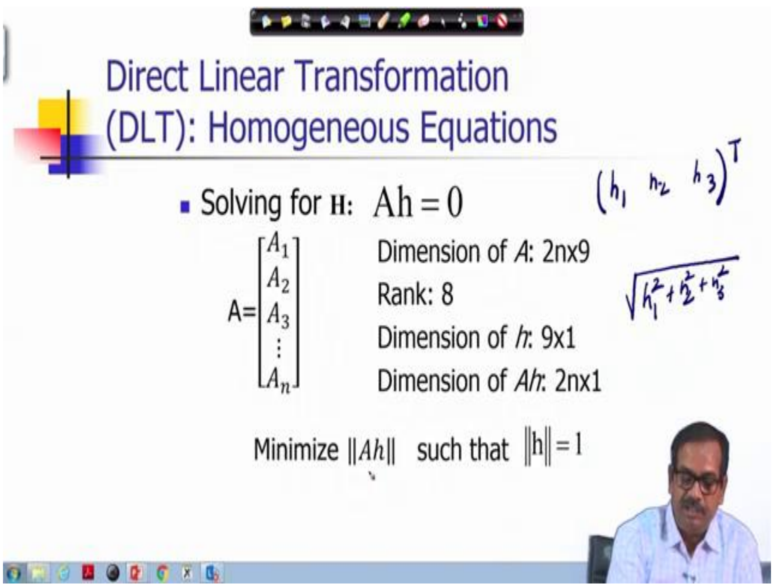

So, there is a method which is more generally applicable and that is called you know solving solution of homogeneous equations. In this case we are not making any assumptions, we are not setting any value to a particular constant to a particular value, any parameter. We consider to solve the whole problem as equals 0. So, this side is the corresponding this specification is 0. So, you would like to get that values of Ah which will minimize this particular equations.

So, the error of error from is your objective functions would be a bit different in this case. You have dimension of A is $2n \times 9$ now, because you have not shifted one of the parameters at the right hand side and setting it value to 1, we considered the whole vector as a 9 dimensional vector, forming the elements of transformation matrix and each point will give you a pair of equations A 1 A 2 A 3, those are the rows corresponding to equations.

So, dimension of A would be $2n \times 9$, dimension of h is 9×1, so dimension of Ah is $2n \times 1$, the whole thing is $2n \times 1$. So, the dimension of 0 is $2n \times 1$. So, our objective is that we would like to minimize the norm of Ah, such that the norm of h equals 1 because there is a scale factor involved, so we have to put a constraint on h to minimize this value, and this constraint; it is a constant optimization problem where we have to keep the norm of h equals 1. So, if you were wondering what is meant by norm of a vector it is simply the magnitude of that vector. So, consider a 3 vectorial form representation say you have $(h_1, h_2, h_3)^T$. Say, this is a three dimensional representation I am using the row vector representation. So, norm would be simply the sum of h squares and you take the square root of this; that means, the magnitude of that vector. So, we are minimizing, also will give you a vector of dimension $2n \times 1$. So, we are minimizing the norm of this subject to this.

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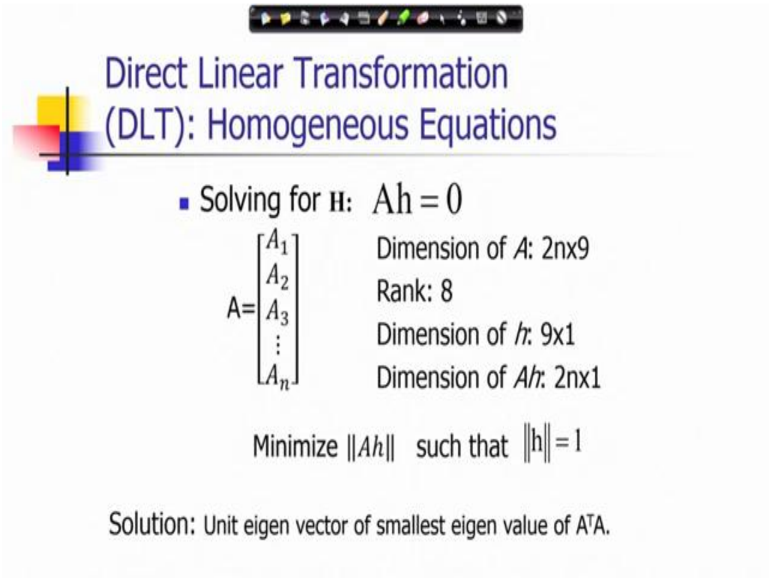

So, there is a solution for this equation, it is a well-known solution. This is the unit Eigen vector of the smallest Eigen value of $A^T A$. So, using this result, so I am not going to discuss this particular theory by which you get this solution. We will be using this particular fact that if I can compute the Eigen vector correspond to smallest Eigen value that Eigen vector is a solution of this equation.

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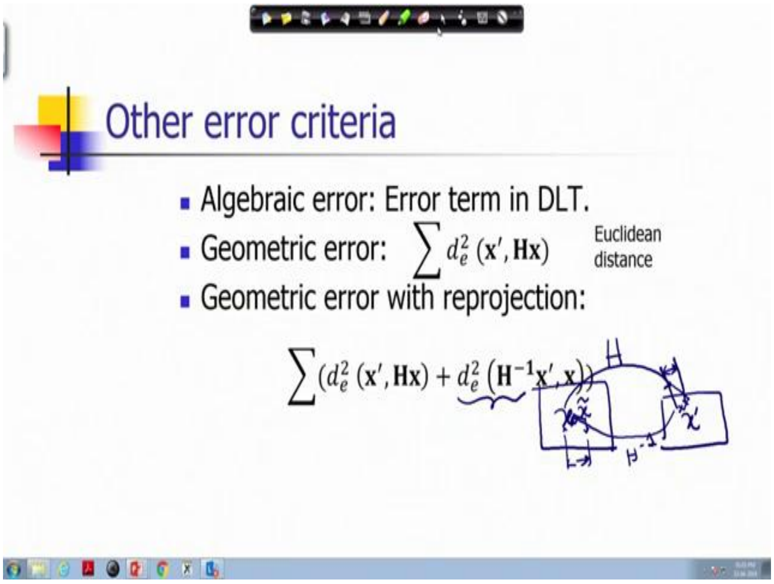

So, this is one way of solving this problem that you can solve using least square error estimate for homogeneous, set of homogeneous equations or set of non homogeneous equations as we have discussed earlier. But there could be different other error criteria. Like the error term what we used in the previous methods which is known as direct linear transform methods, those errors are known as algebraic errors, those are the least square errors that is how we have defined, that sum of square errors, its square of deviations. There could be geometric error, like you can find out the distance between the transformed point, true observed point in the transformation space and the estimated transformation; transform point and square of all these distances can give you an error.

So, the idea is something like this. Say you have the original space x and you are transforming it to the transformation space $x^{'}$. So, when you perform H operation. Now, this may be your true observation points, true and after estimation your point may be estimated at this point. So, the distance between these two will give you a component of error. And you are going to minimize some of all these distances. So, this is one form of geometric error.

And this is the Euclidean distance we use. Geometric error with re-projection where we will be using also the same estimation; that means, here what we will be considering, we will be also applying H^{-1} on the transformed points. We will be also applying H^{-1} on the observed transform points and get the estimated point in the original space say $x^{'}$. Say, this is the point and say this is your observed point and get the distance between them. So, that would give you the component from this inverse mapping. So, this is a geometric error with re-projection.

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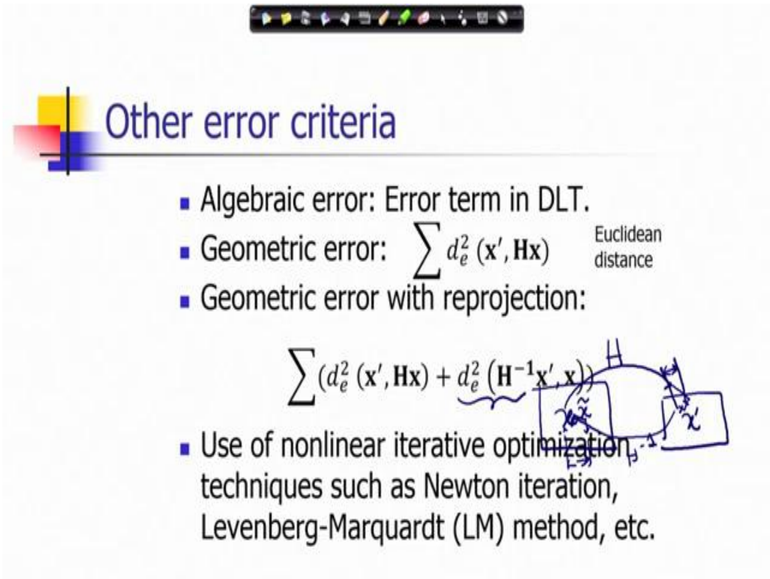

And there are different methods like non-linear iterative optimization techniques, such as Newton iteration, Levenberg-Marquardt method, etc those methods could be used for solving this problem.

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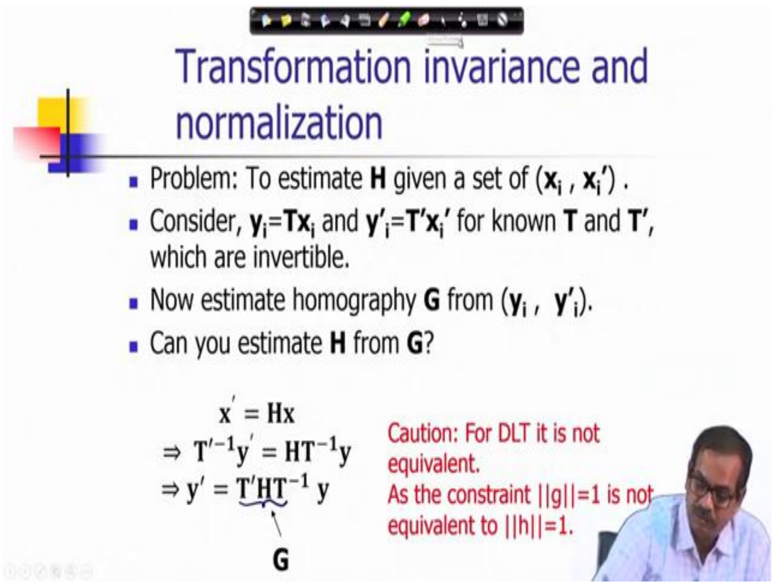

There is another interesting fact, that should be noted. we can apply even transformation on these points, on the point set of points which are observed in the two spaces and even we can estimate the homography even after transformation. So, let us understand this particular fact.

Say, consider $y_i = Tx_i$ and $y_i^{'} = T_i^{'}x_i^{'}$. So, these are the transformations T and T' which are applied in two spaces, both in the original and the transformed spaces. So, now you compute the estimate the transformation matrix between these transformed, set of transform points. So, we can see that even from there you can estimate H. So, this is a relationship that is related with these point x by Hx. So, you apply the transformation relationships with $x^{'}$ T and T' with a corresponding set of points. So, you can see that, so $x^{'}$ is $T^{'-1}y^{'}$. x is $T^{-1}y$. So, y' is $T'HT^{-1}y$. So, this thing can be considered as the transformation matrix. This is what is the transformation G.

So, if I use and y_i can estimate G and from there I can estimate H, but one thing is that if you are using least square estimate then of course, you know you may not get a; the you can get a close estimate of H, but it is not equivalent. So, if I estimate H without transformation and if I estimate G with transformation and convert get back the H from there the results will not be equivalent because of this constant optimization issues.

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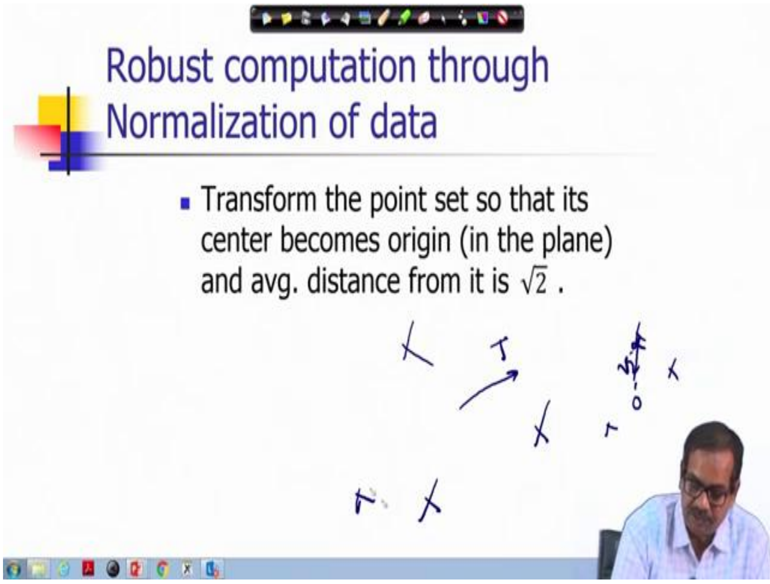

So, one of the example of this transformation which is very often used is that we can transform the point set, so that its center becomes origin in the plane and average distance from it is $\sqrt{2}$. So, we can transform the coordinates, we can scale down or we can scale up, so that finally, the all the spread of the points say if we have the original point like this after transformation, T all these points should be within $\sqrt{2}$ distance of the center of the points.

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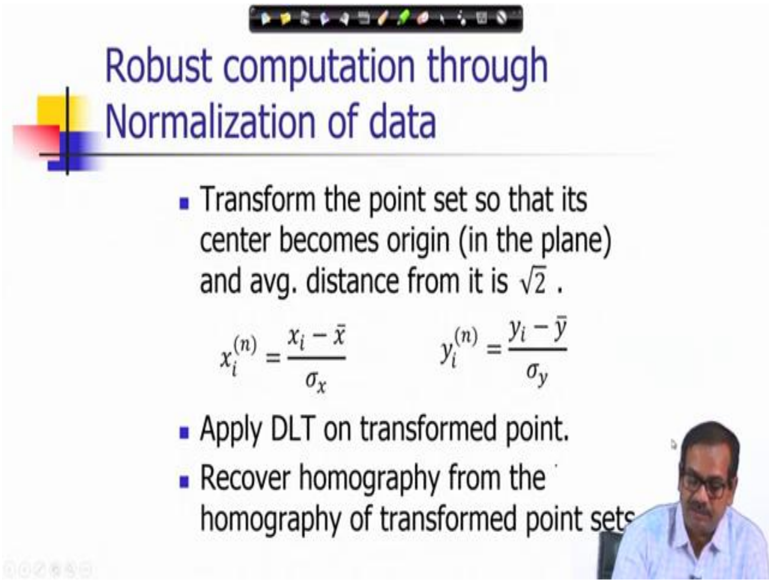

So, this is the idea and there is a particular type of transformation which can perform this job easily. So, take this, that I subtract each point from its center and then divide it by its standard deviation σ_x. With this transformation you can preserve this particular fact, and now you apply the direct linear transformation on this transform point. So, it can recover the homography and this computation is robust because it can take care of the big values that we will be getting at different you know coefficients.

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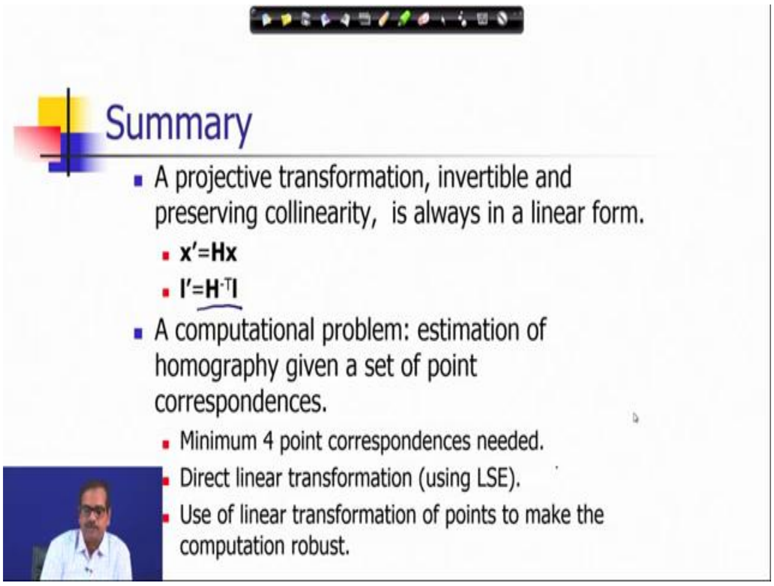

So, just to summarize this particular lecture on computation of homography; what we have seen that a projective transformation, has these properties that it is invertible, it preserves collinearity, and it is always in a linear form. And that information gives you a convenient techniques for estimating projective transformation because you can apply the linear model there.

So, these are the fact with the projective transformation that it is defined as a transformation of a point to another space, another projective space, as another point in the projective space. So the line in a particular original space that is also transformed, and with the transformation H it is related with this particular property, H^{-T} . If I multiply with the original line, line in the original space you will get the corresponding transform space.

And there is a computational problem for estimation of homography in this particular, in this respect which we discussed that is if you have a set of point correspondences then you can compute the homography. And there we require minimum 4 point correspondences to solve it, but if you have more number of observations then you can make it robust and you can apply least square error techniques, like direct linear transformation technique for solving them. So, it makes your computation robust.

Thank you for listening.