Computer Vision
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Lecture - 24
Feature Detection and Description Part - 1

In this lecture we will talk about Feature Detection and Description.

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So far we discussed about camera geometry, stereo geometry where we considered that how to obtain the projection matrix of a single view camera or homography between two scenes or fundamental matrix between two images of the same scene in a stereo camera or in a stereo imaging setup.

Now, there we have considered that the corresponding points of the images are given to us and using those corresponding points we have obtained those quantities. But in this lecture what we would like to explore is how do you automate that process of detection and getting the set of corresponding pairs of scene points. That would be primarily the issue what we will be considering in this particular topic.

So, here for an example, you can see that there are two images of the same scene. There is a structure of an ancient temple and in two different views we have taken this image. So, the question is that how do you match the scene points? For example, we know from this image that this particular part of this image and for the other image they correspond to each other. But how do you precisely define the points of correspondences even in those regions.

So, detecting the regions where they match approximately or crudely, but then again the preciseness of locating the same points of the scene point that is also a requirement and there are several complexities of this problem as we can see that the images they call they are transferred in this case you know you get a view from a different view from a 3 dimensional perspective, but even for a 2 dimension image also there could be various kinds of transformation.

For example, the same image could be translated, it could be rotated say for example, this is the one kind of rotation and even the scale can vary, which means that you can get a shrinked version of this image or you can get the image from a distant viewing point, where objects they look small, but still in that case also you need to identify the corresponding points of the structures.

So, these are the challenges in detecting these points and there are several issues regarding these computations. So, some of these issues are highlighted here like detection. I told how to detect these points even though you detect the structural points first you have to consider that what are the land mark points which are easily detectable even after transformation. So, there is a problem of detection then you need to uniquely characterize those landmarks points.

So, you need to describe the point by its neighboring statistics by looking at the texture around it in the image and finally, you need to match them. So, there are several candidates of such land mark point. So, out of them which pairs they correspond to the same point in the scene. So, in this lecture we will be considering several such issues, particularly the detection and description that is the primary theme of this particular topic here in this lecture. Later on we will also consider the computation of matching of pair of points.

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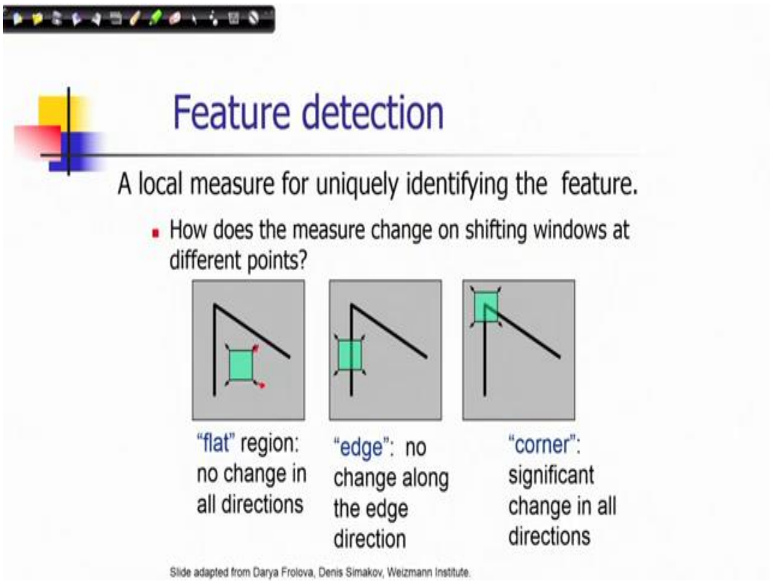

So, while we are considering detecting a feature the idea is that you need to characterize a feature point and which should have some uniqueness with respect to others.

That uniqueness should be preserved even after different kinds of transformation as we mentioned earlier like translation, rotation, scaling etc. So, in this case particularly there are various mathematical techniques by which you we would like to define this uniqueness. So, we would like to consider the local statistics around a point and we call it as a local measure and we desired that this property should be invariant with respect to transformation. So, in this diagram we are trying to show that with the green square block those are the regions of interest say central point of that green squares.

We considered that this is a point of interest and we are trying to find out the statistics around these points and suppose we move this particular window and this is a same point around different image points. So, what kind of structural property that would show us some variations in the measures if you move it? So, you consider this particular aspect that if I consider this particular region which is a flat region.

So, even if we move the windows in different directions still the local statistics they would look almost similar which is a kind of uniform distribution of intensities in particular this example. But when you consider the other image where we are moving in the edges, then if I move along the direction of edge you will not get any change it would look almost similar kind of distributions specially, but if I move along the perpendicular direction of this edge we can find out there is change as we move in the perpendicular directions. But the significant change that you will get even if you move in any direction that we will get in this kind of structure when two edges are meeting here.

In fact, this kind of structure is a corner structure and even a slight movement of your window will disturb the local distribution and that can be reflected by some measure some local statistics we will see later on how can we define this statistics.

So, the summary of this discussion or the highlight of this particular example is that some structures (Refer Time: 08:16) have certain uniqueness in describing their neighborhoods or they could be conveniently characterized by some local statistics. So, these structures are mostly the corners that we can see in this kind of 2 dimensional images. So, let us proceed just to understand what kind of statistics we can define.

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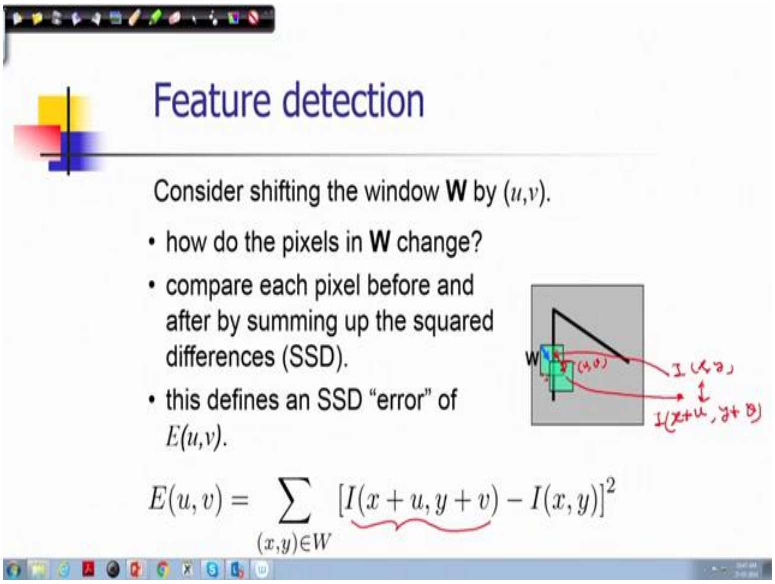

So, consider a window in the same example what you have discussed in the previous slide and the kind of measure which will be changing when we shift the window in different directions. So, we will be considering the intensity distribution around a point and intensity distribution within the window and you would like to see how intensity values are changing because of the changing of these windows. So, how this intensity distributions are changing in this window and there is a particular measure what we can consider during the shifting we can find out the difference between the intensity values with respect to the corresponding point.

So, every pixel when there is a just (Refer Time: 09:53) translator motion. So, every pixel is shifted by a constant vector in a particular direction say (u, v) what has been shown here and if I take the difference between the intensity values between those two pixels in the shifted window and in the original window and if I take the sum of square. So, what is the expected that if it is a uniform region or flat region this sum of square of differences would be very small. They will be almost 0 when it is an edge there would be some difference at those edge points, but when it is a corner this difference would be very prominent.

$$E(u,v) = \sum_{(x,y)\epsilon W} [I(x+u, y+v) - I(x,y)]^2$$

So, you take the difference between these two values and take the square and consider all the pixels in that window for this major you accumulate these differences square of differences for all the pixels in that number that is what is the measured what is described here.

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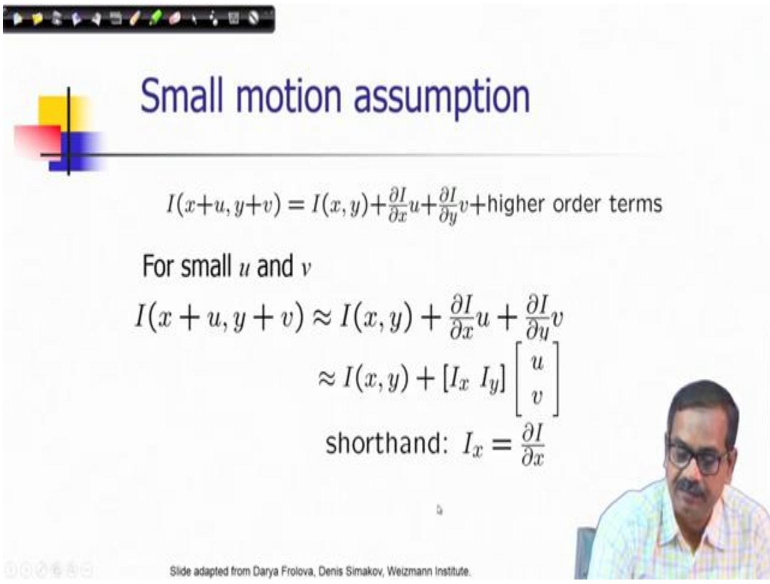

So, we will be expressing this particular measure with respect to the differential geometric corporations and how you can compute this one in a very general situation.

$$I(x+u, y+v) = I(x,y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v = I(x,y) + [I_x \; I_y][{u \atop v}]$$

So, for small u and v this higher order term; that means, those are the terms which are considering higher differential quantities. So, those can be ignored. So, I can simply write in this from, we will use only the first order changes.

And then this can be conveniently represented by this expression. As you can see this is just a representation of the same equation here.

$$I_x = \frac{\partial I}{\partial x}$$

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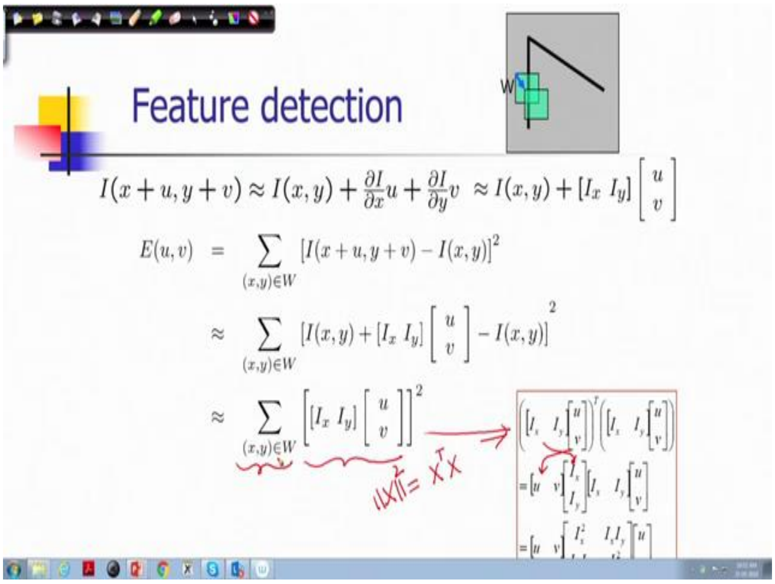

And expanding it further what we can see that how we can write this sum of square of differences. We can write it conveniently in this particular notation simply using only the first order changes along certain directions. So, you can see that only the differential changes are needed for computing this you do not require the absolute pixel values in those windows we can eliminate them by these simple expressions.

In fact, this quantity as you can see that this quantity can be expressed in terms of quadratic expressions of with using the matrix representations. So, and there is a typical representation and what is shown in here that you can see that this could be written as

$$E(u,v) = \sum_{(x,y)\epsilon W} [[I_x\ I_y][{u \atop v}]]^2 = \sum_{(x,y)\epsilon W} X^T X$$

And you can see this is that quadratic expression what I was talking about.

So, this quantity particularly shown here is actually reflecting the local statistics. So, there are three particular measures those are characterizing locally, one is square of the differential in the direction of x then square of the differential in the direction of y those are denoted by u and v directions also and also the differential along x and differential along y. What you should note here that this major is an aggregation over the local statistics. So, it is not a major at that particular point. It is an aggregation. So, we should consider it as a distribution and for any typical distribution is the expectation of those values that would be considered or averages of those values that would be considered to form this matrix.

Now, this matrix characterizes the local statistics of the point and we will see how this matrix will help us in characterizing a feature point.

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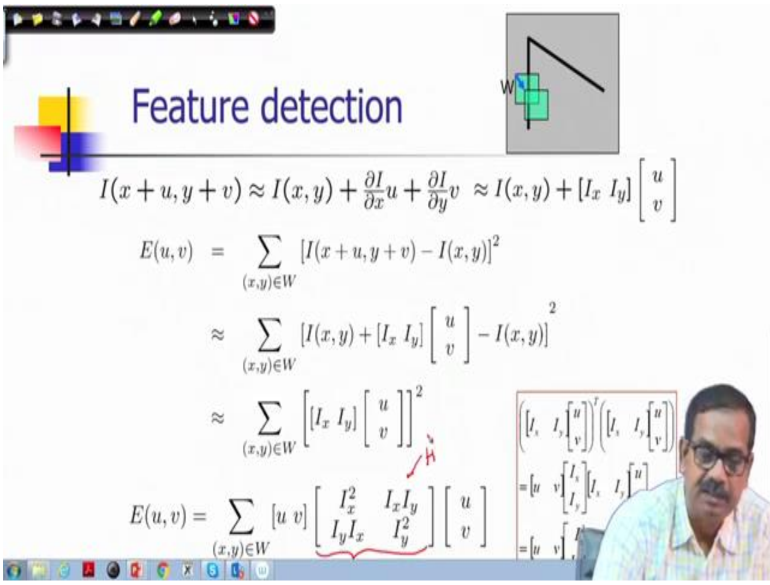

So, this is the summary of this particular expression. Once again it is written in a more prominent form here as we can see that this is the matrix I was talking about.

$$H = \begin{bmatrix} {I_x}^2 & I_xI_y \\ I_yI_x & {I_y}^2 \end{bmatrix}$$

We will denote this matrix as H.

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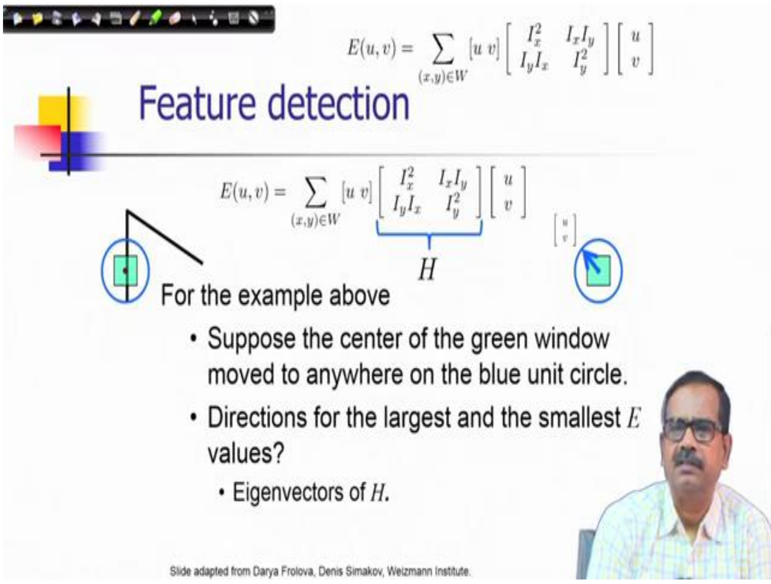

See we will continue with this representation. As the sum of square of differences represented by the function E there can be represented in this form where locally around a pixel you need to measure the differential changes along x direction and y directions take the averages of them.

And you need to take the averages over the square of those changes and individual averages of along x direction and y direction. So, big question once again that if the centre of the green window move to anywhere on the blue unit circle then how this particular quantity changes? So, that is the question we need to ask and we need to find out. So, this is what we are considering that we need there are observing the changes and which are the directions for which these changes would be the largest and also the smallest.

Now mathematically this can be found out. If I perform the Eigen analysis of that particular matrix what I have referred here referred and we denote this matrix by H.

So, we will make an Eigen analysis for this one and the eigenvectors they would give us those directions one of them corresponds to the higher eigenvalue or larger eigenvalue that would give us the changes along the largest changes along the direction of largest change and the smaller eigenvalue you will give us the direction along the smallest change and from the properties of linear algebra that eigenvectors they are orthogonal. So, there are two orthogonal directions that you would get.

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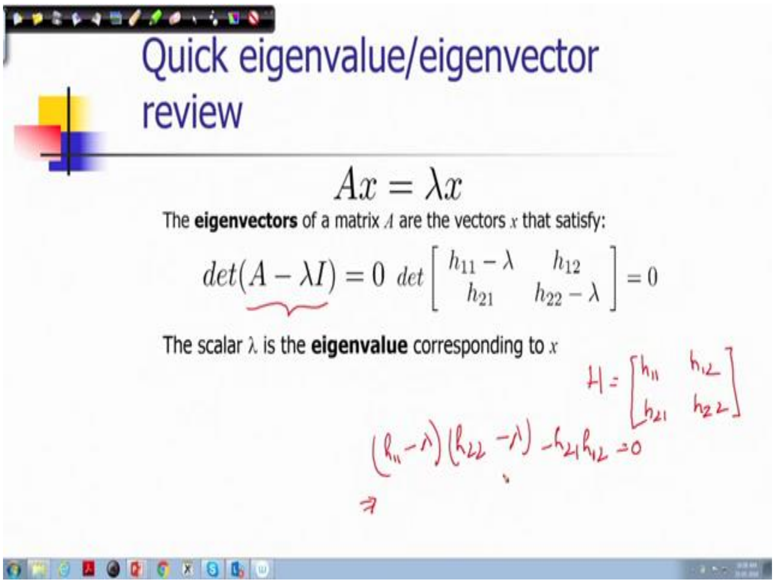

So, just a quick overview of computation of eigenvalue and eigenvector in this particular case we have a very simple situation because our matrix is just 2x2 matrix and as you know the definition of eigenvector of a matrix A is that you know if I perform this multiplication if I transform a vector x in the same dimensional space with this transformation A, I should get the same vector in the same direction with a the change of the magnitude will be there.

So, it's a scaled vector that you would get and that scale value is the eigenvalue and the vector which is not changing its direction that vector is called eigenvectors. So, one of the ways that you can find out these eigenvalues is that as you can

$$Ax = \lambda x$$

$$\det(A - \lambda I) = 0, \det\begin{pmatrix} h_{11} - \lambda & h_{12} \\ h_{21} & h_{22} - \lambda \end{pmatrix} = 0$$

So, there will be two values of lambda and particularly if the matrix is symmetric then you would get the real values there. So, we would see these summarizations of this competition. So, this is what the competitions of determinant in these cases are shown here.

Now, there will be coefficients of λ^2 and also the constant term there and it is in the form of that equations like $Ax^2 + Bx + C = 0$ and then you know what the solutions are.

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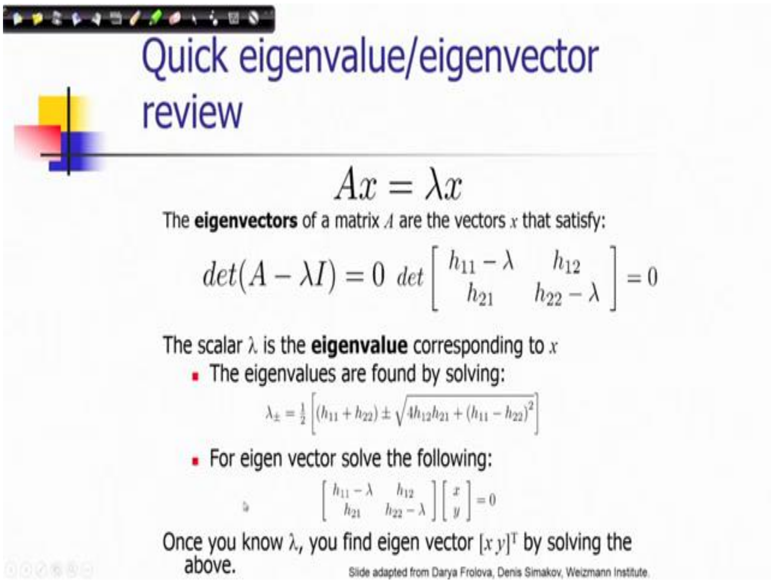

In fact, I will just show you that solution here of this in terms of this elements. So, this is the solution of these equations and you can see that in this case you are denoting this two values one is lambda plus which is the larger value. So,

$$\lambda_{\pm} = \frac{1}{2}[(h_{11} + h_{22}) \pm \sqrt{4h_{12}h_{21} + (h_{11} - h_{22})^2}]$$

We need to solve this particular equation and one of them would be redundant or you can rather use the y equal to sum constant value assuming that is not 0 and then you can find out the corresponding eigenvector.

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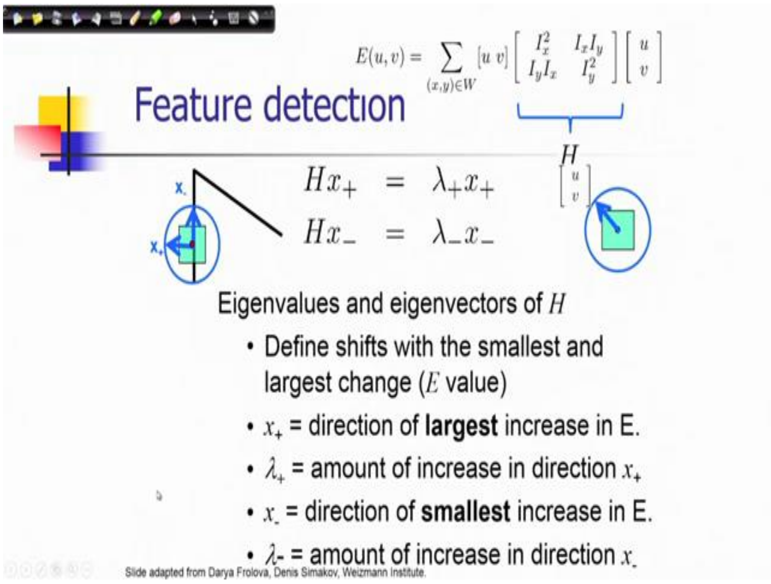

So, once you know the lambda then you find eigenvector by solving the other. So, this is the review of the competitions of eigenvalue and eigenvector given this kind of matrices. Now let us see that how these quantities help us in characterizing the point. So, once again we have shown here the matrix H and we can compute the larger eigenvalues λ_+ and the smaller eigenvalue λ_- corresponding eigenvectors are denoted here x+ and x-.

So, if you want to define the shift with the smallest and the largest change of E value. So, x+ is the direction of largest increasing in E, $\lambda_+$is amount of increase in direction x and x- is the direction of smallest increase in E and λ_- is the amount of increase in direction x-.

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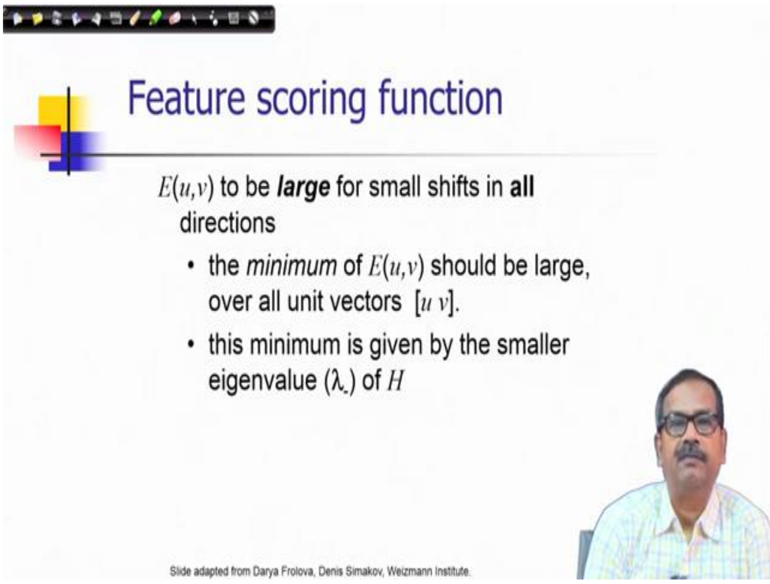

So, our objective is that we should define a feature point where these values of E(u, v) should be large for small shifts in all directions, because that is how we are trying to characterize a point. We told that local measures will be disturbed even if small shift which means is this value should be large in that the difference should be very large. So, u, v should be large, so which means that if I consider the direction of the largest change and smallest change.

So, just to ensure that it should be large the smallest change should be also very high. So, that is how the smaller eigenvalue is more important in this case to identify or to characterize a feature point here. So, this is what the minimum of u, v should be large over all unit vectors and this minimum is given by the smaller eigenvalue lambda minus of H that we discussed.

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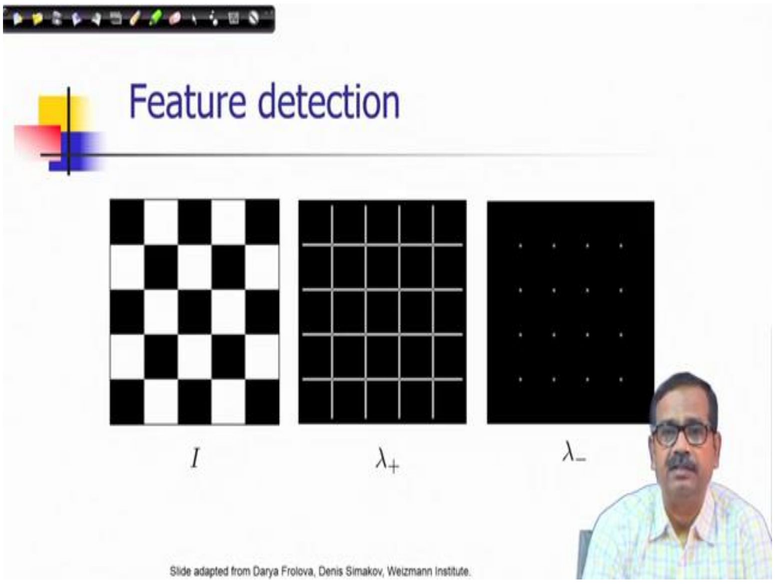

So, some examples here are taken from the slides which has been shown here that is they are adapted from the Weizmann institute slides by Darya Frolova and Denis Simakov. In fact, all the preceding slides those are adapted from those lecture slides, just to acknowledge those sides.

Now as we can see in this very nice example that there is a chess board pattern and if I perform the Eigen analysis of that sum of square of differences at every point then we can get a distribution of larger eigenvalue which is shown as λ_+over this space, so which is given by this.

So, you can find out that we can see that actually all the uniform regions this larger eigenvalue is almost 0. So, that is why it is black whereas, in edges there quite prominent. In fact, they should be more prominent in the intersection points which are the corners just because of optical illusion. Now they look little darker, but it would be clear if I consider the smaller eigenvalue if I plot the smaller eigenvalue then we will see actually those intersection points those are highlighted. So, even for edges smaller eignevalues are also almost 0.

So, only those intersection point of edges which we can called as a corner points of those black and white squares you can find out that those corner points are prominently highlighted and this is how we can detect the detect those corner points in this particular example.

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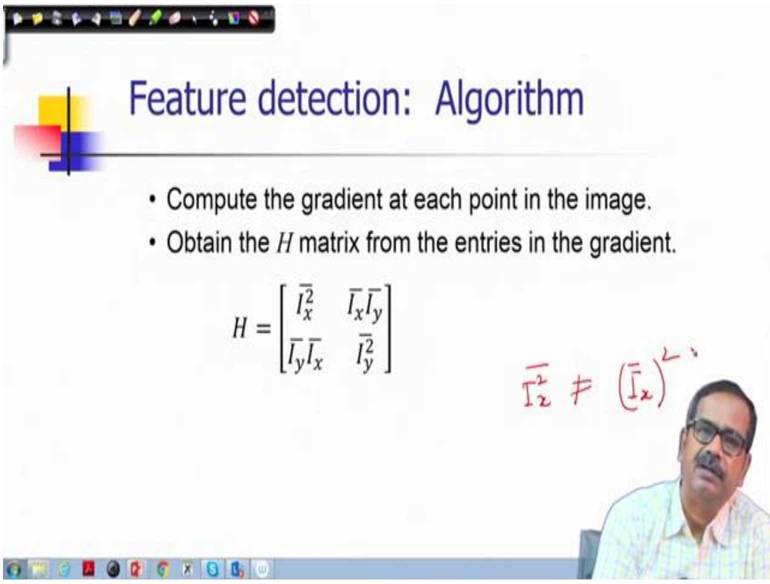

So, we can now design and algorithm based on this discussion for detecting features which are actually corner points in an image. So, what we can do that we can compute the gradient point at each point in the image and then we can obtain the matrix H from the entries in the gradient as we discussed. So, in fact, this is how the elements of H is defined. As I mentioned that you need to compute the gradient along x direction and y direction and also compute the square of the gradients because (Refer Time: 28:00) when you take the averages you need to take the averages over the square of the gradients not just you now you just take the average over only on gradient on x and then you make a square.

$$H = \begin{matrix} I_x^2 & I_xI_y \\ I_yI_x & I_y^2 \end{matrix}$$

So, you have to note this particular point because when you implement this algorithm you have to be careful, otherwise you can see that this determinant of this H would be 0 always. So, that would be a problem for characterizing it.

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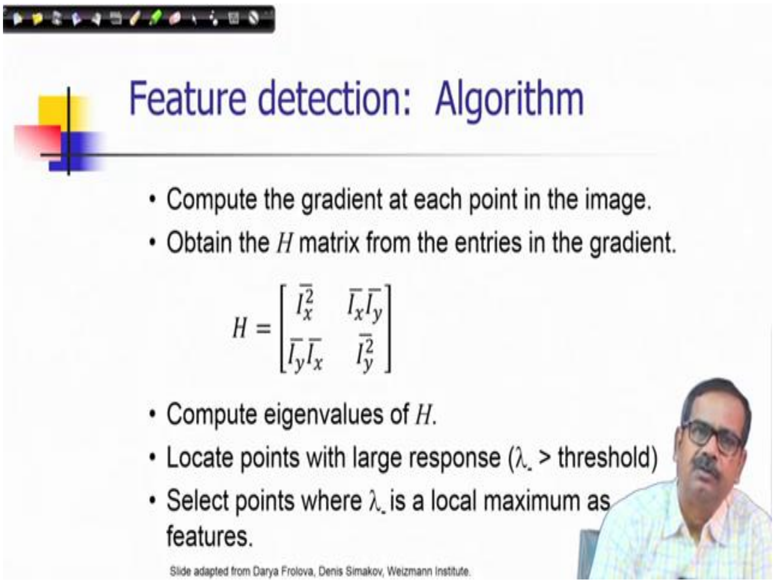

And then we should compute the eigenvalues of H as we discussed. Locate the points with large response of minimum eigenvalue and we can define a large response by considering some empirically chosen threshold that if it is greater than such in certain threshold we can considered that is as a point, but that is not a very precise characterizations there could be even the neighboring points of a particular corner point we can have this large response.

So, just to precisely locate the corner point we need to consider the local maxima of those responses. So, that would give us the feature points.

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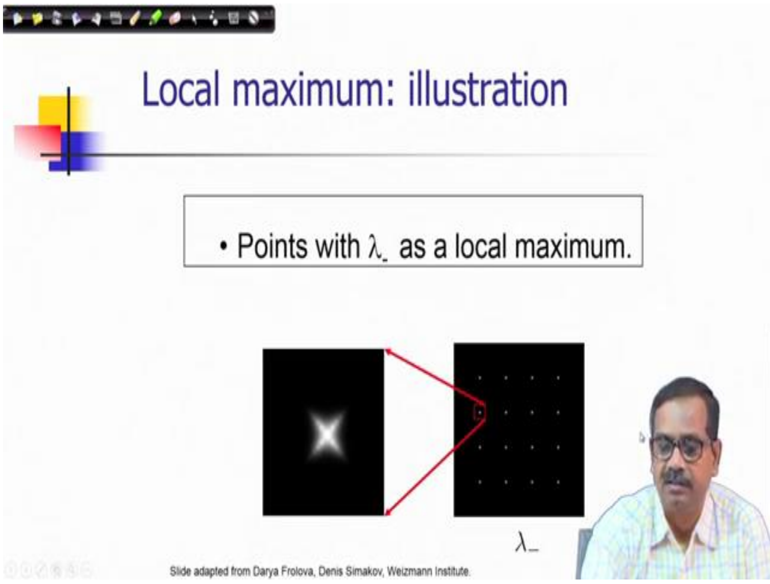

So, this is a once again a very nice example from the slides, it’s like as you can see in this computations what would look appeared is a very tiny dot in that resolution of images. If I zoom this image and look at the distributions of the pixels in a smaller neighborhood in a larger resolution over the display then effectively I can see a distribution of intensity values that you can see here a kind of a star pattern as what you can see in the central part. In fact, they are lies a local maxima.

And that is the corner points and that is how precisely it is defining a particular feature point and we will be considering that local maxima. This is importance of computation of local maxima. So, points with λ_{-} as local maxima that should be considered for the feature point.

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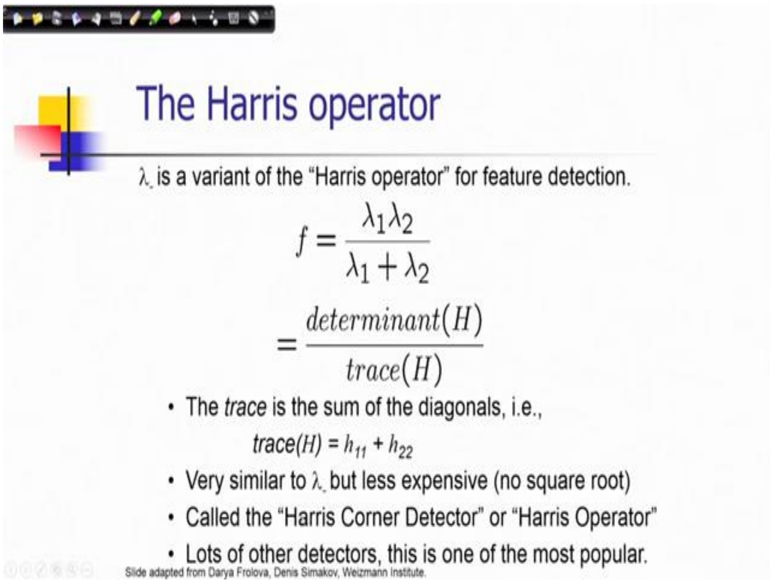

So, this gives us this computation gives us the famous Harris operator and in this case we are considering the λ_{-} as the major, but there is a variant in computing this major instead of computing directly λ_{-} we can compute similar measures which will give you the proportional quantity. So, one of this measure which he has been shown here by the symbol f is you are considering the product of eigenvalues and which is normalized by the sum of the eigenvalues.

$$f = \frac{\lambda_1\lambda_2}{\lambda_1+\lambda_2} = \frac{determinant(H)}{trace(H)}$$

You can see that since both the eigenvalues should be large this product should be very large and then it is normalized with respect to the overall eigenvalues and the determinant. So, these computations can be conveniently carried out without doing any square root operations or anything. This is the same as computation of the finding computing the determinant of the matrix H, which will give us the product of eigenvalues and the sum of the eigenvalues, would be defined by the trace of the matrix H which we have defined. So, using this measure we can look at those points. So, this is called Harris operator, because by applying this operation, by computing these measures and by computing the local maxima of this measure we can detect those features.

So, this is the reason why Harris corner detector is efficient than computing directly the eigenvalues. So, there could be so many other detectors, but this is one of the most popular detectors in the literature.

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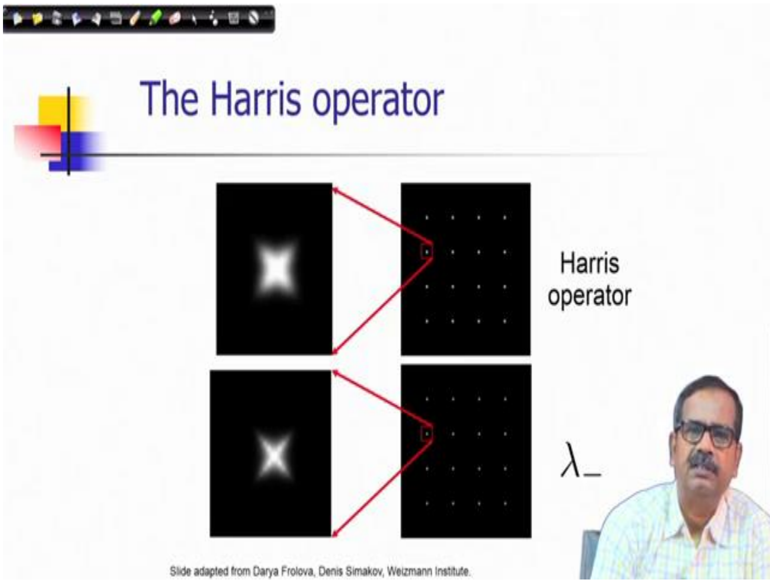

So, it just shows that how these two operations are almost equivalent you can see at the top the pair of images they are showing the distribution of those functional values using Harris operator where it looks little flattered though, but still the local maxima is retained and the sharper one is of course, the eigenvalue smaller eigenvalues, but we have also discussed the advantages of Harris operator, because here you do not require any square root operation simply you can compute detect determinant and trace of matrix and get the ratios and that would give you this operation.

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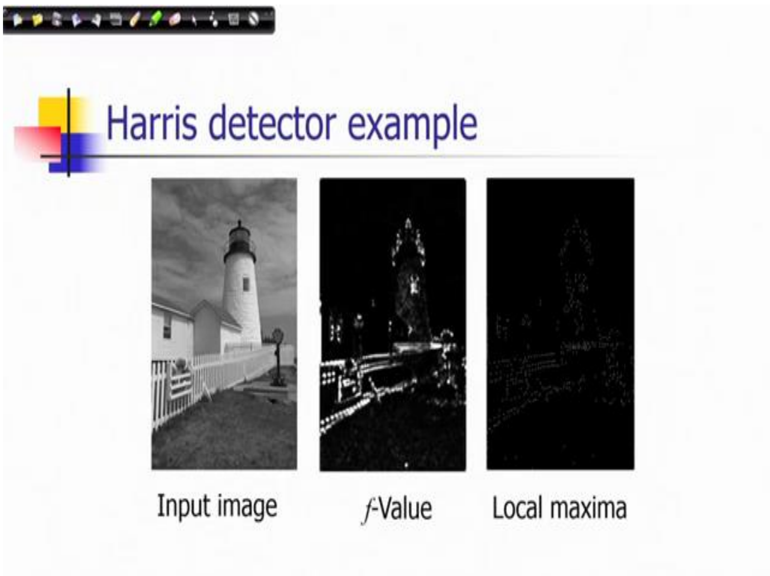

Some examples of this detector, it has been shown here say input image has been shown and then the distribution f value has been shown and if I get the local maxima we can get some precise locations of those points which are like corners in the intensity distribution there are sharp changes in those particular points.

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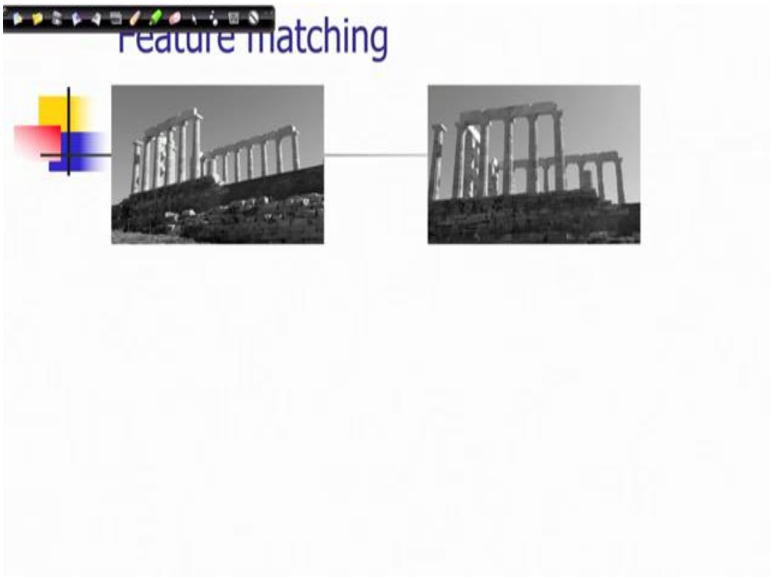

So, let us stop here in this particular lecture, we will continue this discussion of you know feature detection and description and the motivations for obtaining the descriptor that is feature matching that we will be discussing in the next lectures.

Thank you very much.

Keywords: Harris corner detector, local maxima, match scene points, feature detection.