

Computer Vision
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Lecture - 35
Color Fundamentals and Processing Part – II

We are discussing about fundamentals of color perception and in the previous lecture we discussed how this perception is a result of interaction of the energy which is being received in our retina, and also the response of the corresponding retinal cells. And this could be factored into three vectors or into three factors, which can identify a particular characteristics of an incident energy or characteristics of the spectral distribution of the incident light in our visual system.

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Standardizing color experience

- To understand which spectra produce the same color sensation under similar viewing conditions.
- Color matching experiments.

(A) Primary lights (R, G, B), Test light (T), Bipartite white screen, Subject, Surround field

(B) Surround field, Primary lights, Test light

Foundations of Vision, by Brian Wandell

So, that is how we experience color. And to standardize this experience; that means, to calibrate with respect to sources and then understand color perception under any kind of color source or any feature or response from any arbitrary color sources that would be our objective next. So, for that, we need to do color matching experiments.

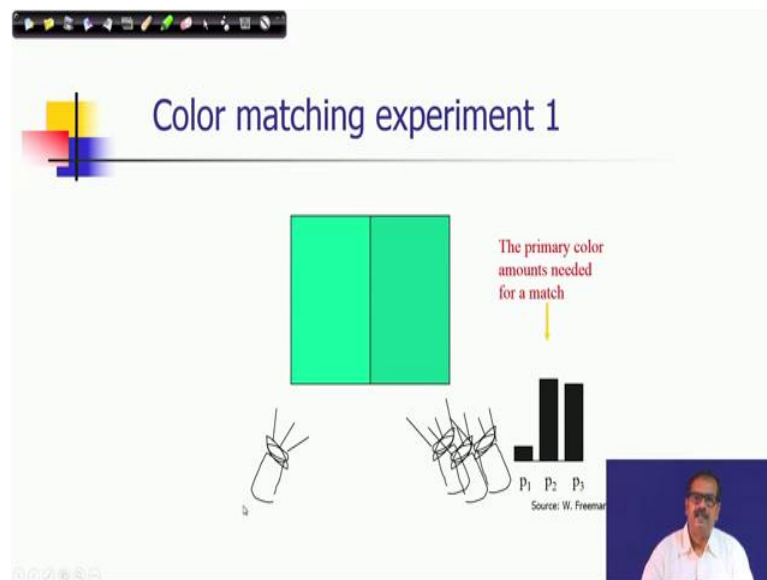
So, let me discuss what is meant by color matching and how these experiments are carried out. So, here is an example of a setup for performing color matching what you can see that, there are three primary sources of lights and of three different wavelengths and there are three pure wavelength sources of lights and we call them primary as we

have seen that a color could be considered as a finally. The color is represented by a combination of three different color components into red, green and blue zone of the spectrum. So, we have chosen such primary color sources in that way and consider a test light which needs to be calibrated with as a factors of these three primary sources, that test light is T.

And if I display this test light, this is once again a say Pure monochromatic color source of a particular wavelength. And if I display it on a screen and the screen is designed in this way there is a circular zone and here in that screen and there is of course, a surrounding field to provide you the contrast and the test light is projected here and this part is the combination of this red, green, blue, that is projected here.

So, you can vary the relative you know strength of this energy sources and you can note down that what in what proportion they are mixed and they are superimposed here. So, that proportion itself could represent the same test light as a factor of these primary colors. So, this is known as color matching. So, you have to vary the proportion in such a way that these two colors there should not be any distinction between them.

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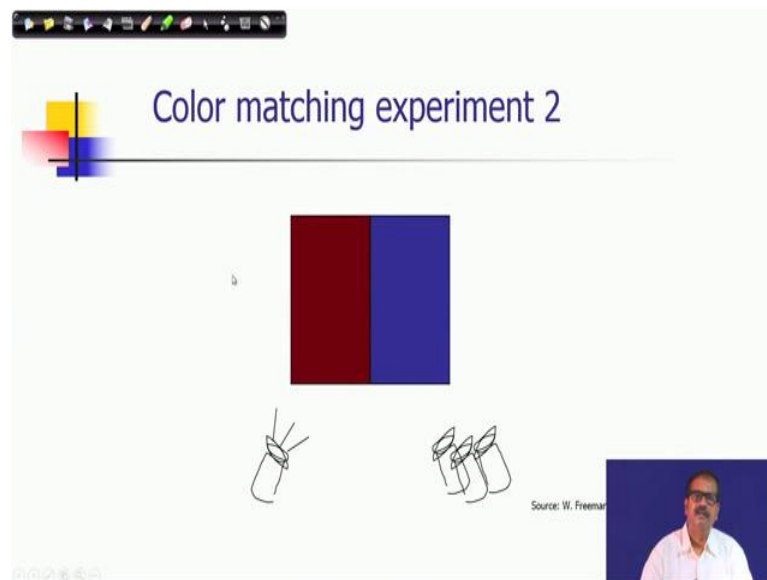


So, we can describe an hypothetical color matching experiment diagrammatically once again it is showing the same setup what I described earlier. So, this is that test light and if I project it for example, it produces a color like this. consider there are three primary sources of red, green, blue and a certain combination produces this. Say this is the

combination of these primary sources, it shows the relative proportion of the amount of energy emitted by these primary sources and they are superimposed here and that is the color.

Now, if I vary one of them for example, p_2 I could see that this color is going near to this throughout visual sensation toward visual observation and further increase may cause exact matching of this two. So, then we say the, relative distribution of these three primary colors, is representing this particular color. That is how we have standardized the representation of this color with respect to these primaries through this experiment. And so, this is the primary color amounts needed for a match. So, consider repeating this experiment for every kind of light sources of different wavelengths.

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There is another scenario that also I should explain here and this is we are considering a type two scenario of this experiment. Here you can see that this is a color of the test light of a particular wavelength and again this is the color produced by the superposition of different primary colors, in some certain proportions.

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Color matching experiment 2

P_1 P_2 P_3

Source: W. Freeman

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Color matching experiment 2

P_1 P_2 P_3

Source: W. Freeman

Consider this is a proportion which is producing, as you can see that the primary p_2 is a very small amount and in fact, if we further reduce it we will find that this color becoming nearer to this one, but still it is not the same now in fact, what we can do there. Consider you add instead of projecting primary from here now, it is add primary from this part.

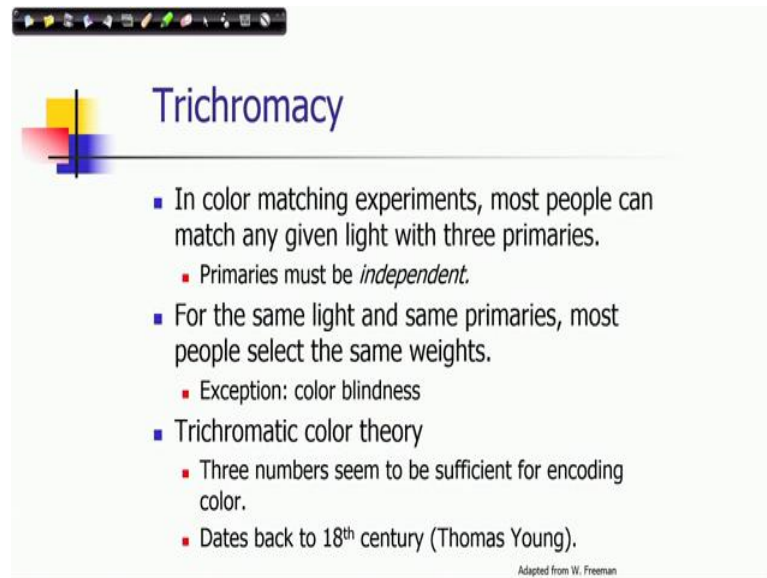
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The slide is titled "Color matching experiment 2". It features a central red square. To the left of the square, text reads: "We say a 'negative' amount of p_2 was needed to make the match, because we added it to the test color's side." Below this text is a diagram of two color mixing tubes. To the right of the square, text reads: "The primary color amounts needed for a match:". Below this text is a bar chart with three bars labeled p_1 , p_2 , and p_3 . The p_2 bar is significantly shorter than the p_1 and p_3 bars. Below this bar chart is another diagram of two color mixing tubes. At the bottom right of the slide is a small video inset of a man speaking. The source is cited as "Source: W. Freeman".

Which means, say if I add a primary here then we can see that these two colors they become same, which means that as if you are subtracting the p_2 from these combinations p_1 and p_3 these proportions. And then you are subtracting p_2 with this proportion from this combination, then it is producing the same sensation.

So, we say negative amount of p_2 as needed to make this match and this is the another kind of experiment where you see that it is not only additive superposition, but also even the negative matching, negative additions from the other side. theoretically you can get the equivalent sensation in this way and you can obtain a figure obtaining proportions like this. So, you can represent in this in this fashion that this p_2 in it needed to be subtracted from p_1 and p_3 . So, in the color matching experiments you note down all such proportions including their addition or subtraction for every wavelengths you can get a chart art for different colors for, different wavelengths and that would give you the color check.

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Trichromacy

- In color matching experiments, most people can match any given light with three primaries.
 - Primaries must be *independent*.
- For the same light and same primaries, most people select the same weights.
 - Exception: color blindness
- Trichromatic color theory
 - Three numbers seem to be sufficient for encoding color.
 - Dates back to 18th century (Thomas Young).

Adapted from W. Freeman

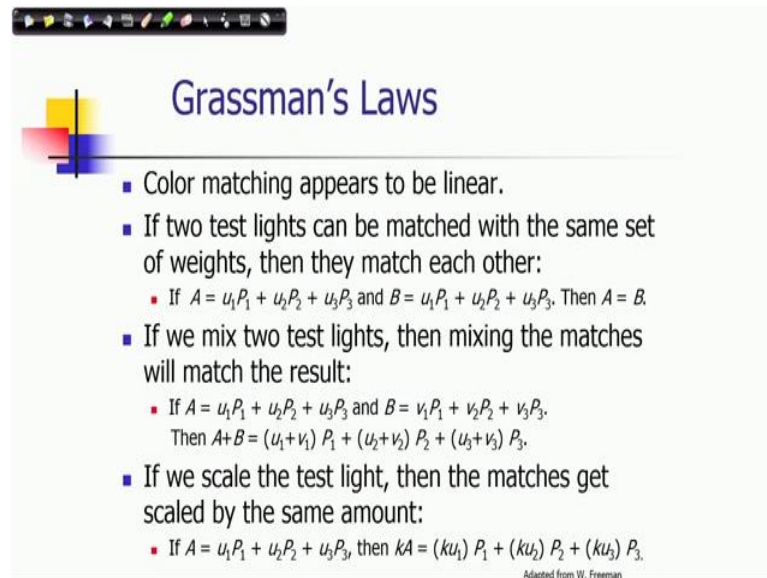
So, by standardizing this chart, we can have a trichromatic representation of any color which means using three primary colors and this feature of color representation is known as trichromacy.

So, it is the fact that most people can match any given light with three primaries and primaries must be independent which means that, none of the ; two other color with addition or subtraction whatever may be that with any proportion that should not be able to produce the particular color. So, that is how the independent, it is this definition is the analogy is with that linear independence relationships among the vectors. And for the same light and same primaries most people select the same weights that is also observed experimentally.

So, that is how the uniqueness of color representation across our perceptual systems of the varying human visual perceptual system so, that unique representation is established through this kind of empirical studies. Of course, there would be exceptions when there are color blindness. So, there would be different kinds of perceptions for that.

So, we are talking about normal vision and there it is an uniquely represented given primary with that same proportion to represent a color. And this gives you the trichromatic color theory, which means that three numbers seem to be sufficient for encoding color, it is a very old theory it dates back to 18th century it is proposed by Thomas Young.

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A presentation slide titled "Grassman's Laws" with a decorative graphic of overlapping colored squares (yellow, red, blue) and a black crosshair. The slide contains a bulleted list of three laws of color matching. The first law states that color matching is linear. The second law states that if two test lights can be matched with the same set of weights, they match each other, and provides the equation $A = u_1P_1 + u_2P_2 + u_3P_3$ and $B = u_1P_1 + u_2P_2 + u_3P_3$, then $A = B$. The third law states that if two test lights are mixed, the matches will match the result, and provides the equation $A = u_1P_1 + u_2P_2 + u_3P_3$ and $B = v_1P_1 + v_2P_2 + v_3P_3$, then $A+B = (u_1+v_1)P_1 + (u_2+v_2)P_2 + (u_3+v_3)P_3$. The fourth law states that if a test light is scaled, the matches get scaled by the same amount, and provides the equation $A = u_1P_1 + u_2P_2 + u_3P_3$, then $kA = (ku_1)P_1 + (ku_2)P_2 + (ku_3)P_3$. The slide is adapted from W. Freeman.

Grassman's Laws

- Color matching appears to be linear.
- If two test lights can be matched with the same set of weights, then they match each other:
 - If $A = u_1P_1 + u_2P_2 + u_3P_3$ and $B = u_1P_1 + u_2P_2 + u_3P_3$, Then $A = B$.
- If we mix two test lights, then mixing the matches will match the result:
 - If $A = u_1P_1 + u_2P_2 + u_3P_3$ and $B = v_1P_1 + v_2P_2 + v_3P_3$. Then $A+B = (u_1+v_1)P_1 + (u_2+v_2)P_2 + (u_3+v_3)P_3$.
- If we scale the test light, then the matches get scaled by the same amount:
 - If $A = u_1P_1 + u_2P_2 + u_3P_3$, then $kA = (ku_1)P_1 + (ku_2)P_2 + (ku_3)P_3$.

Adapted from W. Freeman

And as we can see that there are a lot of similarities with respect to the linear space and also the color space. So, Grassman proposed this laws of color matching using this property of linearity. So, it is considered that it is a linear property and which means that if two test lights can be matched with the same set of weights, then they match each other. Say you consider there are three primaries p_1, p_2, p_3 and this is the amount to which is needed to produce A color a say u_1, u_2, u_3 there is a representation. $A = u_1P_1 + u_2P_2 + u_3P_3$.

And if there is another color which is also represented by the same values same proportions and same values actually u_1, u_2, u_3 then these two colors must be same. And if we mix two test lights and mixing the matches will match the result which means once again you get two colors A and B and having their different proportions of primaries and if I superimpose, the color what you get that would be represented by this factor. So, their property of linearity is also that property holds during the super positions of two colors also.

this property also tells you that if we scale the color scale the test light, then also the matches get scaled by the same amount. we see the similar kind of color, but intensity varies with that color. So, these are the three features of Grassman and three laws of Grassman which which tells that know a color could be represented in a linear space.

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The slide is titled "Linear color spaces" in blue text. On the left, there is a small graphic showing overlapping colored squares (red, yellow, blue) and a vertical line. On the right, a diagram shows a red dot and a blue dot connected by a straight line. Below this, text reads: "mixing two lights produces colors that lie along a straight line in color space." In the center, there is a bulleted list:

- Defined by a choice of three primaries
- The coordinates of a color are given by the weights of the primaries used to match it.
- *Matching functions*: weights required to match single-wavelength light sources.

At the bottom left, a diagram shows a triangle with vertices colored red, blue, and green. To its right, text reads: "mixing three lights produces colors that lie within the triangle they define in color space." In the bottom right corner, there is a small video inset of a man with glasses and a white shirt. The slide also features a standard presentation navigation bar at the top and a small "Adapted from" logo at the bottom right.

So, that is how the linear concept of linear color spaces they have been considered and it is defined by a choice of three primaries. So, there could be different color spaces given different sets of three primaries, because we will get different vectorial representations So, RGB is one common choice, but there could be different other primary choices.

So, the coordinates of colors are given by the weights of the primaries used to match it that is the representation and we need a matching functions; that means, we need these know standardization. So, a monochromatic light source with respect to this coordinate representation- three vector representation of a color.

And then we can apply this linear superpositions principles suppose you would like to have a mixing of two colors. So, if we represent them by that three vectors and any color if you consider a linear combination of these two, they will be represented by a color which could be also represented as a point in this particular straight line.

And if I consider three primaries and any color any combination of this color can be represented by a point in this particular triangle. So, these are all positive weights or the some of the proportion sum of the weight should be equal to 1. it is a convex combination we are considering and then the point should lie in this case within this triangle.

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How to compute the color match for any color signal for any set of primary colors

- Pick a set of primaries, $p_1(\lambda), p_2(\lambda), p_3(\lambda)$
- Measure the amount of each primary, $c_1(\lambda_0), c_2(\lambda_0), c_3(\lambda_0)$ needed to match a monochromatic light, $I(\lambda_0)$ at each spectral wavelength λ_0 (pick some spectral step size). These are the color matching functions.

Source: W

So, we have described this experimental techniques that how to compute the color match for any color signal for any set of primary colors. So, now we will be considering the representation of a color signal because as you see that so, far we have represented light of a monochromatic wavelength which means a pure color and then you have represented by the three vectors.

But again the light could be considered this color or the color signal could be considered as linear combinations of different wavelengths at different proportions. So, for each wavelength we can have its representation from the color chart; that means, this is the three vectorial representations say for a λ , this is the amount for primary p_1 , amount for primary p_2 and amount for primary p_3 . And so, we can measure the amount of each primary needed to match a monochromatic light.

So, in this way we can consider that whatever relative amount of energy is there in the spectrum of a light source, in the same proportions all these three vectors needed to be mixed and that would represent this color signal. So, color signal would be also finally, represented by a three vector form.

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Using color matching functions to predict the matches for a new spectral signal

A monochromatic light of λ_i wavelength will be matched by the amounts $c_1(\lambda_i), c_2(\lambda_i), c_3(\lambda_i)$ of each primary.

And any spectral signal can be thought of as a linear combination of very many monochromatic lights, with the linear coefficient given by the spectral power at each wavelength.

$$\vec{i} = \begin{pmatrix} I(\lambda_1) \\ \vdots \\ I(\lambda_N) \end{pmatrix}$$

Color signal

Relative energy

Wavelength (nm)

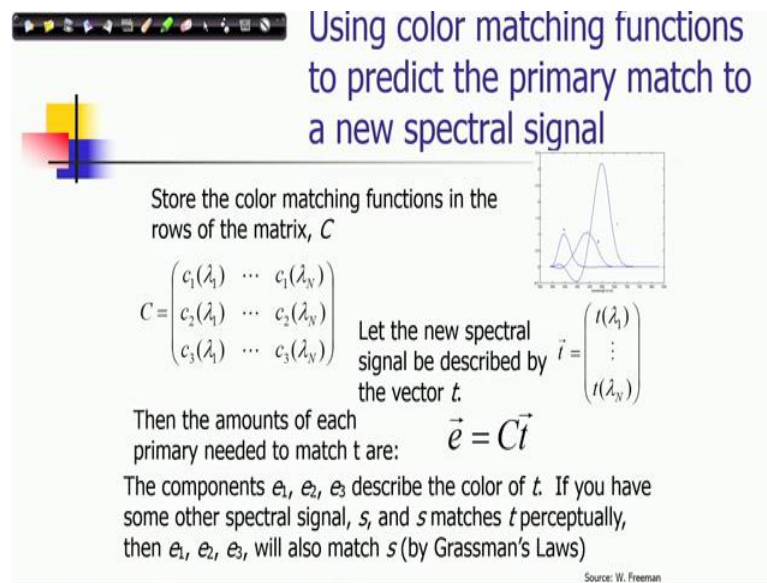
Source

And using this linear superposition principle we can compute them, if we know the corresponding color responses are three vectorial standard representations of these wavelengths so, that is what we will be describing here. So, a monochromatic light of λ a wavelength, it could be matched by the amount say c_1 , c_2 and c_3 for each primary and consider any spectral signal, it is thought of as a linear combination of so, many monochromatic lights and this linear combination is can be represented in terms of a vector.

So, this is the amount of the energy which is there in the wavelength λ_1 in that light source and consider there are wavelengths from λ_1 to λ_n . we can discretize the range of the wavelengths and also use the amount of energy for each wavelength which is present in that color signal. So, this is a representation of the color signal in terms of spectral power distribution. So, this is what is spectral power distribution and that is discretized in this form.

So, now, for each one, once again λ_1 is represented by the amount of primary color. So, it is normalized representation. So, if I multiply. So, each one should be multiplied by this $c_1 \lambda_1$.

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Using color matching functions to predict the primary match to a new spectral signal

Store the color matching functions in the rows of the matrix, C

$$C = \begin{pmatrix} c_1(\lambda_1) & \dots & c_1(\lambda_N) \\ c_2(\lambda_1) & \dots & c_2(\lambda_N) \\ c_3(\lambda_1) & \dots & c_3(\lambda_N) \end{pmatrix}$$

Let the new spectral signal be described by the vector t .

$$\vec{t} = \begin{pmatrix} t(\lambda_1) \\ \vdots \\ t(\lambda_N) \end{pmatrix}$$

Then the amounts of each primary needed to match t are: $\vec{e} = C\vec{t}$

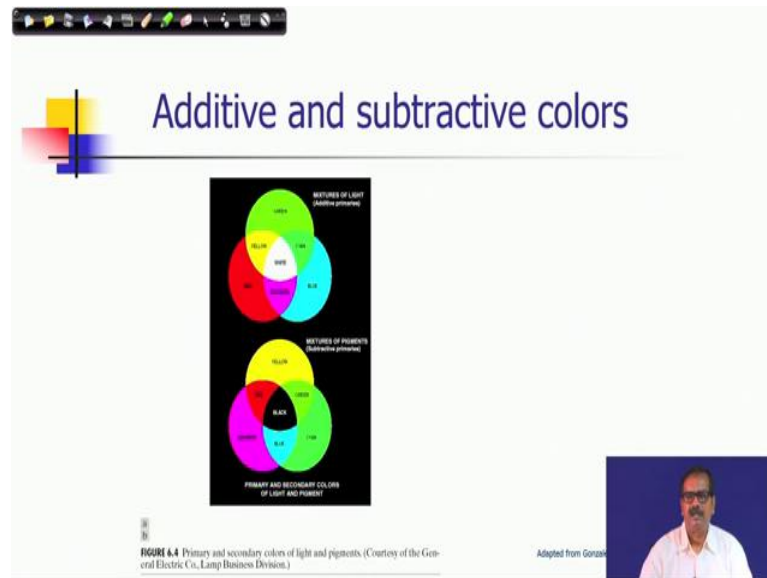
The components e_1, e_2, e_3 describe the color of t . If you have some other spectral signal, s , and s matches t perceptually, then e_1, e_2, e_3 , will also match s (by Grassman's Laws)

Source: W. Freeman

So, we can consider this, say they store the color matching functions and then this is the representation of the color signal and now as I was mentioning you multiply this matrix with this particular combination. So, this matrix(C) is a $3 \times N$ as you can see and this (\vec{t}) is $N \times 1$.

So, finally, what you get? You get 3×1 vector for each energy, each component you will get for example, $t(\lambda_1)$ will give you $c_1(\lambda_1) \times t(\lambda_1)$ for the c_1 component part of it and then $c_2(\lambda_1) \times t(\lambda_1)$ for this c_2 component and $c_3(\lambda_1) \times t(\lambda_1)$ for the c_3 component. And for every wavelength you are finding out these components and then again add them component twice that is what it is simple matrix multiplication that is what it is explaining. So, in this way you can apply Grassman's law to get the representation of a color signal by three vectorial form or three components.

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So, we will be considering now production of or renditions of colors in the technology and you can see that now there are two different types of technologies available. As we have discussed that a color could be represented as a superposition of primary colors now this superposition does not mean always an additive mixing, there could be also subtractive component.

But the problem is that even in today's technology we cannot you know have both the properties either the principle should be on additive mixing, principle of the technology or it could be only on subtractive principles. So, there are two types of systems for color renditions either it is an additive system or it is a subtractive system.

So, this slide particularly it is showing that this is a additive system, where the red, green, blue color, they the primary colors and a different proportions these colors when they are added all of them are positively added there is no subtraction, then you produce different kinds of colors like you can have yellow here, you can have cyan, you can have the magenta color here.

But on the other hand in the subtractive system primary colors could be magenta yellow and cyan, and which means that in the magenta the red color gets absorbed and then you have the sensation. So, it is absorption of red color, but which will produce a sensation of yellow, absorption on green which will produce the sensation of; of magenta and this is

a sensation of yellow. And the absorption of blue which will produce you the sensation of sand and using them you can produce different kinds of colors.

So, this principle is not you know subtractive principle by which these colors are produced. So, example of this kind of system is the printing system. So, when you print colors on white pages, we use a subtractive principle whereas, when we you display colors in a computer monitor or any display screen or any cathode ray tube using just normal televisions. So, there we work with the principles of additive superposition.

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Linear color spaces: RGB

- Primaries are monochromatic lights (for monitors, they correspond to the three types of phosphors).
- *Subtractive matching* required for some wavelengths.

RGB matching functions

$p_1 = 645.2 \text{ nm}$
 $p_2 = 525.3 \text{ nm}$
 $p_3 = 444.4 \text{ nm}$

The slide features a color bar on the left, a graph of RGB matching functions on the right, and a small video inset of a man in the bottom right corner. The graph shows three curves representing the spectral sensitivity of the human eye to red, green, and blue light, with peaks corresponding to the primary wavelengths listed.

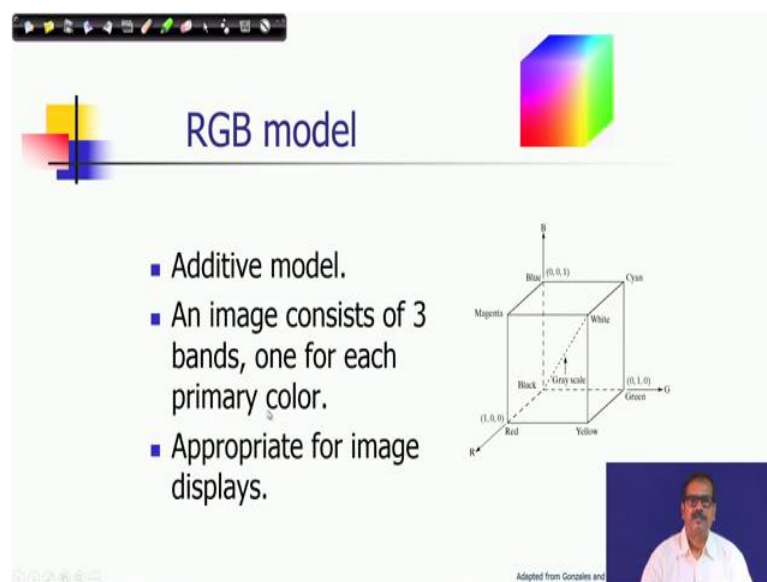
And we can talk about know different colored spaces linear color spaces, the common color space is an RGB color space because when we captured the color information in cameras, this is the space in which we capture the color information of a point. Because we use red filter, green filter and blue filter in the camera when the optical energy is incident on a lens before that we apply those filters. So, you get only the energy in the red zone or green zone or blue zone of the spectrum accordingly, and then these three components are independently captured for the same light source for the same incident energy and that is how a color is represented.

And that space is a natural red, green, blue space or RGB space and in the RGB matching function if I show it, over as a function of wavelength which has been displayed here, you can see also the corresponding wavelength which is representing red, green and blue which is shown here.

So, one of the thing you should observe here that it is not always additive mixing for every wavelength. For some wavelength for example, you consider this red matching response for some wavelengths it is subtracting. So, that is why in the RGB color space every color cannot be produced by only additive color.

So, that is one kind of property and that is the reason why every display additive display cannot produce all sorts of colors, there are set of sets of colors which cannot be rendered, because they require subtraction and which is not possible by the technology what is giving that perception or producing that color for our perception. So, this is the thing we should note that for some wavelengths you record subtractive matching.

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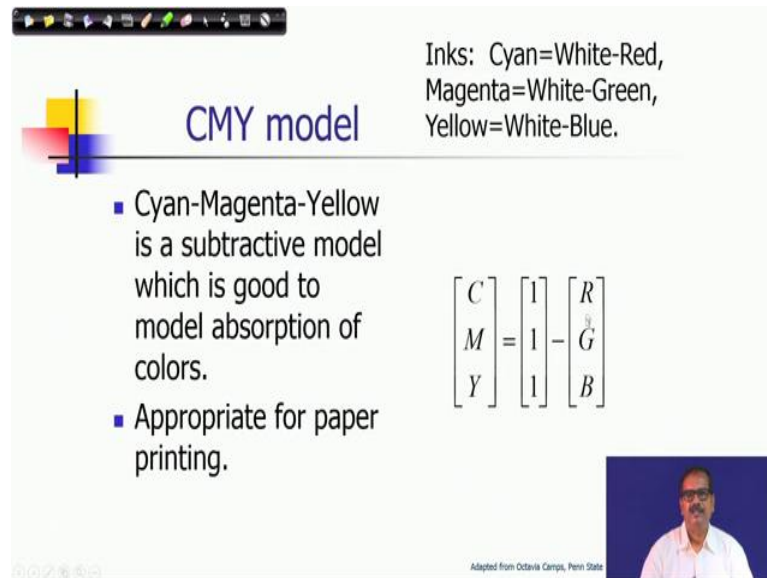
The slide features a title 'RGB model' with a small graphic of overlapping colored squares. To the right is a 3D color cube. Below the title is a bulleted list:

- Additive model.
- An image consists of 3 bands, one for each primary color.
- Appropriate for image displays.

To the right of the list is a 3D coordinate system with axes labeled R (Red), G (Green), and B (Blue). The origin is labeled 'Black'. The axes are labeled with coordinates: (1, 0, 0) for Red, (0, 1, 0) for Green, and (0, 0, 1) for Blue. Other points on the cube are labeled: White at (1, 1, 1), Yellow at (1, 1, 0), Cyan at (0, 0, 1), and Magenta at (1, 0, 1). A diagonal line from the origin is labeled 'Gray scale'. At the bottom right, there is a small video inset of a man speaking and the text 'Adapted from Gonzalez and'.

So, just to summarize an RGB model could be represented in this normalized form where you can consider 1 is the highest intensity values and 0 is the lowest intensity value and then you can represent every vector within that normalization. And it is an additive model an image consists of three bands one for each primary color and it is appropriate for image displays.

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Inks: Cyan=White-Red,
Magenta=White-Green,
Yellow=White-Blue.

CMY model

- Cyan-Magenta-Yellow is a subtractive model which is good to model absorption of colors.
- Appropriate for paper printing.

$$\begin{bmatrix} C \\ M \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

Adapted from Octavia Camps, Penn State

Whereas the other subtractive model the primary colors are cyan, magenta and yellow and can see that they are actually complementary colors of red in this way the complementation is explained here.

$$\begin{bmatrix} C \\ M \\ Y \end{bmatrix} = 1 - \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

So, you can see also. If I represent the red, green, blue primary colors those components, if I normalize them within the range 0 and 1 corresponding cyan, magenta and yellow also can be represented from there for that color in the subtractive model. Because in the subtractive model what you are doing. If I produce this amount of cyan means this amount of red will be absorbed in this particular treatment.

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CIE chromaticity model

- The Commission Internationale de l'Eclairage (estd. 1931) defined 3 standard primaries: X, Y, Z that can be added to form all visible colors.
- Y was chosen so that its color matching function matches the sum of the 3 human cone responses.

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.6067 & 0.1736 & 0.2001 \\ 0.2988 & 0.5868 & 0.1143 \\ 0.0000 & 0.0661 & 1.1149 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1.9107 & -0.5326 & -0.2883 \\ -0.9843 & 1.9984 & -0.0283 \\ 0.0583 & -0.1185 & 0.8986 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Adapted from Ottawa Camps, Penn State

And this is appropriate for paper print. There is another standard color space and which has been provided by the chromaticity model and which has been provided by a standard body of international body of color which is known as here that commission internationally de l'Eclairage, I mean this is in the French pronunciation my pronunciation is not correct.

So, we call it in short CIE, I will refer it to that. So, international body of color and this is established in 1931 and it defines three standard primary colors the objective is that in this space you do not require any subtraction, you can produce all color using addition, but the thing is that it is a hypothetical space. So, you cannot have light sources which will produce this kind of primary colors. But, mathematically we considered that these are the three primaries, three hypothetical primary colors. And in this case Y component was chosen such that the color matching function matches the some of the three human code responses.

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.6067 & 0.1736 & 0.2001 \\ 0.2988 & 0.5868 & 0.1143 \\ 0.0000 & 0.0661 & 1.1149 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

So, the example is that, it is a, linear transformation of any vector in RGB space that gives you X Y Z now this is a primary color. So, X is this combination of RGB will produce this X and this combination of RGB will produce Y and this combination of

RGB will produce Z as you can see that this factor is more than 1. So, it is not possible to produce because we have normalized all RGB component to 0 to 1.

So, this primary color is not possible to produce all these primary colors not possible to produce physically, but mathematically this transformation is defined. And now the advantage of this transformation is that : use the same matching functions with the matching experiments whatever you have with RGB, if I transform all those matching functions using this transformation you will find all of them becomes positive.

All the RGB components, even some of them is negative in the matching function due to our experiments, but after transformation you will find all of them becomes positive in this case. You should note that this transformation is an invertible transformation and as you can transform an RGB vector to XYZ space. So, we call this space as CIE XYZ space or simply XYZ color space, you can again transform it back to RGB space.

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1.9107 & -0.5326 & -0.2883 \\ -0.9843 & 1.9984 & -0.0283 \\ 0.0583 & -0.1185 & 0.8986 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

So, inverse transformation is shown in the above fashion, and you can see that this is the matrix and your corresponding XYZ component is shown here and if I multiply this with RGB, if I multiply this with XYZ, then you can get the corresponding RGB component. So, let me give a break here for this in this lecture. So, we will continue this topic of discussing about color fundamentals and later on processing with color images in our subsequent lectures.

Thank you very much for your attention.