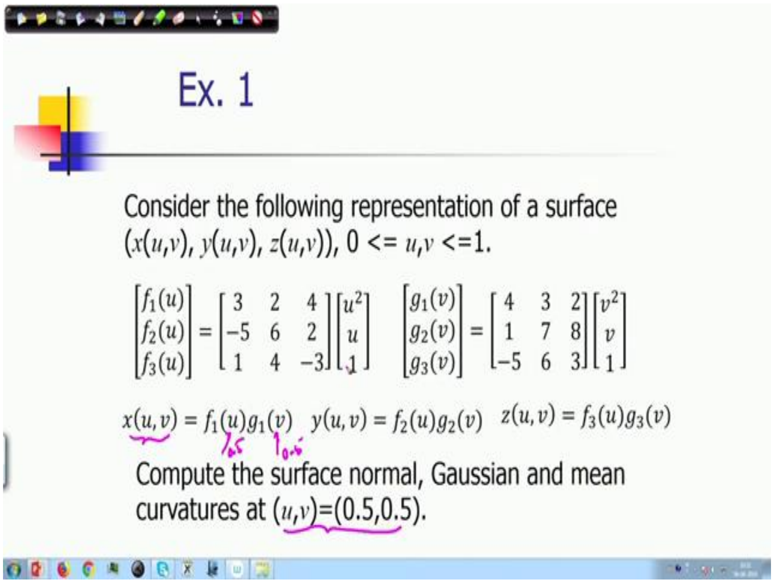
Computer Vision
Prof. Jayanta Mukhopadhyay
Department of Computer Science and Engineering
Indian Institute of Technology, Kharagpur

Lecture – 44
Range Image Processing – Part IV

(Refer Slide Time: 00:26)



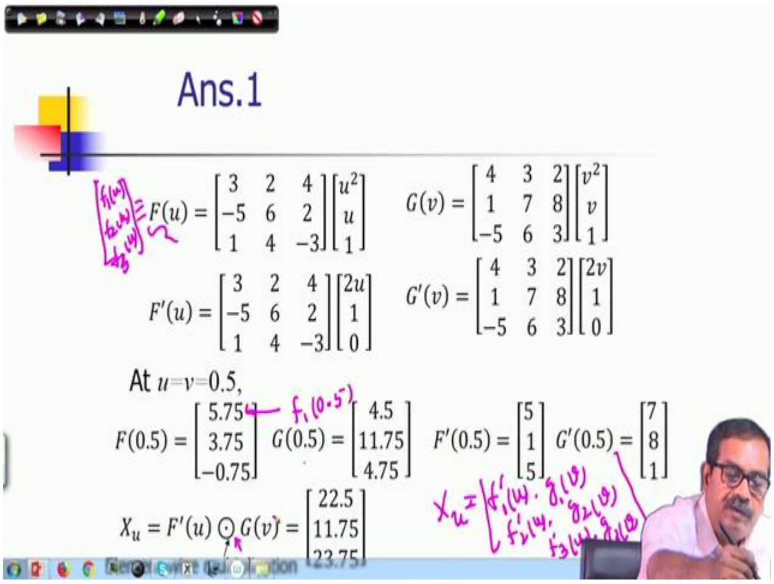
We continue our discussion on computing local features of range data or surface local features of local geometry or range data. For understanding this process let us discuss a particular problem in computation of this features and let us solve this problem and I hope that would give you a better understanding of the computation.

So, you consider this exercise that a parametric surface which has been described in this form. You note that in the problem definition statement, I have used the brief notations by using matrix algebra. So, all the partial functions of u and v they are independently defined and the function of x y parametric functions of the corresponding coordinates, they are separable in terms of the function of u and v.

So, that is how it is given. And what we need to do we need to compute the surface normal Gaussian and mean curvatures at (0.5 , 0.5) of the parameter values. So, this is our this is the point. So, this surface point we need to compute and these values. So, you should note that with this parameters value what is a co-ordinate that can be compute it by using this functions.

So, you replace u by 0.5 and v by 0.5, then you can get the coordinate of x by this function. So, you can use this values and you can get these coordinates. So, let us proceed that how this computation. So, we will be exploiting this matrix notation to perform the derivative,s first derivative some second derivatives because matrix is a linear operation. So, we can see that those delivery operations also become simple in the structure.

(Refer Slide Time: 02:35)



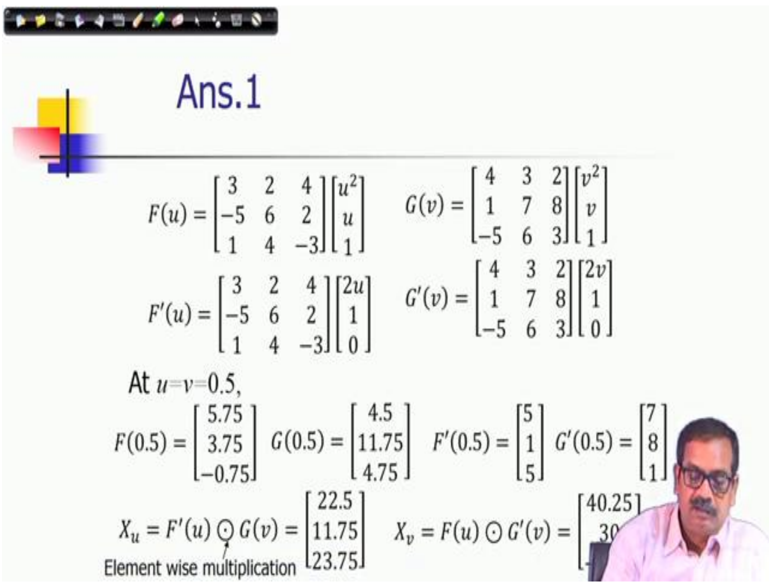
So, let us considered the representation of the functions of u and v in the form of a column vectors and which is given F(u). So, this is a column vector which is equivalent to like F(u) is expressing $\begin{bmatrix} f_1(u) \\ f_2(u) \\ f_3(u) \end{bmatrix}$. So, this is what F(u) similarly G(v). The advantage here with this notation is that, I can easily compute first derivative of all these functions; I can express in terms of matrix itself.

So, simply I have to take derivatives of corresponding elements of matrix which is a function of u in this case. So, as you can see, this($F'(u)$) is a first derivative of the function F(u). Similarly, you compute second derivative of function G(v). So, you put the value of 0.5 you can get the first derivative values of the functional values of F(u) and second derivatives for v equals to 0.5 also you can get these values.

So, at u equal to v equal to 0.5 the, the functional values are given by this. So, it is computing all the things together like $f_1(0.5)$, $f_2(0.5)$ and $f_3(0.5)$; these are the corresponding values. So, this(5.75) value is actually $f_1(0.5)$ and so on. So, this is a interpretations of this computation and similarly we can get G(0.5), we can also get the derivative with respect to u $F'(u)$ and get the value at 0.5.

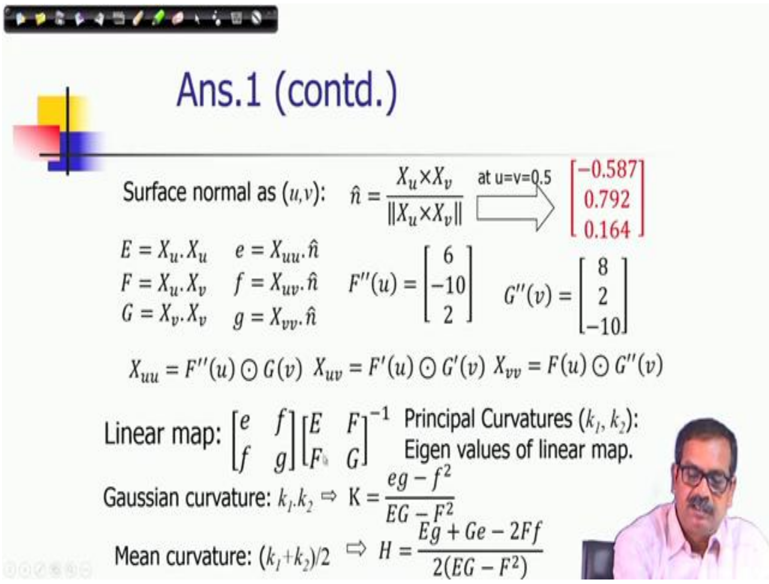
This is a value get the value with respect to derivative of G at both are 0.5. So, now, this is the vector which is giving you the X u with respect to u the surface point. So, this is giving you in depend this function is in independent of v. So, what you can do? You are multiplying it means your X_u is considered as $\begin{bmatrix} f_1'(u).g_1'(v) \\ f_2'(u).g_2'(v) \\ f_3'(u).g_3'(v) \end{bmatrix}$, . So, this is the vector and this point wise multiplication with respect to G(v) which is given by $\begin{bmatrix} g_1(v) \\ g_2(v) \\ g_3(v) \end{bmatrix}$ column vector that is expressed by this notation.

(Refer Slide Time: 05:34)



Similarly, you can compute the X_v in this passion.

(Refer Slide Time: 05:39)



So, from these two, we can get the normal because you are getting two tangents along u curve and v curve. So, you take the cross product and normalize it then you get the surface normal.

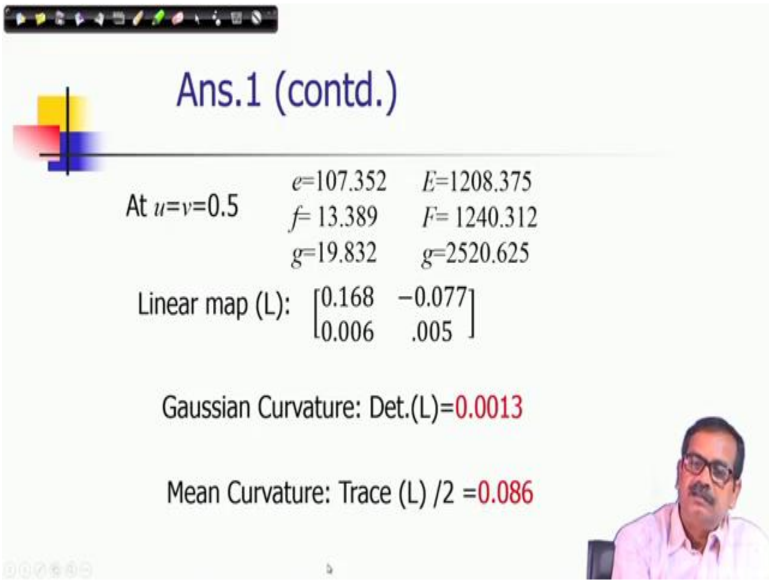
$$\hat{n} = \frac{X_u \times X_v}{\|X_u \times X_v\|}$$

So, surface normal at this point is given by this value. So, this is a unit vector unit normal vector at that point. Now to compute the mean curvature and Gaussian curvature as I have already discussed that you have to compute the elements of linear map and then also the linear map itself the matrix linear map and the trace of that matrix half of the trace will give you mean curvature and determinant of that matrix will give you the Gaussian curvature.

So, we are computing this particular entities here and we have already seen how the first derivatives with respect to u and v for surface points could be completed. Similarly we will be computing the second derivative. So, this is a linear map and you have to come to those elements and from linear map, we can get principal curvatures eigen values of linear map or as I mentioned the Gaussian curvature as this and also mean curvature as this.

So, let us compute $\begin{bmatrix} e & f \\ f & g \end{bmatrix}$ that is related to second fundamental form and here also we are using that convenient form of derivatives with respect to on the only on the column vectors of u functions of column vectors of u. So, $F''(u)$ or second derivative of F(u) that would give you the X_{uu} and element wise multiplication with G(v) that would give you the X_{uu}. Similarly, so this computations first you compute $F''(u)$, second derivative in $G''(v)$ and these are the element wise multiplications of respective elements.

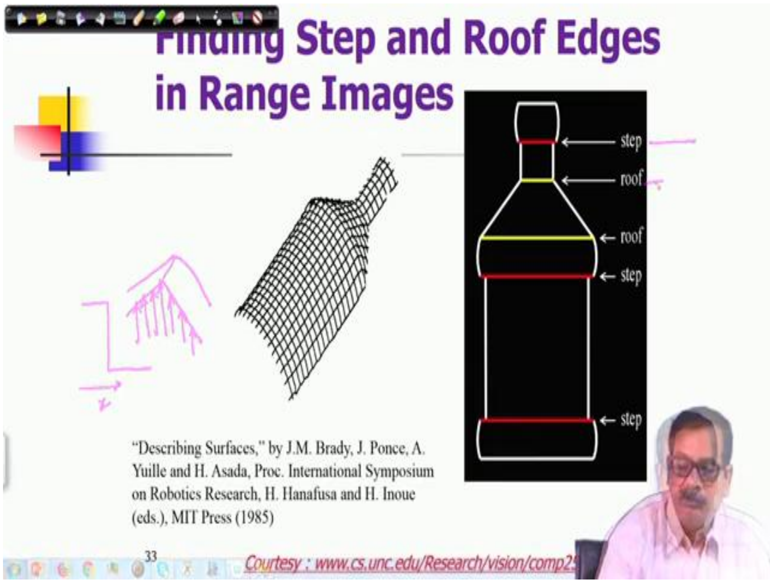
(Refer Slide Time: 07:42)



And then the values what you will get you use those values then you will get the elements of second fundamental form in this way. Similarly, we already obtain the, we can get also first fundamental from these are the values and by using them, you can construct the linear map matrix. This is a linear map matrix. So, if you take the trace and take the half of it which means you add them and you would take the average.

So, the value would be so, Gaussian curvature is determinant which means y you take the determinant of this matrix it is coming like this and trace is coming like it 0.086 that is what is mean curvature, I think this is what was a problem statement and this is how we could solve them.

(Refer Slide Time: 08:38)



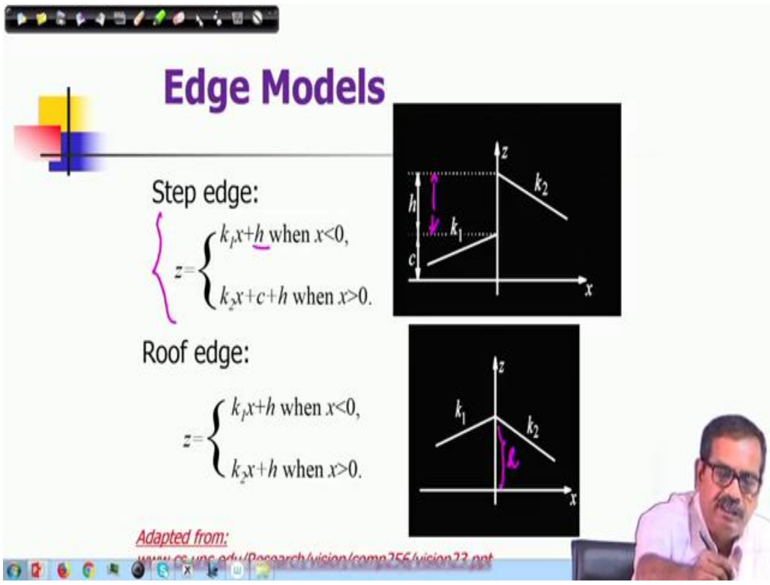
So, next we will be discussing another kind of processing with the range images and in this case, we would like to compute the edges in the range image. Now the difference between the intensity image and range image is that in the intensity image, a primarily the edges are all like a very discontinue there is a short discontinuity short changes of brightness value at the edges.

So, I should say it is a almost like zeroth order discontinuity that is happening in the edges and we call those edges as step edges very short discontinuity. Whereas, for the case of range image, there is a continuous change of the functional value and those edges you have a differential change, the discontinuity is not so, sharp and it is like an kind of an plane planar. It is a intersection of two planes in the functional map and that is producing image and that is how the depth also is varying.

Now those kind of edges are called roof edges. So, if I draw them for the simplicity see this is a short change along when we vary this edges that is what is step edge whereas if I consider a change like this is a value these are the two planes and you are observing the depth observing the height, then you see that this change is continuous and this structure this kind of edges called roof edge.

So, the in this diagram also it has been shown in this bottle, this is an example of step edge. This transition is an example of step edge whereas, this transition is an example of the here to here this transition is example of roof edge. So, in the range image you have both types of edges and that is why the characterization of these edges are important because they are bit different and their processing should be different.

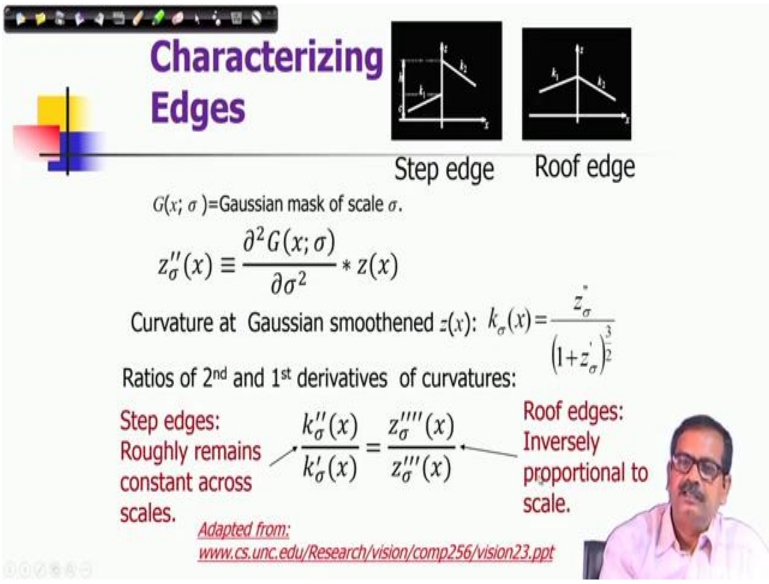
(Refer Slide Time: 11:05)



So, we will be considering to analyze these two cases. Now I will not go into details of this analysis what I will provide you the result of analysis which will provide the summary of data analysis which helps us in developing an algorithm for extracting edges so, the characterizations the properties that I will be describing. So, this is an example how this edges are model you can see the step edge. There is a sharp jump of the functional values a jump of edge that is you can see here the first function which has been shown here as a step edge and this edge is showing this jump.

So, this is the jump which has been shown here. So, it is a certain jump where as for the roof edge you have no jump. So, this is an edge here and there is a continuous gradation continuous decrease of the value in a linear form in a professional form as you move along x. So, with this kind of functional definitions of edges so, this analysis is done with respect one dimensional function.

(Refer Slide Time: 12:31)



So, you can characterize these two situations in this way. Suppose you smooth this particular functions using a Gaussian mask and then you take the double derivative, you can take higher order derivatives including first derivatives second derivatives of this smoothed signal.

So, you are it is possible to compute them because this is one of the techniques or trick so, that is used for computing gradients instead of directly computing gradient. So, we compute gradients in a smoothen signal to handle noise to make it first you smooth the signal . noise should be reduced in that case and then you perform this kind of operations or derivatives. Otherwise furious changes will cause high you know errors in computation of gradients.

Now, there are some advantages of having Gaussian mask because either you can smooth the Gaussian smooth it using Gaussian mask and then take the derivatives or instead you take the derivatives of the Gaussian mask and then apply convolutions to get the result in operations. So, that is how you get the second derivative operations.

So, what is very characteristics in this analysis? They have observe that ratios of second and first derivatives of curvatures they behave differently across scales; that means, as you vary the smoothing factor sigma the scale of Gaussian mask, they have certain invariance properties or they have certain interesting properties that analysis as in shown. So, this analysis is given by I think this is a paper given in this paper.

So, if you would like to go through the details you should read this paper, I will be just describing this particular this results I will be describing here. So, just let me elaborate once again. So, curvature at Gaussians smoothened function is given in this form. it is the usual definition of curvature of a function , one dimensional functional function of a single variable.

$$z_{\sigma}^{''}(x) \equiv \frac{\partial^2 G(x;\sigma)}{\partial \sigma^2} * z(x) \qquad \text{Curvature at Gaussian Smoothened z(x):} \; k_{\sigma}(x) = \frac{z_{\sigma}^{''}}{(1+z_{\sigma}^{'})^{\frac{3}{2}}}$$

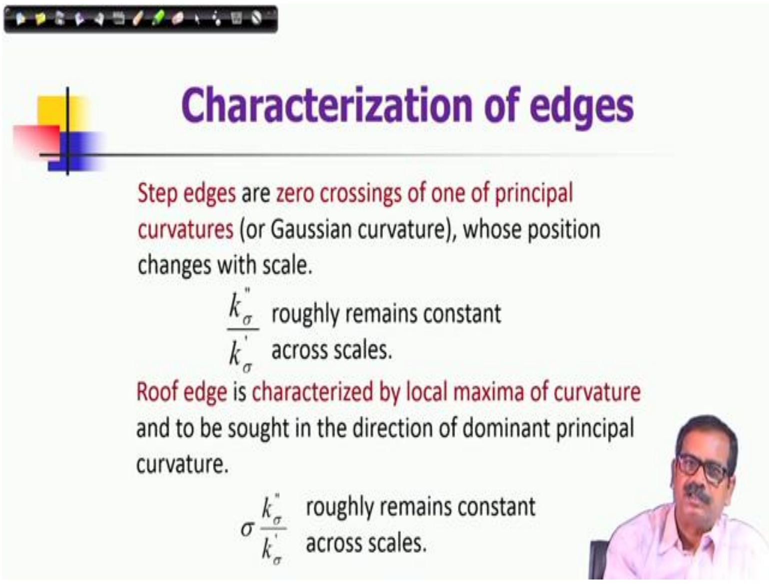
As you can see it is a ratio of second derivative and or other in the numerator there is second derivative and in the denominator we have the 1 plus of first derivative raise to the power of 3 by 2. This is a very standard expression of computing curvature for a function of one single variable.

Now, if I would like to take ratios of second and first derivatives of curvature, then this ratio can be computed also from the functional from the function z it itself as a ratio of fourth derivative and third derivative of curvature. So, this is how a curvature is related with this original function; curvature is proportional to the second derivative. So, second derivative of curvature is proportional to the fourth derivative of function and first derivative of curvature is proportional to third derivative of function.

So, this is how the ratios could be compute it directly from the functions by taking the ratio of fourth derivative and third derivative of the function and there all smoothed across by this scale sigma. So, the characterization of step edge is that this ratio remains roughly same across scales whereas, for roof edges this is inversely proportional to scale. So, when this is there is a real edge point if we observe this properties we can say that edge point is genuine and we can delete spurious edge point which does not you know hold this property.

So, t this is one of the very key findings or key observations from the researchers and they have used it, but; however, to detect the edge points we have to use the curvatures.

(Refer Slide Time: 17:02)

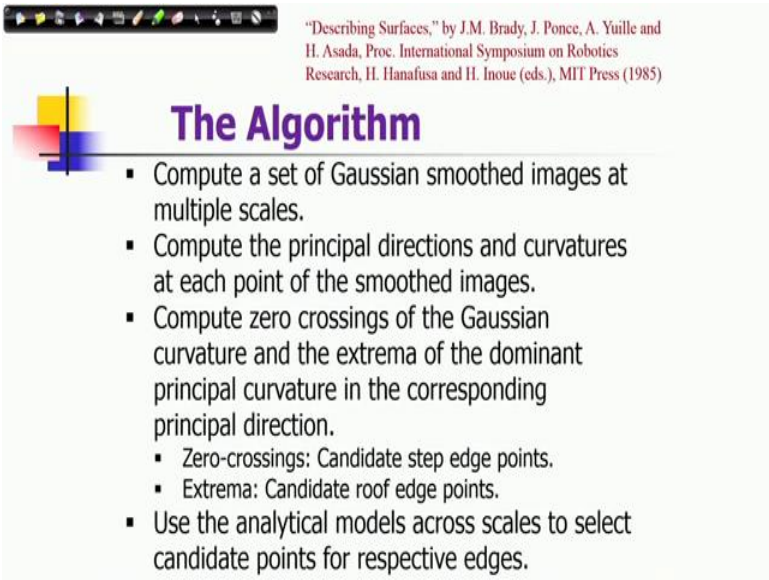


For the step edges it is a zero crossings of one of the principal curvatures that would give you give which will give you a candidate step edge point. And then you observed across scales whether that point preserves the property of invariance of ratios of ratios of those two quantities, what I mentioned that ratios of second derivative of curvature and first derivative of curvature. So, they should be constant. So, that is a thing.

So, this should be roughly remains constant across scale whereas, for roof edge it is characterized by local maximum of curvature and that to be also sought in the direction of dominant principal curvature. So, you should note here that dominant in the sense of magnitude that you have to consider which is the dominant and this roughly remains constant across scales. So, this one; that means, earlier it was inversely proportional to scale.

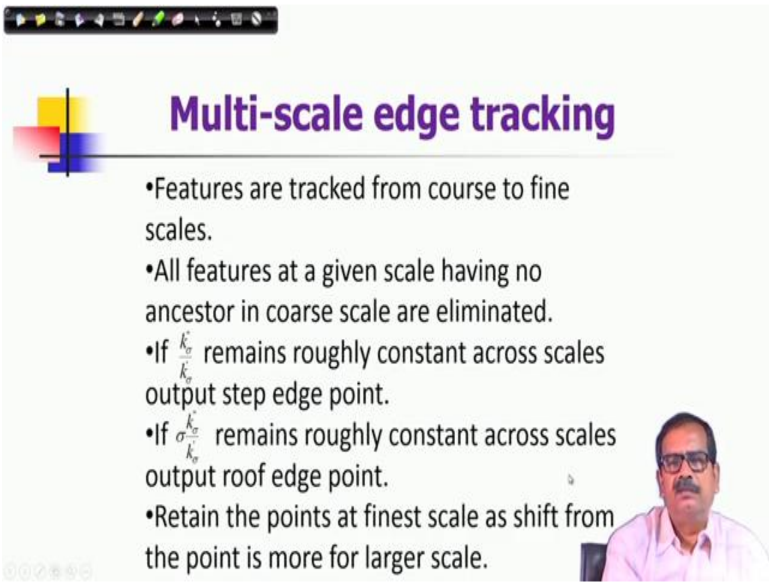
So, if I multiplied with scale so, this should we constant. So, these four know observations can be combined together and can be used in developing an algorithm of edge detections.

(Refer Slide Time: 18:23)



So, the algorithm says that we can compute a set of Gaussian smoothed images at multiple scales, then compute the principal directions and curvatures at each point of the smoothed images compute zero crossings of the Gaussian curvature and the extrema of dominant principal curvature in the corresponding principal direction. So, zero crossings, they would give you the candidate step edge points and extrema would give you the candidate roof edge points. And then use the analytical models across scales to select candidate points for respective edges.

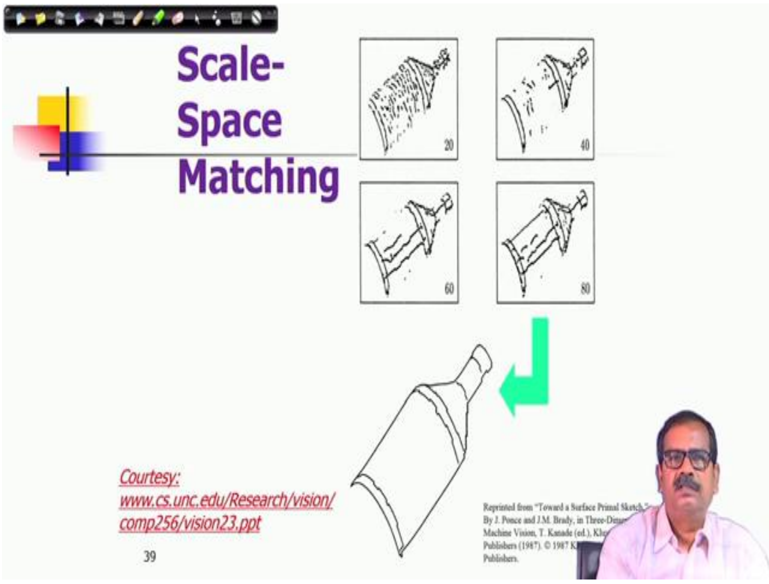
(Refer Slide Time: 19:05)



So, this analysis gives you a mechanism of multi-scale edge tracking. So, what it says that you need to track those edge pixels at different scales of images which means images which are smooth smoothed by different scales of Gaussian functions.

So, if we can observe that still it is retaining at higher scale also this property is retainwhich means the ratios of the curvatures second derivative on first derivative of curvature, they remain constant across scales for step edge point or the scale multiplied by these ratio remains roughly constant across scales output across scales for roof edge point, then we select those points and we retain those points. But those points they locations, they retain which are given at the finest scale because shift has happened due to the smoothening operations.

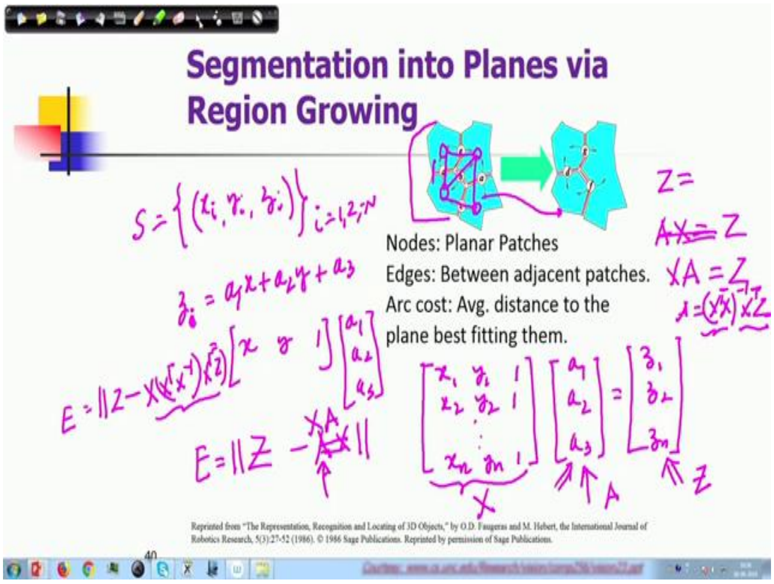
(Refer Slide Time: 20:08)



So, this is one example that is once again the results reported in that vapor. So, it is shown here. You can see that edge points of differently smoothed images are shown here with scale 20, scale 40, scale 60 and scale 80 and as you move across hire scale, you will find that edge points are getting fitted out because those are the points which are still maintaining those invariance properties as we noted.

And finally, when you consider the result at the locations of the at the point then you are simply getting the these are the edge points, but you are getting from this operations.

(Refer Slide Time: 20:57)



So, that was about the detection of edges in range images and now I will consider another operations; another processing with range images which is segmentation of range images. Mostly the segmentation of range images there are different algorithms, but I will be restricting ourselves to consider only segmentation of planar patches. If I consider small planar patches and then integrating those planar patches, we can get large surface from them that is one kind of approach and one of this approach is given here of course, it is considering on a planar facets.

So, what it does that first you will locally fit very small patch with a equations of plane. And then for each patch it forms a node of a graph between all the neighboring patches, you can let me explain it say you have fitted this patches and these are the neighboring patches. So, you consider also the cost of fitting of the total patch. So, this cost we will give you a average distance to the plane best fitting them.

So, this cost of fitting. So, if I merge this, then you have a cost of fitting and that would give you the average fitting. So, let me explain that how a plane could be fitted. See we have already discussed in model fitting. I will simply bring that particular technique here. So, you know that equation of plane can be suppose I have given you what is given as problem here.

You have given here a set of points if I consider them as just a set of points. say set of N points and this is a set ($S = \{(x_i, y_i, z_i)\}_{i=1,2,\ldots N}$)and your objective is to find a plane, fit them with a plane which means a plane can be expressed in the form of equations say $z_i = a_1x + a_2y + a_3$ t that is how a plane could be written. You know that familiar equation of $ax + by + cz + d = 0$. So, I will just expressed that equation in this form where it is a function it is z as a function of x and y which is a usual form of range data.

So, this I can write it in the matrix form as $\begin{bmatrix} x & y & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ and that is equal to z. So, if I have given so, many points I can write a set of linear equations say x 1 y 1, one that should be equal to a 1 x 1 a 2 plus a 2 x 1 plus a 3 should be equal to z 1 x 2 y 2 1 z 2 like this and this is a x n y n one say this is a z n.

$$\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ & \bullet & \\ & \bullet & \\ x_n & y_n & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 \\ \bullet \\ \bullet \\ z_n \end{bmatrix}$$

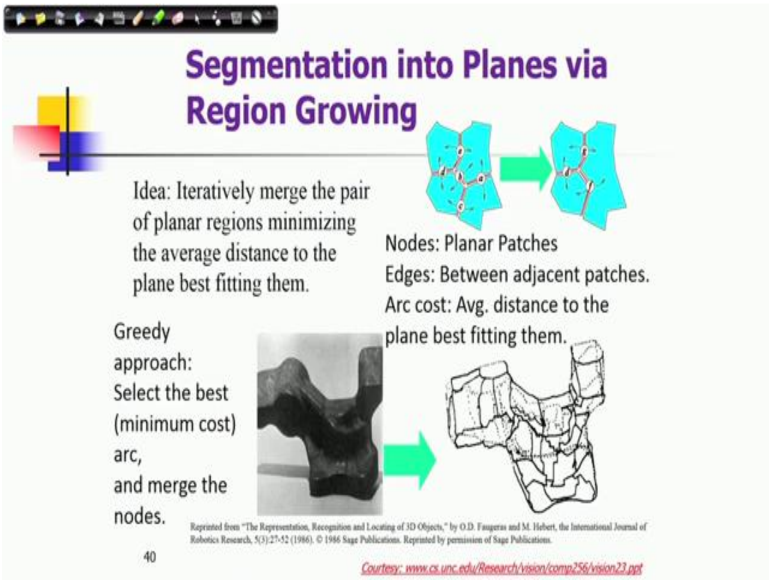
$$XA = Z$$

So, what is your problem your objective is to compute a_1, a_2, a_3 to give a plane equation and what you need to minimize. So, let us consider this matrix is Z this matrix is X and this is A. So, you have to minimize this norm $\|Z - AX\|$, you have to compute find out that A which will minimize this particular norm. This is that least square error estimate and we know that we can solve it using non homogeneous method of solving least square error estimate. And many a time I have used this particular analysis say I will write it as AX = Z and I would like to compute sorry it is not AX, it is XA. I.e., $E = \|Z - XA\|$

So, I will write it as XA equals Z and then A should be equal to X transpose X inverse X transpose Z. $A = (X^T X)^{-1} X^T Z$ So, this is a equation this is a pseudo inverse and the error would be you replace A by this form and you will get the corresponding error. So, error can be computed as so, Z minus X then X transpose X inverse X transpose Z and you can simplify these expressions. So, you will get error. $E = \| Z - X(X^T X^{-1}) X^T Z \|$

So, this error is the arc cost; this error or planar fit is a arc cost between two nodes. So, for every for this graph, you have nodes you have say four nodes and they have corresponding edges all neighboring edges means your fitting them with planes and the arc cost is this one. So, in this way you form a graph and as you can see, it is we will describe it later on that in this method the edge which is having minimum cost that is considered and those nodes are fitted and now you transform this graph into this form. And you continue go on doing this work.

(Refer Slide Time: 28:03)



So, just to summarize this particular operations. So, what we are doing what is the idea is that you iterately the merge this spare of planar regions to minimize the average distance to the plane best fitting them. And we apply a greedy approach, we select the best minimum cost arc and merge the nodes and we do it iteratively. Go and doing it till we find there is no more patches to be mask because there is a threshold of error. We cannot should not merge with the error is too large.

So, this is an example of an image and whose range data is available and if I perform these operations, you will get this kind of planar fits. This is taken from that paper itself which is a representation recognition and locating 3D objects by Faugeras and Helbert and it was also published in 1986. So, this is one technique of segmenting range images. We will continue this discussion I will discuss another technique which is quite fast and effective in the next lecture. So, let me stop here.

Thank you very much for your listening.