Computer Vision
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Lecture – 23
Stereo Geometry Part - VIII

We will discuss special case of epipolar geometry, under the same topic of Stereo Geometry and that is affine epipolar geometry. So, we discussed earlier also how an affine camera differs from a general projective camera. In an affine camera your centre of projections or camera centre that lies at infinity which means that the projection rays, they are all parallel to sudden directions and the basic projections basic imaging mechanism it takes place through that parallel projection.

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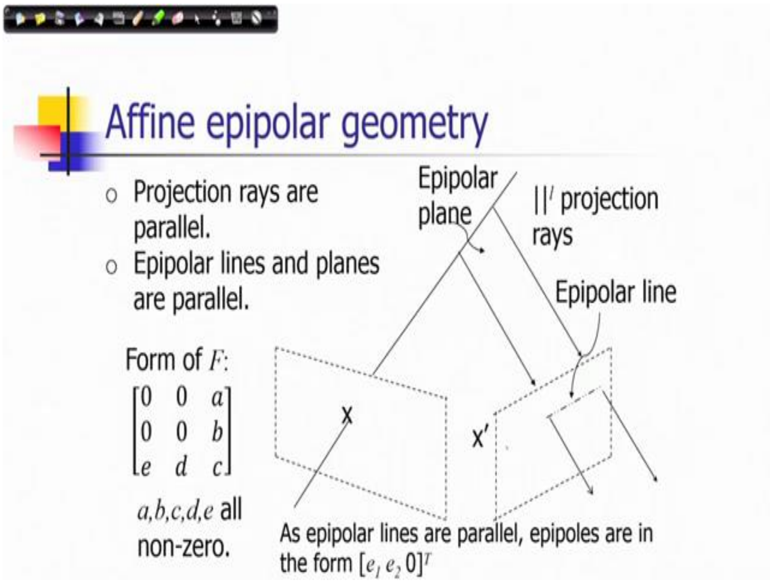

So, let us consider a stereo system which is formed by two affine cameras. You can see here in this diagram that there are two such cameras and there are direction, so it is a parallel projection ray which is forming images in the second camera. Similarly, in this direction ray for the first camera. And the camera centers this is lying at the infinity, this parallel projection, so the intersection of this projection rays will be at infinity which will define the camera centre, effectively that is the direction of these rays itself.

So, how the epipolar lines are found? Epipolar lines are all parallel lines. Here we can see that for the same projection ray, if I take the projection of the same points lying in this projection ray then all those image points will form an epipolar line. And since all the rays are parallel here all the parallel rays in the first camera, epipolar lines also should be parallel.

And the form of fundamental matrix particularly when you have this kind of geometry it get simplified you can see the structure here, you have only 5 non-zero elements.

$$[e']_x = \begin{bmatrix} 0 & 0 & e'_2 \\ 0 & 0 & -e'_1 \\ -e'_2 & e'_1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & b \\ -b^T & 0 \end{bmatrix}$$

See all the 4 elements in this part they are 0, and the other elements are non-zero. And so there are only know 5 parameters, but since once again it is their scale is involved in this representation, so there are 4 independent parameters. So, here projection rays are parallel and epipolar lines and planes are also parallel. That is the characteristics of an affine epipolar geometry.

And the other thing is that as epipolar lines are parallel, epipoles are in the form of e1, e2. So, we know that because the intersection of epipolar lines that would be also at the point at infinity which means the value at the third dimension representing scale should be 0.

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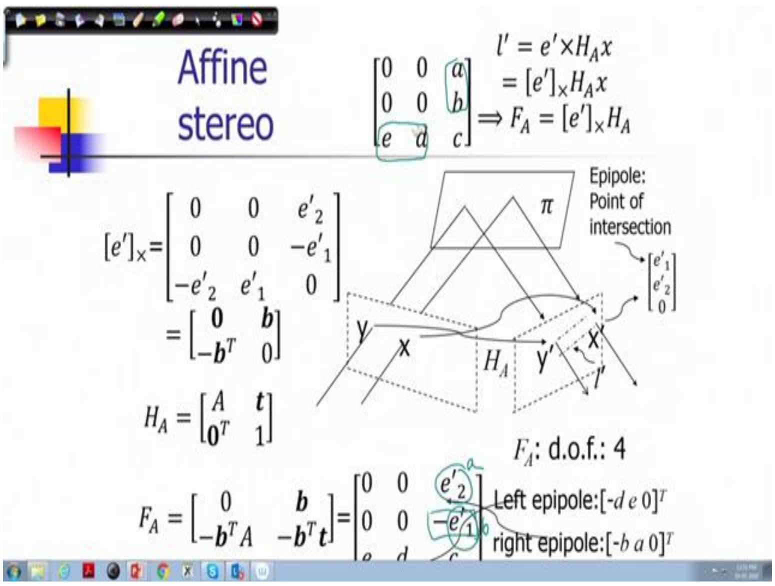

So, let us again elaborate this particular property. Consider this affine stereo x is corresponding to x‘, y is corresponding to y’ and they are existing homography between them when all these points are lying on a particular plane pi. So, that is the plane induced homography.

$$H_A = \begin{bmatrix} A & t \\ 0^T & 1 \end{bmatrix}$$

And so epipole a point of intersection is represented as e1’, e2’0 we have seen that this is the structure. And, following the same construct the epipolar line is given by e‘XHAx because this is the homographic transformation of y to y’ which will lie also on the epipolar line and epipole also will lie on the epipolar line. So, e’XHx equal to 0, so FA should be e’XHx. And since epipoles have these construct, so all those 4X4 sub matrix of the affine fundamental matrix in the upper side that becomes 0, because of these structure.

$$l' = e'XH_Ax$$

$$= [e']_x H_A x$$

$$F_A = [e']_x H_A$$

So, particularly you notice that the right epipoles they these are given by this particular column. So, the values of the right epipoles it has occurred in this fashion. So, e2’ comes here and e1’ comes here so. Similarly, we will see also the values of the left epipoles will be at the row [-d, e, 0]. Right epipole is given by [-b, a, 0]. So, from the structure of affine fundamental matrix epipoles are very easily determined that we can see.

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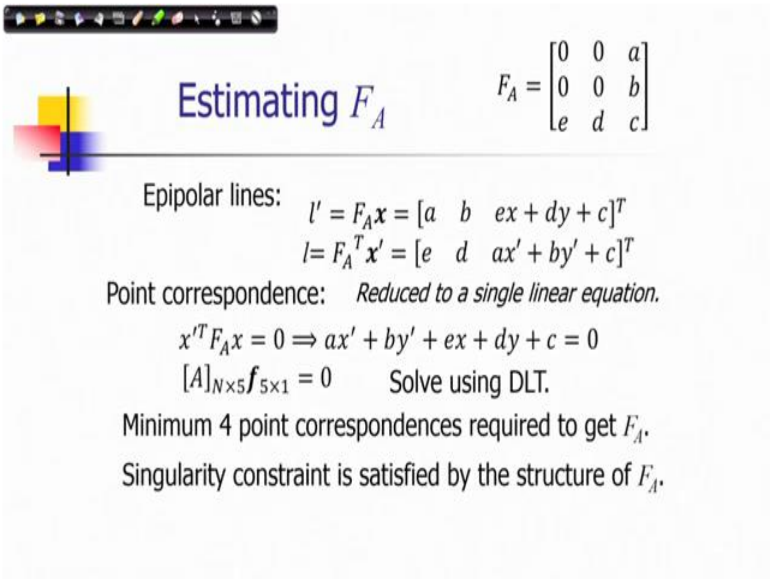

$$F_A = \begin{matrix} 0 & 0 & a \\ 0 & 0 & b \\ e & d & c \end{matrix}$$

$$l' = F_A x = [a\ b\ ex + dy + c]^T$$

$$l = F_A^T x' = [e\ d\ ax' + by' + c]^T$$

For estimating this affine fundamental matrix we have to use the corresponding epipolar lines. So, this equation becomes a in a in a much more simpler form and you can represent the set of linear equations again in the matrix form in this way and you can solve using direct linear transform.

Since, there are 4 independent parameters you require only 4 point correspondences to get FA and from the structure itself singularity is satisfied because of the structure that all the upper sub matrix 2 x 2 sub are all of them are 0, so if you take the determinant you can check that the determinant value itself is 0 in this channel structure. So, you do not have to perform any special operations to input singularity.

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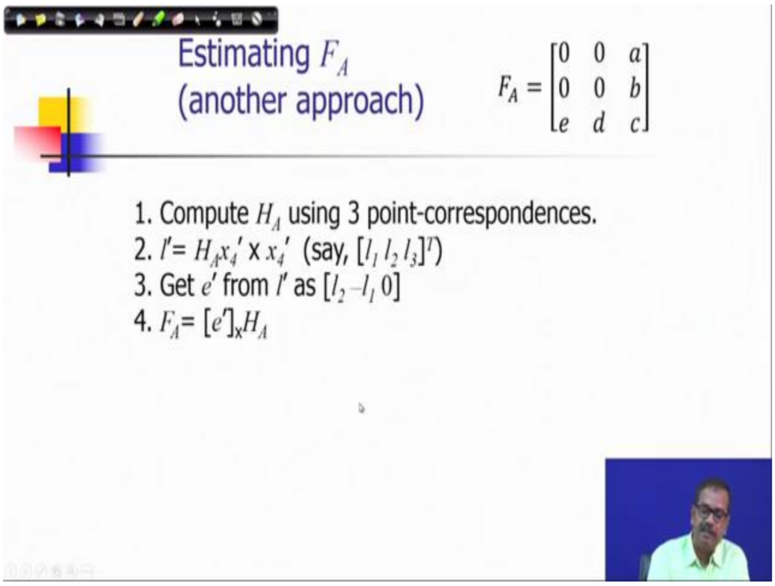

So, the approach was using the set of correspondence points where you can find a linear equations exploiting this constraints of epipolar geometry, and you have seen how it can solve those problems and solve those equations which becomes a simpler for the case of affine fundamental matrix estimation.

Now, we can use another approach for estimating the fundamental matrix. Here we can compute the first we can compute the homography induced by a plane by using 3 point correspondences. So, minimally we require 4 point correspondences. So, we have seen already this homography would be affine homography and for that 3 point correspondence is sufficient to compute it. So, we can do that and then we need to compute the epipolar line.

Now, in this case it is sufficient to compute the epipolar line l' because from there itself we can get the epipoles. The directions of epipolar line itself will give you that epipoles because that is what its direction cosine is given in this from [l2 -l1 0]. So, actually, epipoles is the point at infinity along that directions and this is direction of the line. So, in this way epipoles can be obtained.

$$l' = H_A x_4' X x_4'$$

$$F_A = [e']_x H_A$$

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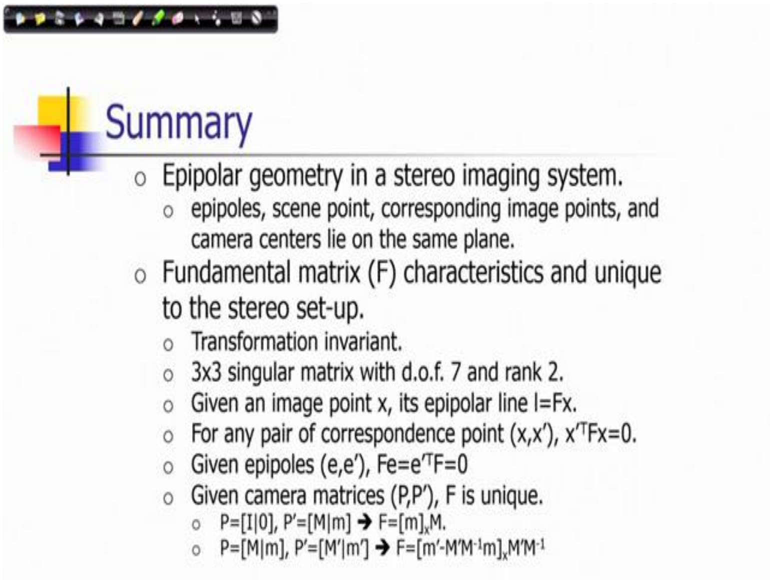

So, we have come to the end of this particular topic of stereo geometry. So, let me summarize it is different features that we discussed. First we discussed epipolar geometry in a stereo imaging system and there we have seen that epipoles, same point corresponding image points and camera centers, all of them lie on the same plane. Then fundamental matrix characterize its stereo system and it is unique to the stereo setup it has various properties like transformation invariance is one of the properties. That means, transformation of image points will give me back the fundamental matrix. You can use the transform points to estimate it.

Then it is a 3X3 singular matrix with degree of freedom 7, that is the number of independent parameter is 7 and since it is singular its determinant has to be 0, rank has to less than 3 and in this case for a fundamental matrix its rank will be 2. So, it is function that it transforms an image point into its epipolar line in the second camera plane, second image plane. So, and this is the relationship that you have to multiply fundamental matrix with image point in the homogeneous coordinate representation, then you will get a line in the homogeneous coordinate representation in the image plane.

And for any pair of corresponding points you get this relationships

$$x'^T F x = 0$$

If we get epipoles e' then this is the relation

$$Fe = e'^T F = 0$$

So, they are the zeros of fundamental matrix. So, epipoles are the 0s of the fundamental matrix. e is a epipole in the reference camera plane, so it is right zero of F and e' is the epipole in the second camera plane and it is a left zero of the fundamental matrix. On the other hand, if I change the convention of reference camera that show up them then F transpose will be the fundamental matrix and all these results are consistent with that representation.

So, given camera matrices P and P' and F is unique that we discussed. And particularly these computations we should note that if we have the projection matrices in this from, that projection matrix in a canonical form of [I|0] and P' as [M|m], this is the representation. Then fundamental matrix is given as mXM. So, we discuss the definitions of these notations in our lecture.

Or, you consider a very generic representation I have given you the expression, I did not really expand or discuss in my lectures, but please go through it both note this expression. So, what we will find that from P and P' we have computed the epipole and which is given by this

$$F = [m' - M'M^{-1}m]_x M'M^{-1}$$

So, this is now the epipole is given. That means, you have to take the image of the centre of the first camera. So, image of the centre of the first camera is given in this form. And then the himography at infinity is given $M'M^{-1}$. So, that is how we get this fundamental matrix.

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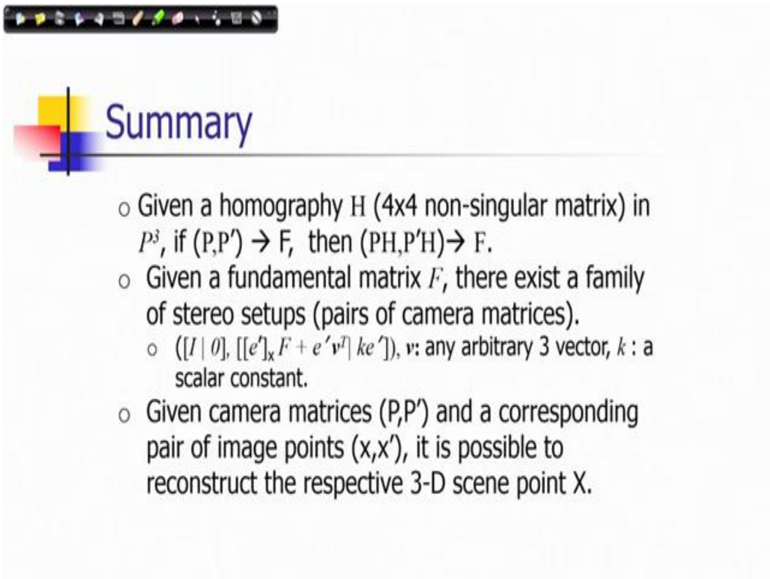

Then we discussed that how projection matrices could be related with fundamental matrix though a pair of projection matrix give uniquely one fundamental matrix, but a fundamental matrix can lead to a family of projection, pairs of projection matrices because now if you have a homography in the three-dimensional space like which means it is a 4 X 4 non-singular matrix homography of a 3D projective space.

$(P, P') \to F, (PH, P'H) \to F$

Then given a fundamental matrix F that exist a family of stereo setup or pairs of camera matrices, that we need to note. And this is one example of a family of matrices considering the reference camera is given always in the canonical form [I|0], then you can express this family given that fundamental matrix by its epipole and any arbitrary 3 vector and a arbitrary scalar constant, as you can see in that expression which has been shown here in this case. Then, given a camera matrices (P, P') and a corresponding pair of image points (x, x') it is possible to reconstruct the respective 3D scene point X.

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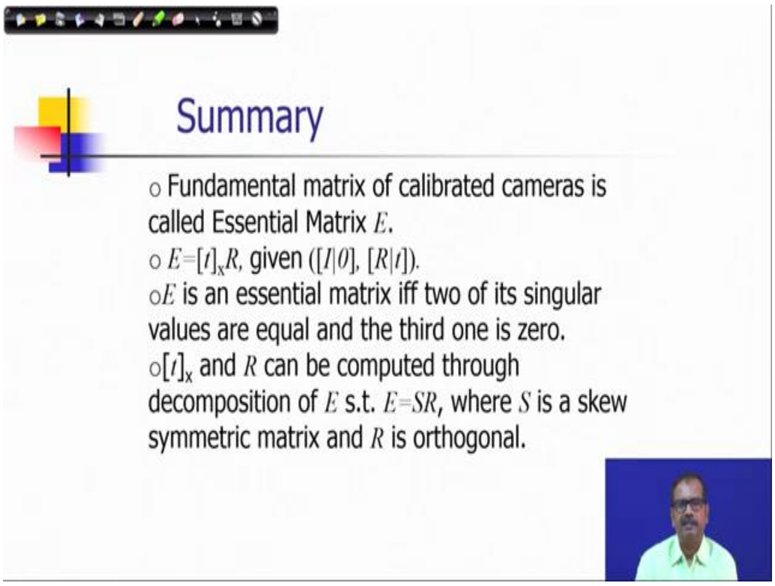

We have discuss 3 approaches for this and one of them was using the triangulation geometry, the second one was using geometry preposition error is in non-linear optimization technique, and third one is algebraic form of representation of triangular triangulation approach. And that is how we can derive the structure.

Then, let us summarize also the facts regarding essential matrix which is a fundamental matrix of calibrated camera, and this is given in this structure. That is, if I considered a stereo system where projection matrices are given in this form that reference camera is canonical camera canonical matrix I 0 and the second camera is related by the rotation and translation of rotational axis and translation of R h. So, in that case essential matrix is given by those parameters only. And one of the properties of essential matrix is that two of its singular values are equal and the third one is 0.

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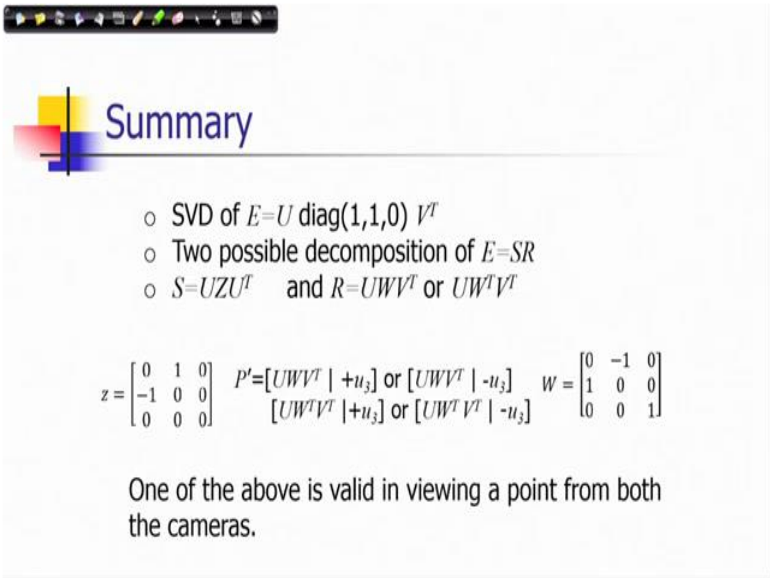

Then you can compute that it components t prime cross and R by matrix decomposition of E such that know E =SR, where S a skew symmetric matrix and R is orthogonal. And if you have this decomposition, then you can compute also its projection matrices which are shown here, that apply singular value decomposition of essential matrix. And, there could be two possible decomposition of this essential matrix which is shown here in this form that use the singular value decomposition matrix U and V and there are two special matrix Z and W which is shown here in this form.

Then the expression of S and R is given as you can see.

$$S = UZU^T, R = UWV^T \ or \ UW^TV^T$$

And from there we can compute P'. So, note here this u3 is the third column of the matrix U. They are the singular value decomposition, which means this is a column vector corresponding to singular value 0. And one of the above is valid in viewing a point from both the cameras.

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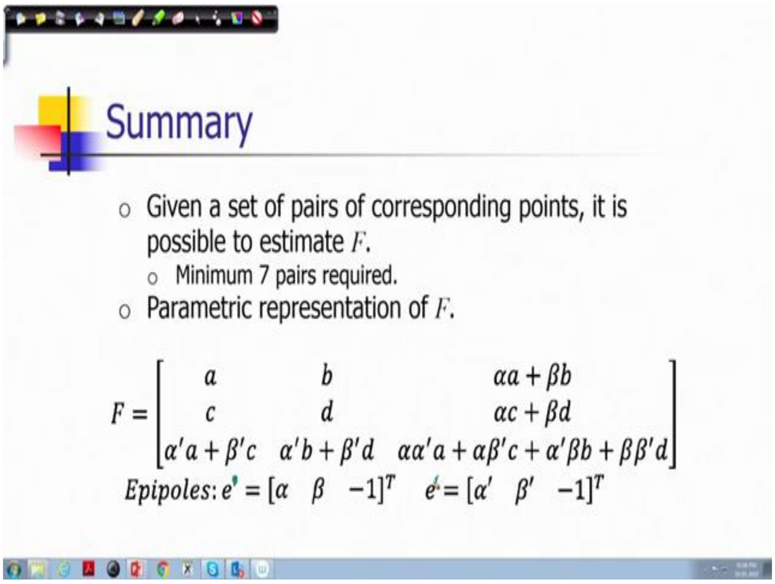

And then we discussed also estimation of fundamental matrix. So, given a set of pairs of corresponding points it is possible to estimate fundamental matrix. Minimum 7 periods of points required, we have discussed one particular method where we have seen that how the eigenvectors corresponding to 0 eigenvalues they are used in this case, 0 eigenvalues, so eigenvectors of a system.

Then parametric representation of a fundamental matrix is given in this form. This is the particularly this from also contains the epipoles, you should note that know there is a little bit of ambiguity in representing e prime and e, e’ should be e here and e should be e’ here. This should be e’.

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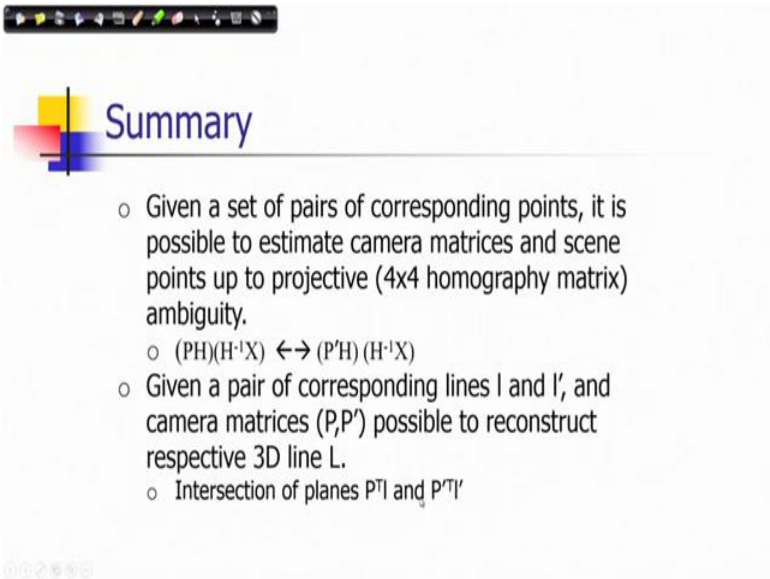

And given a set of pairs of corresponding points it is possible to estimate camera matrices and scene point up to projective ambiguity which is 4 X 4 homography matrix.

$$(PH)(H^{-1}X) \rightarrow (P'H)(H^{-1}X)$$

Then given a pair of corresponding lines l and l' and camera matrices P and P' it is possible to reconstruct respective 3D line L. We discuss that also. So, in this case you can compute the plain formed by the lines l and also camera centre we know that that is $P^T l$, and also the plane found by l' and camera centre of P' which is $P'^T l'$ and then two planes they intersect at a line which is that corresponding three-dimensional line, so that also you can reconstruct. So, this is what intersection of planes $P^T l$ and $P'^T l'$.

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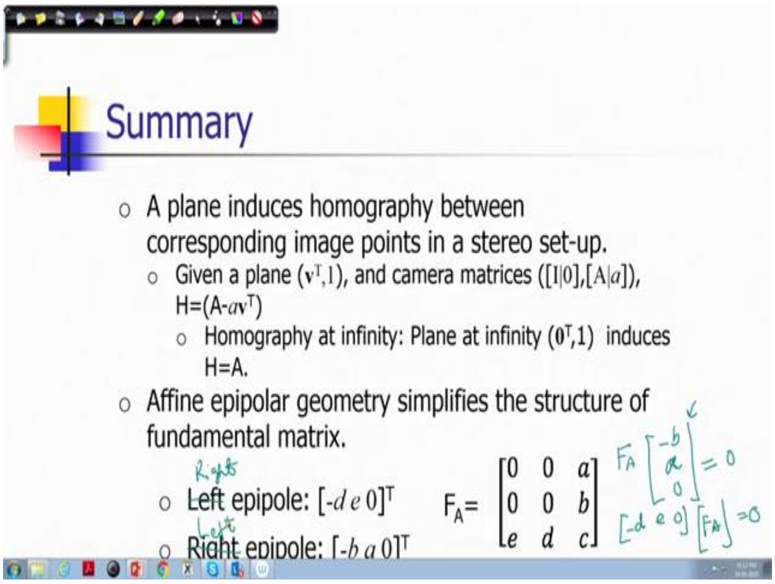

Then, we discussed about a plane induced homography. So, a plane induces homography between corresponding image points in a stereo set up. So, this was the expression for the homography for a particular description of the camera matrices and the plane. So, plane described by say (v^T, l) transfers gives the directions of normal of the plane and camera matrices are given by [I|0] that is the canonical camera matrix of the reference camera and [A|a] that is a general camera matrix. Then, homography induced by that plane is given in into this form.

We also discussed about affine epipolar geometry which simplifies the structure of fundamental matrix.

$$F_A = \begin{matrix} 0 & 0 & a \\ 0 & 0 & b \\ e & d & c \end{matrix}$$

So, the epipoles they are given by is the right epipole which means the left epipole is at [-b a 0] and the right epipole is [-d e 0]. And there we are having the same confusion. Left epipole is the FA [-b a 0] that would be 0 which is the left epipole. So, it should be right epipole. And similarly [-d e 0]FA that should be equal to 0 which is actually may added. So, please note this correction once again here also.

Left epipole is the epipole at the left image plane and right epipole is the epipole at the right image plane. Thing with this we have come to the end of this particular topic and we will be discussing our next topic on feature detection in the next lecture.

Thank you very much.

Keywords: Affine epipolar geometry, fundamental matrix, left epipole, right epipole