Computer Vision
Prof. Jayanta Mukhopadhyay
Department of Computer Science and Engineering
Indian Institute of Technology, Kharagpur

Lecture – 31
Feature Matching and Model Fitting Part - III

We are discussing about Feature Matching and Model Fitting. In the last two lecture lectures, we discussed about the techniques for matching feature vectors and also efficient computation of that problem. In this lecture, we will be considering the issue on model fitting.

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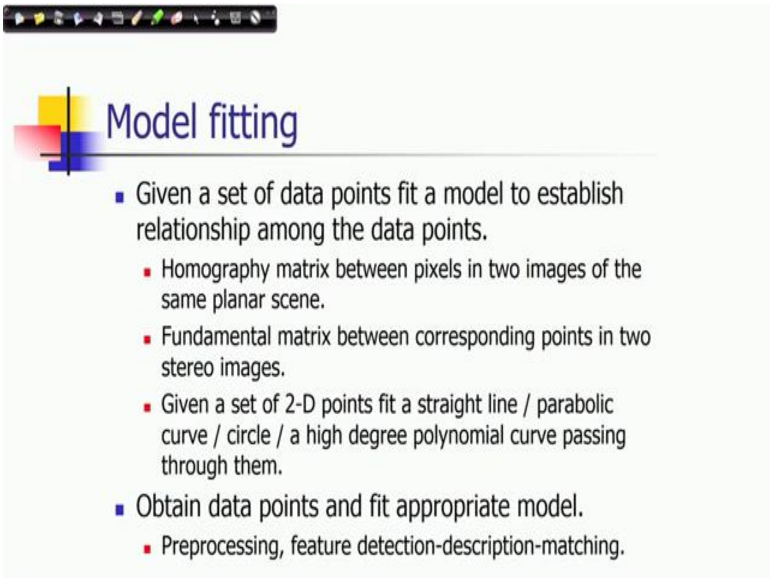

So, the computational problem for this fitting model is that given a set of data points we need to fit a model to establish relationship among the data points. For proposing the compression problem first we need to have a set of data points. So, we should obtain data points and then we need to fit an appropriate model to explain those data points. As we mentioned to establish relationship among the data points we need to fit an appropriate model.

So, in the last few lectures we discussed how we can obtain this data points by detecting feature points and then matching feature points in multiple views in different images, you can establish correspondences and there are various problems on which this correspondences are to be required for fitting a model. Some of the examples could be, these are also we have discussed in our previous topics. For example, if that between corresponding pixels in two images there exist this homography, then how to compute homography matrix. That itself is a model fitting problem. We have already discussed it solution. So, in this lecture we will consider a bit more general issues involved in this kind of computational problem.

Another example could be again computation of fundamental matrix between corresponding points in two stereo images. Even if I give you a set of 2D points how can you fit a straight line or parabolic curve or circle or a high degree polynomial curve passing through them. There has to be some knowledge about this kind of train or relationships about the model and using that knowledge only you try to get the precise model visual with this points.

For obtaining data points we need to apply various image processing techniques with respect to data points from generated from an image from images, like there are various pre-processing techniques and feature detection description matching that we have already discussed in previous lectures.

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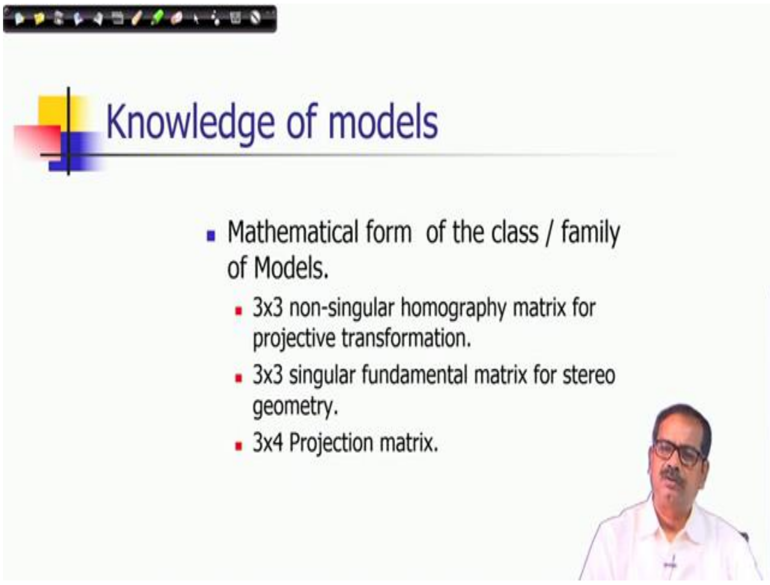

So, as I was referring that knowledge of models is crucial for defining a computational problem. What should be the mathematical form of the class or family of models? For example, we know the mathematical form of a homography relationships among the image points between two sins, between two scenes or images of the same three-dimensional scene, we know that for a two-dimensional projective transformation the only form of transformation matrix should be in the form of a 3 X 3 non-singular matrix. So, using that knowledge then we have solve that problem that we have already discussed in a previous topic.

Similarly, the form of fundamental matrix in a stereo geometry and its role that has that we know and applying those relationships, we can derive a fundamental matrix from the set of corresponding points. We should have the knowledge of the structure of that fundamental matrix, its properties that it is a singular matrix and also how this matrix relates the corresponding points that request to be used while fitting a model.

Projection matrix of a camera, its form also we have seen how this from could be derived, in the form of a 3 X 4 matrix, where it maps a three dimensional sin point to an image point in the projective space or in the homogeneous coordinate systems. So, finding out or providing a set of corresponding points between three-dimensional sin points and there corresponding image points, we can derive a projection matrix by using this model fitting techniques. This also be discussed in a previous topic.

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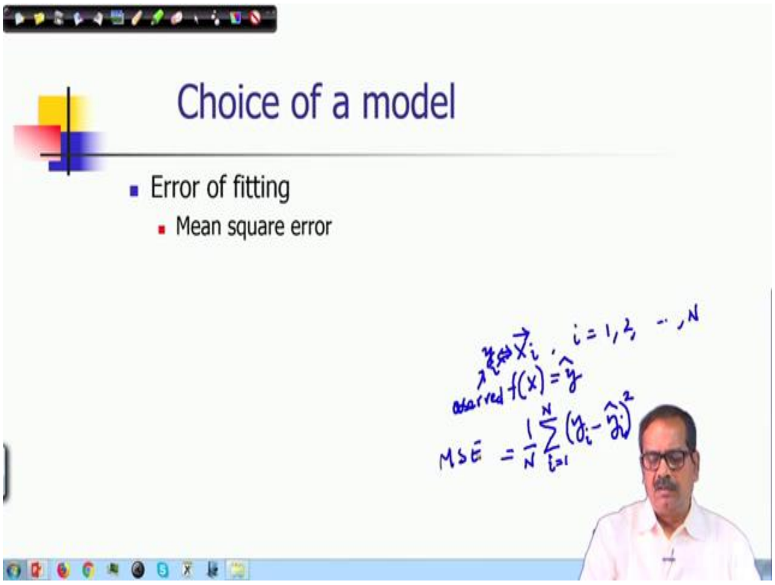

So, choice of a model is very important. It comes from the knowledge or analysis of a particular system or analysis how the data are generated, how the data has been obtained, from there you can get that information and you can choose an appropriate model. But there are situations where you may not be able to precisely define a structure of model.

So, you may have to guess even there are also some model some intuitive grace some intelligent grace we can make, but still precisely you do not know how many number of independent parameters that would take, that would play into that model. So, how to ascertain that when you choose a model using some kind of intelligent and intuitive grace work. How that model is appropriate in your data fitting?

So, there are certain checks and bounce for that. Particularly, you need to consider the error of fitting in that case. There are various kinds of errors which were used. We have also used and mean square error between the predicted value and the observed value in our previous model fitting techniques of fitting homography matrices or fundamental matrices or projection matrices. So, similar mean square error could be, similarly we can define mean square error in various other contexts.

Just to explain that what is meant by mean square error though it may be very clear to you through our different exercises, just to elaborate that fact. Suppose, you have some data, let me consider your measurement is a scalar quantity which is y and which depends upon a feature vector. So, corresponding to a feature vector you have a measurement y and you postulated that there exist a functional relationship between the feature vectors and also your measurement that is the model you would consider. You, suppose the form of these functions and apply different techniques to derive this function.

So, now there are when you actually apply a model, then the value what you get that is a model predicted value and this is the value which is the observed value, this is an observed data. So, the error between observation and prediction could be define as square error; the difference if I since I have assumed in this particular case this is a scalar value, so I can simply take the difference. You can extend this concept; when your observation is also a vector and you can consider norm of the differences in that case.

However, so mean square error is that you have so many observation. Suppose, you have n observations and I represent mathematically this pair in this form that means, there are n observations. So, in that case mean square error would be for each observation I will find out the corresponding predicted values which is shown in this form. So, I will sum all these square deviation, square of the errors and take the mean that is what the mean square error is.

$$MSE = \frac{1}{N}\sum_{i=1}^{N}(y_i - \overline{y_i})^2$$

In fact, you have used this error in obtaining homography matrix, fundamental matrix or projection matrix in previous computational problems, there mean square error is expressed in terms of norm of a difference vector, mean of the norm of difference vectors. Let us now consider other aspects. This is one particular example of mean square error.

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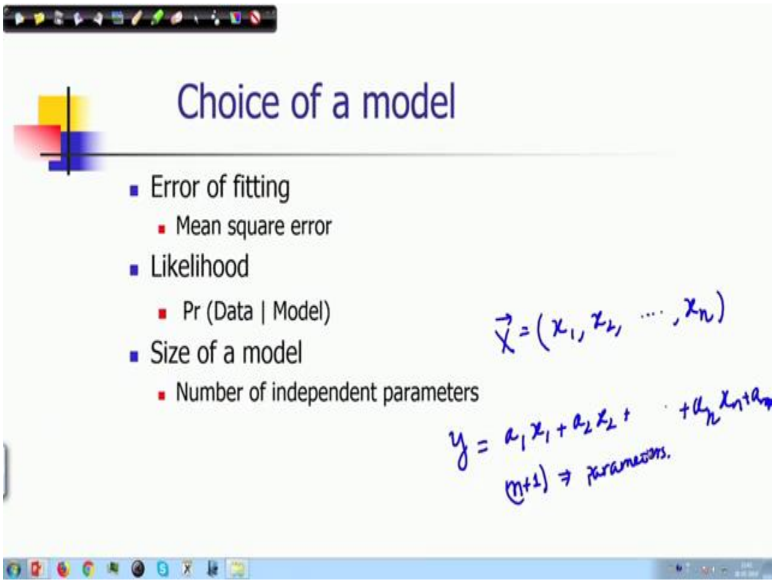

We can also evaluate the strength of a model by computing likelihood of data given a model, which is defined in this form. What is the probability of occurrence of data given a model? So, this measure should be high, as you know it is a probabilistic measure, so the probability value should lie between 0 to 1 and if it is a very high value your model fitting is good that is one kind of evolution of models. And that determines how good is a model in or when you have a competitive models if we can compute their likelihoods their relative ratios can provide you which model should be consider. There are theories for that, I am just providing in the intuitive reasoning in this case.

Size of a model is important. So, in brief we can say a size of a model is determined by the number of independent parameters those are involved in a model. That is just a very short way of defining a size. If you have more number of independent parameters your module is more complex, that is the rough idea. So, you have to choose a particular size.

Suppose, you have simple linear relationships between the observed between your measurements and also the feature vectors, observed feature vectors and in a feature vector suppose there are n components. So, in that case in a linear model we can write the form of

$$y = a_1 x_1 + a_1 x_1 + \cdots + a_n x_n$$

Suppose, your feature vector is n dimensional element, it is represented in an n dimensional space in this fashion and you can also use a constant here, you can use that also in your model. So, we can see in this linear model there are n + 1 parameters. They are the independent parameters.

In this case, you have to establish that whether they are independent or not that depends upon the problem, but let us assume they all independent. So, this is one kind of size of a model. If you consider only a subset of feature vectors, only certain dimensions are related with these observations than your model size also gets reduced. Or, if you want to use the non-linear forms then also there will be coefficients regarding the non-linear terms and your model size will be increasing that as there will be more number of parameters. So, this is some example that depending upon your model description this number of independent parameters they vary and they determine in one way the complexity of a model.

So, the question is what should be the form of a model when as I mentioned that a scenario when you are not sure about the precise structure of the model, as we did in the previous cases of determining of homography matrix, fundamental matrix or projective matrix, the structures were very precisely defined, their properties are also well known or known to us and applying them we have derive them. But in some situations you may not get that then you have to apply intuitive or intelligent gases about this kind of structure, and then observe the error of fitting or likelihood of data based on that model and decide whether you should accept that module or not.

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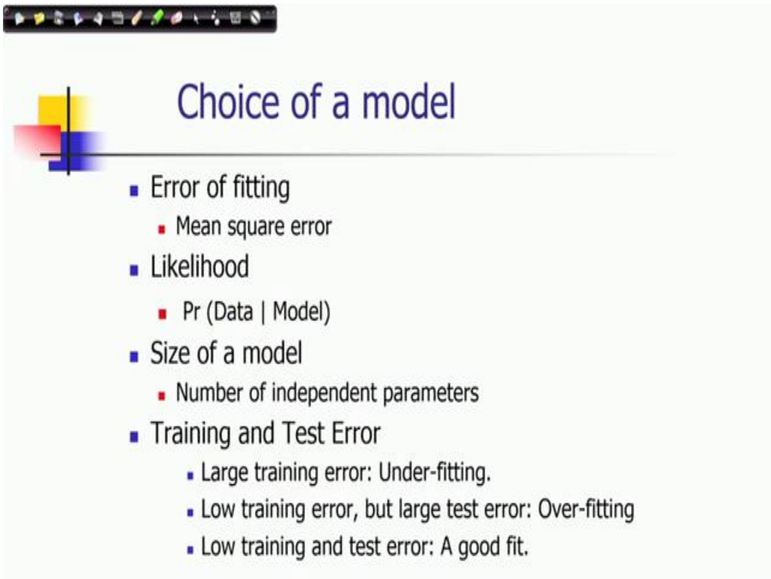

So, regarding this when you are evaluating the performance of a model there are two particular type of errors, those you should note. One is that training error, another is test error. So, training error these term came from the machine learning is perspective. So, when we are fitting a model using a set of data that is the kind of training operations and when you are testing that model with another set of data which is has been kept outside of this training set, but in the test set we know the actual values that means, ground truths values and the we compare those values with the predicted values, then we get also and error while those comparisons that we called test error.

Now, by observing the amount of training error and test error, we can also qualify your model fitting. For example, if your training error is very large that itself reflects that your model is wake it does not explain the data properly, it is an under fitting case which means your number of parameters in the model may have to increased. If it is a low training error that means, in the training you get reasonable you know error term which is very small, but while testing you find actually you are getting large tested which means your model is very data specific and we call that over fitting. It tries to minimize the error just considering that data.

Moment you want to generalize this model over other data sets which are not used in fitting them then actually your model is not performing well. So, this problem is an over fitting case. So, we have to consider that what kind of structure of model you should take. Most likely you have taken too many parameters, you have to work with less number of parameters in that case. Ideal situation is that you get a low training error and also a test error. Then, that gives you the confidence of having a good fit.

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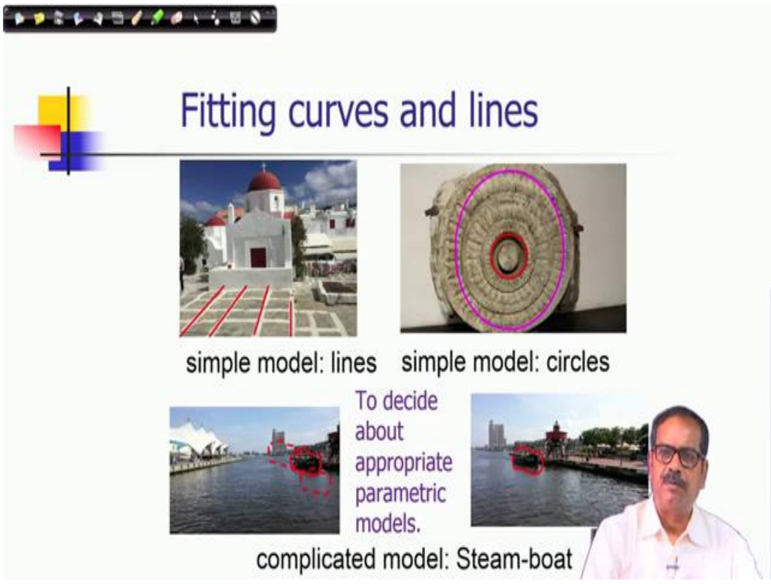

Some examples of model fitting which are observed in image imaging. We have discussed some examples of model fitting regarding homography computations, fundamental matrix computation or projection matrix or camera matrix computation, but from the geometric relationships in a two-dimensional image there are some simple models. As we; I have shown that given the set of points you may have to find out the lines which are which could be formed by this points.

So, the model is that those set of points they lie on a particular line and you are describing that line by their parametric form. It could be a circle or it could be any arbitrary shape consider this scenario and a boundary of an object has been shown in that shape and in another image we would like to see whether that object is present or not. So, using that polygonal models or the boundary described in that kind of shape we are trying to find out whether it exist in other cases or not. But this model is a bit complicated model for any arbitrary shape.

So, the question is that how to decide about appropriate parametric models or appropriate form of a models to relate the data, data points or to establishes relationships among this data points.

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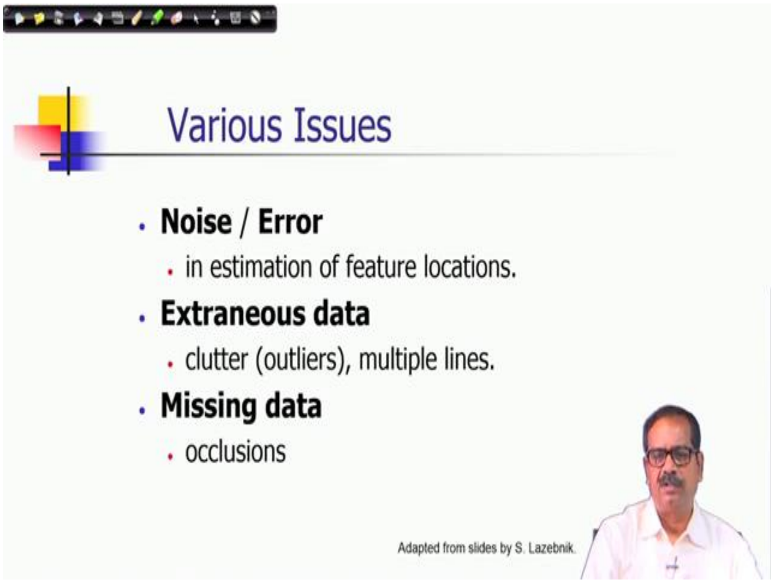

So, there are various issues which are involved in model fitting. Like, your observation could be noisy there could be error. For example, even when you are obtaining data you are applying different computational techniques, different kinds of estimation. So, this estimation will have some error. So, feature locations may not be precisely found, there could be some error in deciding about those locations. There could be some data which is not generated by the process which will fit the model. So, they are called clutter or outliers.

Say, you are assuming the corresponding points between two images there exists an homography and you know that a plane planner, a planner sin induces a homography. So, so assumption is that those points they belong to with a sin points which are lying on the same plane. But it may happen in your observation, in your measurements or in your experiments when you establishes correspondences, when you get those data point some of them may be out of the plane, images of some sin points which are not lying on the same plane. So, if you would like to establish the homography matrix that would provide an error, that would give an erroneous measurements for those models and since you are trying to optimise the overall error that would make a problem.

Then there could be multiple lines. So, your model is a for single line, but actually there are multiple lines and then again the single line model, model for single lines will not be applicable there. Some data could be missing also. For example, some occlusion, so you may not get the it is a kind of partial information that you observe in your data points, it owned give you the full picture of the model in that case.

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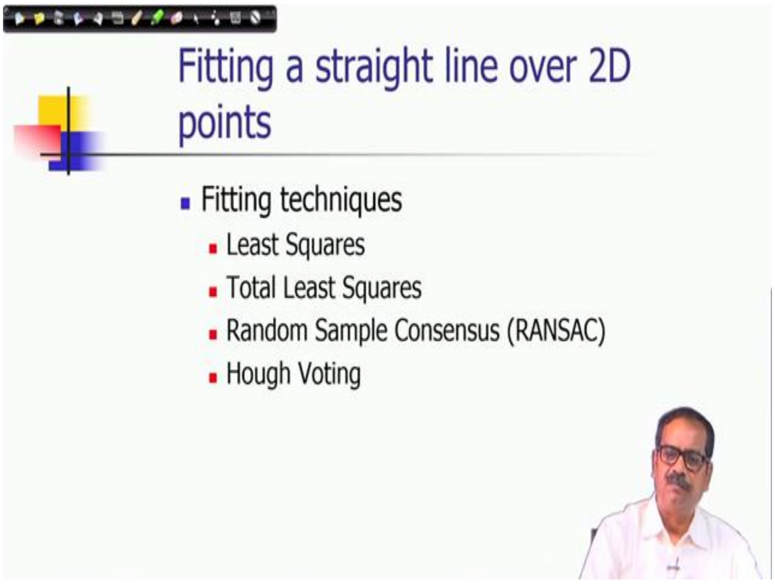

Now, we will be considering a particular type of problem of model fitting, for this lecture to give you some ideas of you know these issues what we discussed and some approaches to handle them, those could be extended in fitting more complex models also. So, would be considering only a very simple model of fitting a straight line over two-dimensional points and we will discuss about some of this techniques as they are shown here techniques of least squares, total least squares, random sample consensus, then Hough transformed techniques, mentioned here as Hough voting technique.

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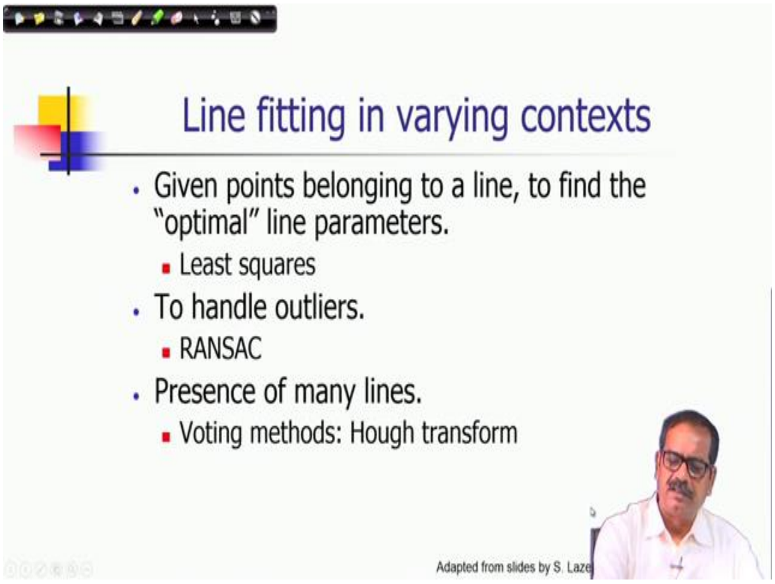

So, actually these techniques they are presented in varying contexts. Suppose, you have a points belonging to a line and you have to find the optimal line parameters then least squares technique they are applicable, they are very useful, they are very appropriate, but suppose they have outliers in that case we should think about some technique like random sample consensus technique or in short we call it RANSAC. We will see it is a very generic approach of fitting different kinds of models, where we expect there would be outliers in the data or clutter in the data as we mentioned earlier.

If we have too many lines and you would like to fit a model then voting methods are appropriate like Hough transform is applied there. So, in this lecture we will this will be discussing on this different kinds of techniques.

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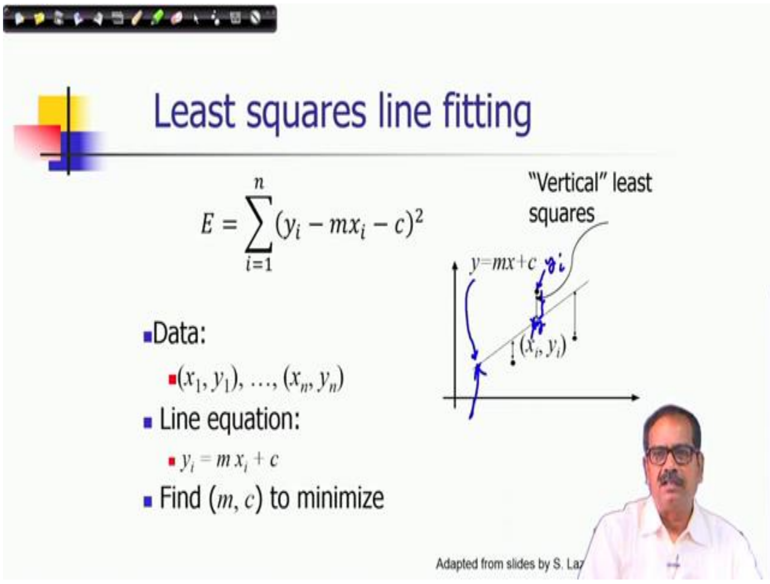

So, let us first considered the techniques of least squares for line fitting. So, the problem here I have shown that you have a data which is given in the form of a set of two-dimensional points. There are n points and you can see that there coordinates are denoted in this from that means, if you have i-th point its coordinate is denoted in my representation as x i and y i. And the relationship between this coordinates, in the model it is a straight line equation relationship. It is not strictly linear relationship, as you know the straight line is not will not give you a linear relationships it is called a find relationships. Colloquially, even that then we call it a linear fit, but actually it is an a find relationships.

Anyhow, so we know how a line is represented in a two-dimensional coordinate system and that is the familiar representation of y=mx+c that is the model. For a particular instance i-th instance we write it as yi=mxi+c. Then, the error of fit can be expressed in this form,

$$E = \sum_{i=1}^{n} (y_i - mx_i - c)^2$$

So, you can see in this particular diagram, see, if this is the equation or the straight line which has been given by this equation y=mx+c, then given xi say this is your value of y and this is your observed data yi. So, what, the deviation is given y, this vertical shift vertical difference. So, we call it vertical error and square of these deviations will give you mean square error. So, that is how the error term is defined. So, we call it vertical least squares because no we are trying to minimize this error. So, we have to find out that straight line that m and c which will minimize the sum of square of these errors, which means the error can be expressed in this form and then we need to solve it for this problem. So, find m and c to minimize it.

So, let me stop here and for this lecture. We have understood what problem we need to solve. We will continue this discussion in the next lecture.

Thank you very much for your attention.

Keywords: Model knowledge, mean square error, fitting curves, occlusions, RANSAC