Computer Vision
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Lecture – 22
Stereo Geometry Part – VII

We are discussing about recovering structure from the set of corresponding pairs of points of a stereo system. And we have discussed how there are different approaches for computing that; computing a scene point given a pair of corresponding point. So, just to illustrate this process we will be discussing a problem for computing the scene point.

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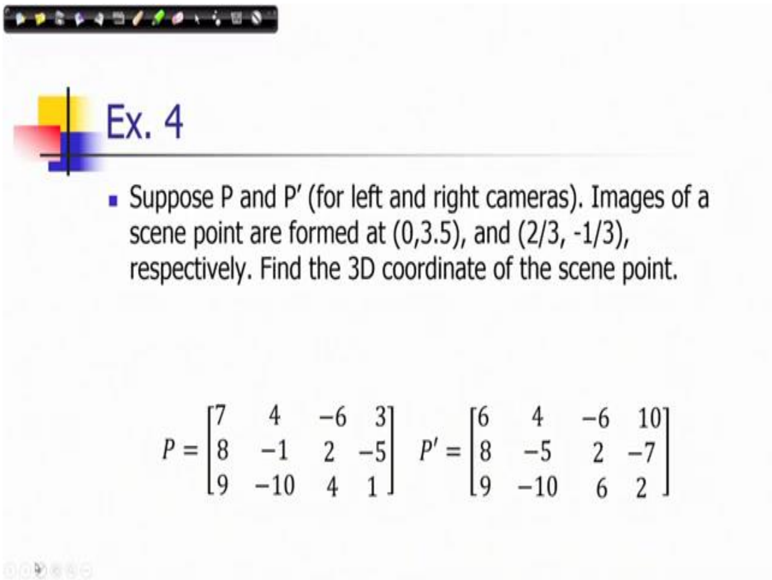

So, consider this particular example that you have two camera matrices P and P’ for left and right cameras. And its images of a scene point are formed at say (0, 3.5) and (2/3, -1/3). So, we have to compute the 3D coordinates of the scene point. You can see the projection matrices of the two cameras P and P’ they are also given here.

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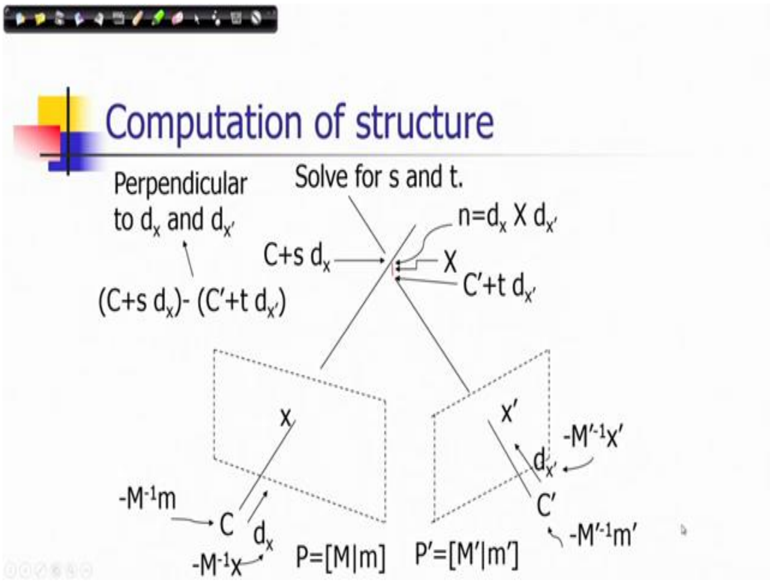

So, we will discuss the solution. So, let us consider how we are going to compute this particularly to get the scene point. So you have projection matrices P and P';

$$P = [M|m] \;\; P' = [M'|m']$$

And we have shown that two pairs of corresponding points as x and x'. So, we will be applying the geometry approach of triangulation; that means, we will form the rays back projection rays for these two cameras and then try to compute its intersection. And due to the noisy observation, you will not exactly get that intersection of this it is expected they should intersect, but you will not get exact intersection. So, what you need to find out that find out a very close estimate that intersecting point. So, you should consider a perpendicular segment for both the lines and then get the midpoint that was the solution.

So, let us see how this computation proceeds. So, C is the centre of the first camera, C' is the center of the second camera and we can compute the direction cosine of the projection ray; given the image point x. Similarly, direction cosine of the projection ray of the second camera; given the image point x'; so, this is what. So, $-M^{-1}m$ that would be the center C, then $-M'^{-1}m'$; that would be the center of the second camera, $M^{-1}x$ is a direction cosine of the projection ray of the first camera and $M'^{-1}x'$ is a direction cosine of the second camera.

So with this computations you get to know all the three dimensional constructs of the straight lines because the points passing through the straight line and its direction cosines.

Now, you apply the three dimensional coordinate geometry to solve the perpendicular segment and its meet point; so there lies the solution. So, one of the property of this perpendicular segment that its direction cosine should be obtained by the cross product of dx and dx' because it is perpendicular to that.

And if I assume; a point lying on the projection ray passing through the image point x and C; as C plus s d x; so s is some it is a parametric representation of that straight line; any point can be; if a s varies; s is a any positive value. Similarly, C prime plus t d x prime is the another point. So, what we need to find out exactly that which s and t will satisfy this constraint that the line joining them; line joining those two points given by this values of s and t will give me a perpendicular directions of both the rays; so that we translate. So, it is perpendicular to this and then you know we need to solve for s and t applying this constraint.

$$C = -M^{-1}m$$

$$C' = -M'^{-1}m'$$

$$d_x = M^{-1}x$$

$$d_{x'} = M'^{-1}x'$$

$$direction\ cosine = d_x X d_{x'}$$

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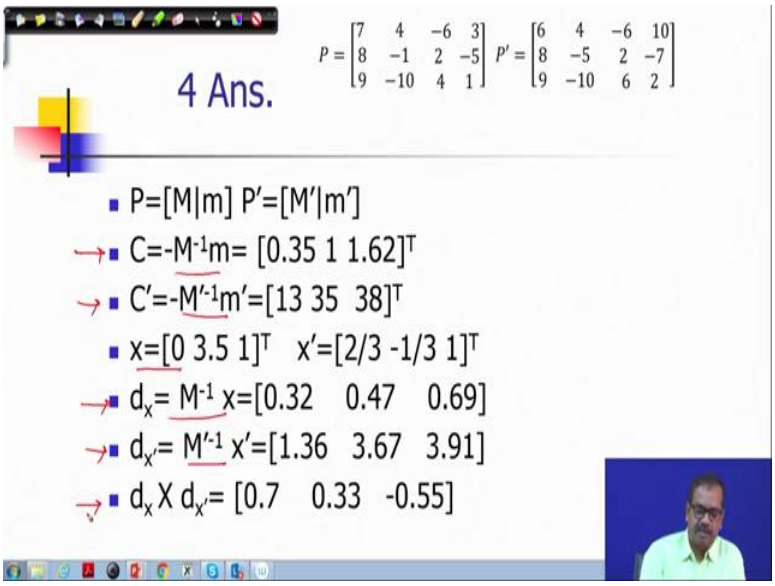

So, these are computational steps what we discussed and here is the summary of all those results. As you can see that we have computed the first centers, this is given by applying this relations. Then you have completed the direction cosines by applying this particular facts. Then this is the direction of the perpendicular segment which is perpendicular to both of these rays.

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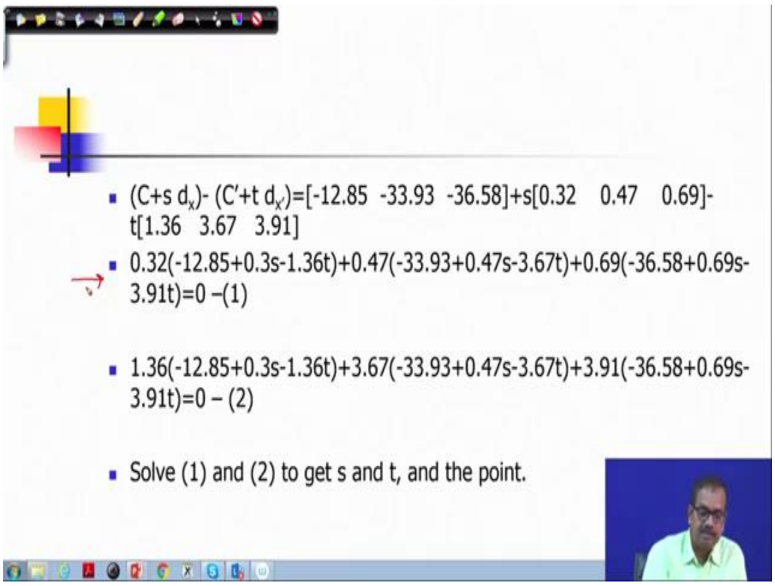

And then we are trying to solve for s and t by applying the constraint that the line forms by this; so the line forms by them this segment it is perpendicular to both the directions of dx and dy'.

So, you get actually two equations; so if you note the direction cosine of dx=[0.32, 0.47, 0.69]. Then in this slide you can see that you have taken the dot product of that direction with this particular expression that should be equal to 0. Similarly from the direction cosine of dx'; it should be orthogonal to this direction. So, the dot product of between these two vectors should be equal to 0. So, this gives me two equations is that a linear equations in terms of s and t; so if you solve and you can get s and t and the point.

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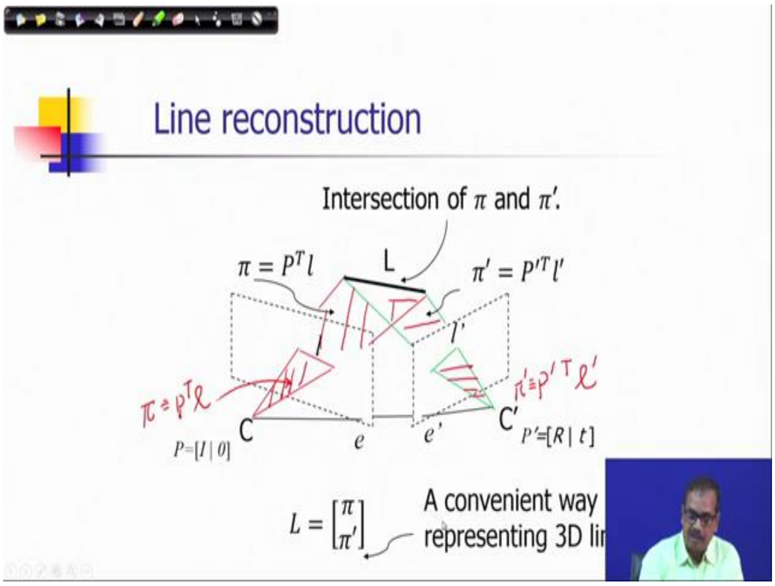

So, this is how we can solve this problem and we can get the structure. Let us consider the other problem of line reconstruction. So, you consider here a particular situation, where you have been given a straight line L and you can get its two images which will be again straight lines in the first image plane and in the second image plane.

Incidentally, as you can see that there will be also an epipolar lines those corresponding epipolar lines. So, this is an image of 3 dimensional line in the first image plane; similarly the image of the same line in the second image plane; both are given by L and L'. So, what we can do here that given this line and given this projection matrix P; we know that equation of this plane is $P^T l$; so this is the equation of this plane.

$$\pi = P^T l$$

$$\pi' = P'^T l'$$

$$L = \begin{bmatrix} \pi \\ \pi' \end{bmatrix}$$

Similarly, equation of this plane is $P'^T l'$. So, now you have these two plane one is say π another is say π'. So, when two planes intersect; they forms a line and that is the solution there is a three dimensional line and that is exactly you will get if you compute this intersection; so this is what we I just mentioned. So, finally, your solution is in this form; so this is one convenient way of representing a three dimensional line.

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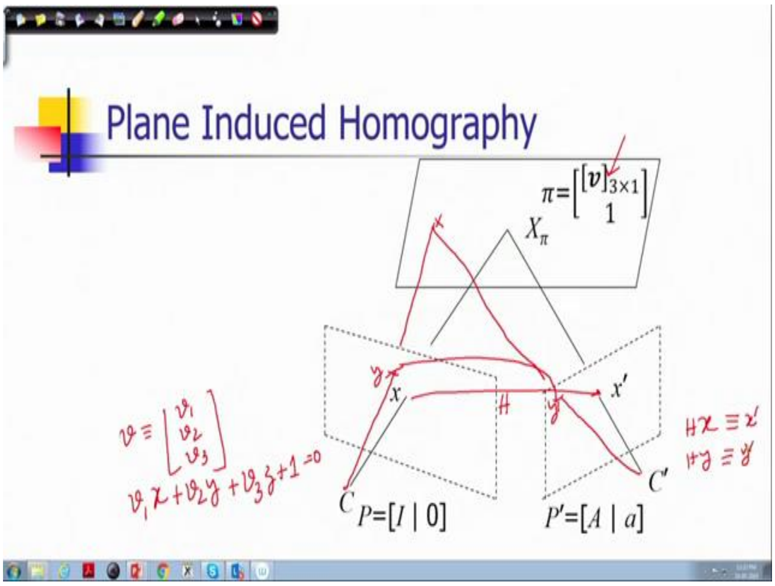

Now, we will be discussing a very important property of a stereo system; an important feature of a stereo system. And you can show that how the projective transformations among the image points they exist in a stereo geometry. So, the homography or the projective transformation are established among the pair of corresponding points for which the scene points lie on a particular plane. So, we call this kind of homography a plane induced homography.

So, take this particular scenario which is been shown in this diagram that you have once again two cameras of projection matrices P and P'. So, P is given by [I|0] that is the projection matrix for the first camera and P' given in this representation [A|a];

And we have a plane which is given by this algebraic representation as you can see this is an equation of a plane. So, representation of a plane in a homogeneous coordinate system of a three dimensional projective space; so this is the 3 vector which is giving you the directions cosines of the plane and your scale value is kept as 1; which means the equation of the plane if v is given as a column vector

$$v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$v_1 x + v_2 y + v_3 z + 1 = 0$$

Consider a point on that plane which is given here x pi and its corresponding image points x and x prime. So, we can show that there exists a homography between these two points. So, you can take another point for example, and also you can form its say images say this is y and say this is y prime; so this is the corresponding point. So, there exist homography H such that Hx will give me x prime and Hy will give me y prime and this homography is called plane induced homography; we can show this things. So, let us check the derivations, clean this particular diagram.

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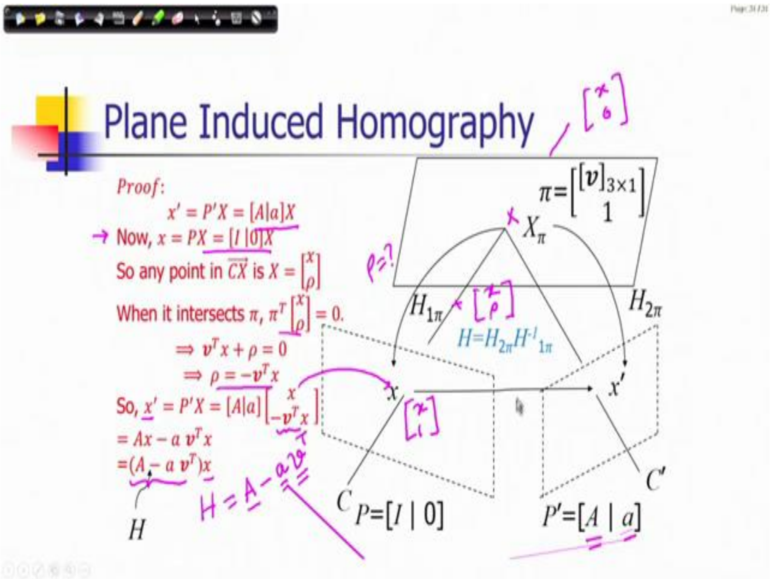

So, we can see that; we have already discussed earlier also that points in a plane and its corresponding image points in a single vis[ion]- camera also they are related by the homography. Because points in a plane can be represented by a three vector by two independent parameters and they are related by a homography; consider that homography is H 1 pi. Similarly, you can have an homography H 2 pi; so from there you can get a homography which is given by this H 2 pi; H 1 pi inverse.

This plane induced homography is can be related with the parameters of the plane which is inducing this homography and also the parameters of the projection matrices. So, here we have shown; we have shown a proof that how it is related the; the basic theorem is that this homography is given by this. That means, plane induced can be expressed in this form where A is the 3 cross 3 sub matrix of the projection matrix of the second camera or P prime. And this vector small a is the fourth column vector; fourth column vector of this projection matrix and v as you know this is the direction ratios of the normal of the plane pi.

So, this has been shown here how it be related like this. So, let me just provide you the outline of this proof as it is given here. Now, you can see that x prime which is the image of the scene point x is given by this P prime X; which is can be expressed in this form whereas, x is just PX; which can be expressed in this form. So, now, any point in this ray can be considered as x rho; as you know that the in the homogenius coordinate system, the scale factor provides you the corresponding you know points and the interpretations with respect to three dimensional space real space is that; it should could be any point in that projected ray connecting the image point x.

You should note that at rho equal to 1; you have the image point. So, when rho is 1; this is represented in this form and when at infinity, this would be represented in this form x 0. So, exactly at what value of rho you are getting this value that can be determined by considering the point containment relationships on the plane. So, if I take pi transpose x rho; this vector that should be equal to 0 and value of rho is given by minus v transpose x and that now we will be using in the expression of x prime.

So, x is related with this capital X that is the scene point is related with the image point x and also the corresponding scale factor which has been computed using this particular fact of point containment in plane. So, x and scale factor is minus v transpose x; so if I do the corresponding submatrix multiplication, I will get it as Ax minus a v transpose x and no ah; so this can be written in this form. So, as you see x prime is related with x; by this linear transformation and which is nothing but we call it a projective transformation and which is what is homography H. So, you will get in this way this is the direct relationships.

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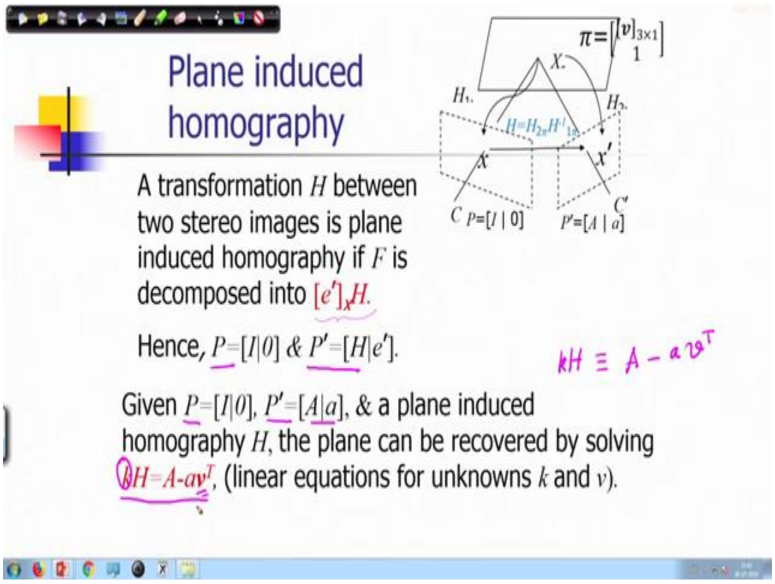

So, using this particular fact we can say that a transformation H between two stereo images is plane induced homography; if you can say in the other way that if fundamental matrix can be decomposed into this form; when e prime e prime cross H; e prime is a epipolar plane epipole epipoles right epipoles. And as you know that the cross product of right epipole with the matrix that is an operation matrix operation basically this e prime cross is a matrix equivalent matrix which is representing that operation; that itself will be we can you will give you F.

We have; earlier we have discussed that actually this homo[graphy]- it could be any homography and that is what it is represented here; previously we have taken only the homography at plane at infinity. So, even if I decompose F in this form of e prime cross H; if we decompose F in this form of e prime cross H, then P and P prime can be ah obtained; can be can be expressed in this form, it is a in the family of ah pairs of projection matrices this could be also one such option.

So, we can also say that if you have two projection matrices P which is in the canonical form of I 0 and P prime which is given in this form as we can see that a plane induced homography H. And then the plane can be recovered by solving this equations because we know that homography would be ah; we have seen that homography is given by A minus a v transpose in the previous case. But naturally this equality we cannot establish unless we know what is the; exact equality can be determined can; can be put when we know exactly what at what scale this equality holds.

So, that is why we have to we write this equation in this form. So, what are the unknowns here? There you have the unknown k and unknown v; so there are you get a linear equations there will be no; no there in the H, there are 8 elements; 8 independent elements independent parameters. So, you can get linear equations for unknowns k and v and that you can solve using this homography. So, this is how you can get the equation of the plane.

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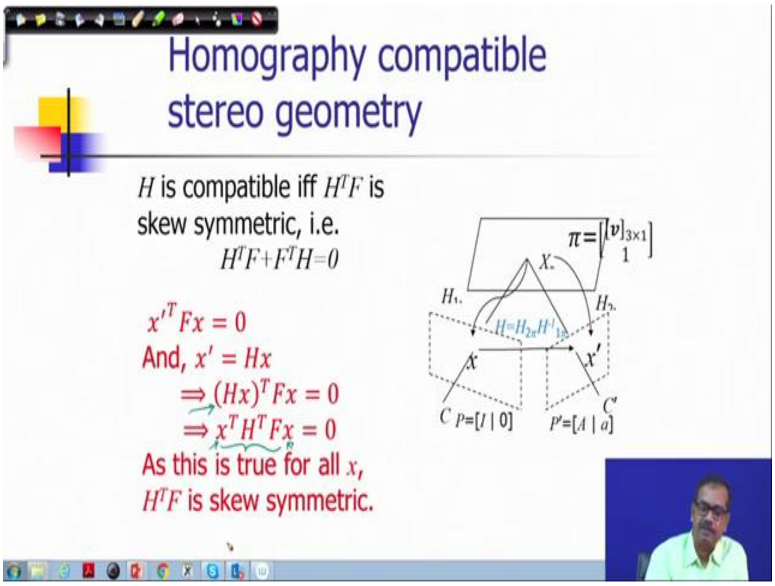

Another feature of the stereo geometry that this plane induced homography or you can find out whether it is compatible to a fundamental matrix or not. Like you have observed; we have discussed how fundamental compatibility of fundamental matrix will pairs a projectional matrices could be tested by observing, whether it is that there is a skew symmetric matrix property; P prime transpose F P should be skew symmetric. Similarly here also you can see that H transpose F should be skew symmetric. So, that is you know there is one of the interesting that is one of the feature of the skew symmetric matrix as the transpose of that matrix is the negation of the matrix. So, if we add the transpose you should get 0.

So, this can be shown in this form that is the proof that since x prime transpose F x equal 0 and x prime is H x. So, H x transpose F x equals 0 and this is true; once again this is true for every x because see these are all same image points; so this is true for every x. So, H transpose F is a skew symmetric matrix from the property of a skew symmetric matrix.

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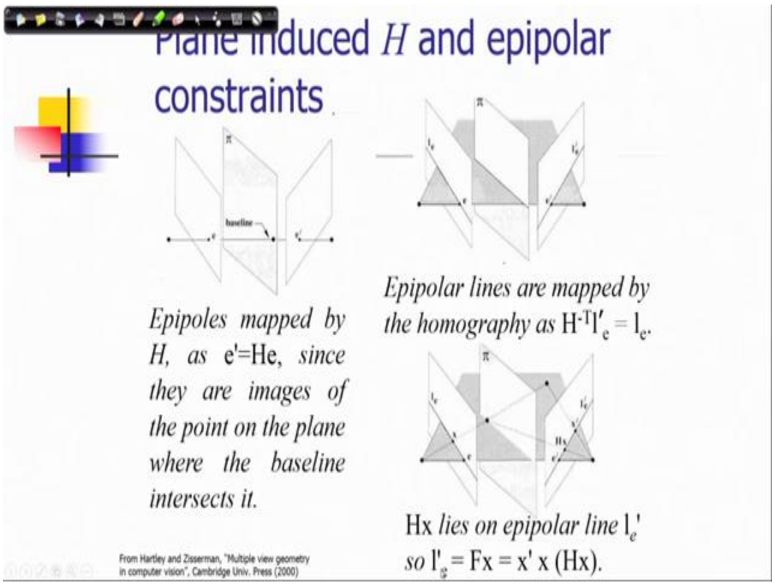

So, these are certain facts of the stereo geometry that once you have a plane induced homography; that means, in this case the corresponding pairs of points they are all formed from the scene points of a plane, so that is explained by that form. See you consider, a plane any plane you consider and in that plane if it is not parallel to the base line; there is an intersection with respect to that plane. So, once again if for that plane if you know there is a homography H; so e prime should be equal to He for that plane induced homography.

Similarly, we have mentioned that epipolar lines; corresponding epipolar lines which are lying on the same epipolar plane. And any particular line lying on the same epipolar plane they also form the plane induced homography. Because once again the plane which is inducing and points are lying in that plane and any intersection of that plane with respect to epipolar plane will satisfy this constraint. So, this is also satisfied; for any pair of epipolar lines this is satisfied.

And H x lies on epipolar line of L e prime, there is another constraint it has been shown that here also you can see that if you have; you have a scene point if you have some scene point here this is the epipolar line. So, epipolar line of a corresponding point can be obtained by performing the multiplication with respect to x F x.

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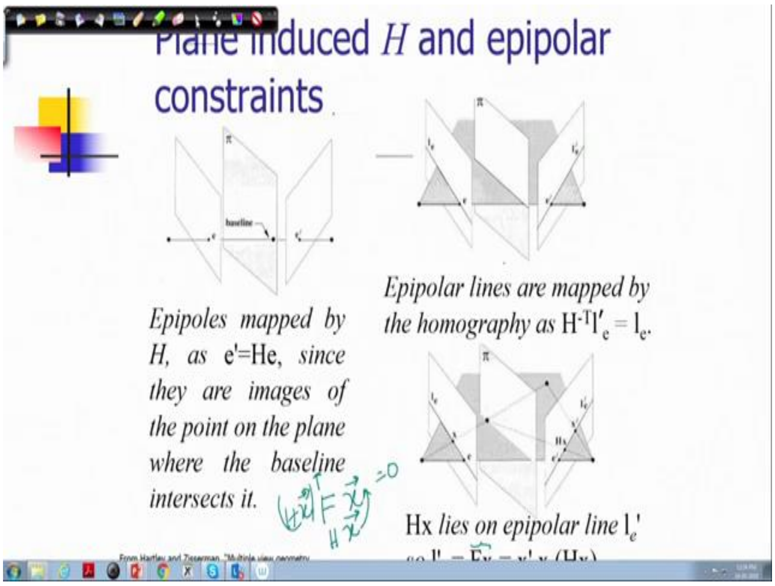

And then; that means, I can obtain the epipolar line by multiplying x and if I perform the homography transformation on x; so this point will also lie here; that means, I can write H x transpose; F x that should be also equal to 0. So, that is the essence of this particular computation.

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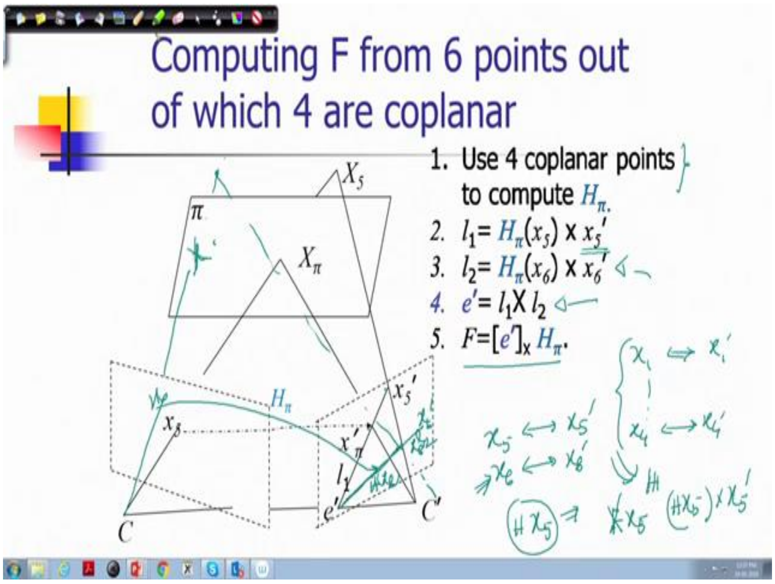

So, we will be considering; we will be exploiting this facts for estimating the fundamental matrix under certain constraint scenarios because that would simplify the computations. Suppose, you would like to compute fundamental matrix from 6 points; out of which 4 are coplanar. So, I can using that 4 coplanar points; we know that they establish a homography. So, their corresponding points; there is a relationship of homography among themselves.

So, we can compute the homography matrix H by using those 4 points and then you can find out the epipoles by computing the epipolar lines. So; that means, you consider the actual corresponding points which is been observed for x 5. So, you have x 5 corresponding to x 5 prime, then x 6 corresponding to x 6 prime; also you had x 1 corresponds to x 1 prime to x 4 corresponds to x 4 prime. So, first you compute the prime induced homography among themselves, you require minimum 4 point correspondences.

So, now if I transform x 5 using this homography; the same point will lie in the epipolar line f x 5 or I should say that; the epipolar line will be formed because F is not known. So, epipolar line will be formed by H x 5 cross x 5 prime. So, that is what is shown x 5 prime and this is the plane induced homography point; so they will form an epipolar line.

In the same way, using the other point you form the second epipolar line. So, you will get another points see you get x 6 prime and it is suppose this is x 6, this is x 6 the; it may be somewhere else, it is not here. So, this point may be here and say this is x 6 prime and after homography transformation; this is H x 6. So, this will also form another epipolar line; so this two epipolar line l 1 and l 2; their intersection will be e prime; that is expressed in this from.

So, if I take the cross product of l 1 and l 2; I will get e prime. So, now fundamental matrix can be defined as e prime cross H pi that is fundamental matrix; so you can compute it. So, using the 6 point no fundamental matrix from 6 points; you can compute using the 6 points, you can compute fundamental matrix.

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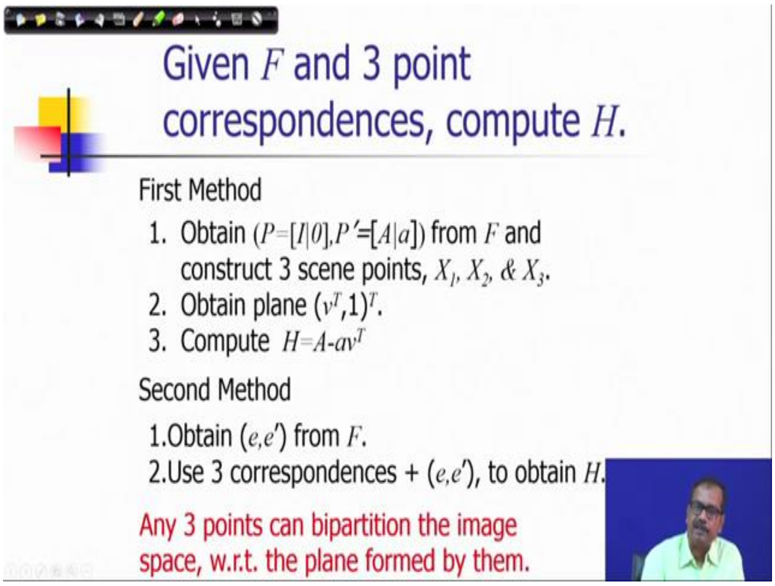

Now, here the problem is the; in the reverse. You have been given a fundamental matrix and 3 point correspondences and then you need to compute H. So, the first method says you can fundamental matrix you can always; no estimate the camera matrices P and P prime using the method what we discussed. And then you can obtain the plane because you have 3 scene points and you solve for the 3 scene points; construct 3 scene points and obtain the plane and then from there you can compute the plane induced homography that is H equals A minus a v transpose given the parameter in this form.

The second method is that; that for that 3 point correspondences, you require another point correspondences from which you can get this homography because minimum 4 point correspondences are required. So, what are those, what is the pair of point correspondences that is the epipoles. So, from F; you can compute this epipoles; that means, you get its left 0 and right 0; those are the epipoles and they are also corresponding points. So, now you have 4 corresponding points from where you can compute homography. So, this is also another interesting feature that any 3 points can bipartition the image space with respect to the plane formed by them.

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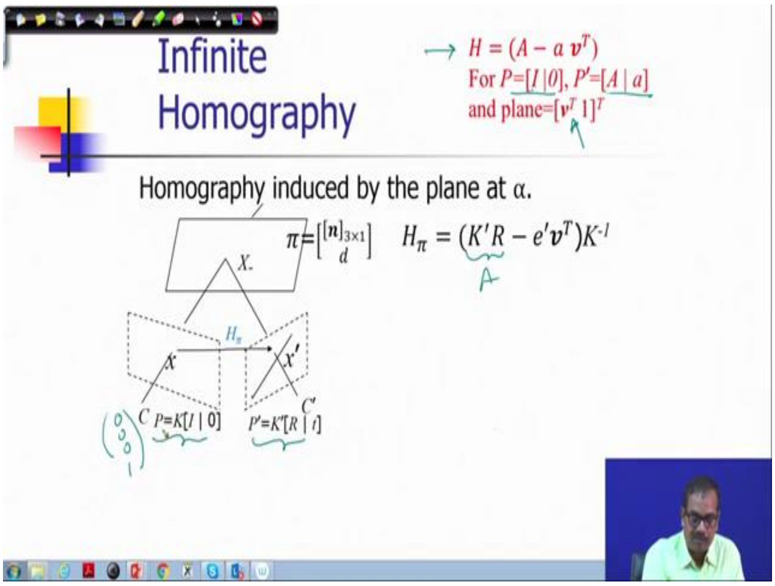

Infinite homography, we discussed earlier also and let us discuss it in the context of this plane induced homography. So, you consider any general plane induced homography which is the results are shown here at the top; here you can see that this is the general result of homography given your projection matrices in this form and also the plane three dimensional plane; the representation in this form.

So, in this case of course, this is given in the form of n d; so that we can relate it. So, you just if you just scale this normal vector by d that is equal to b; so let us see how this. So, now your plane in this tomography following this particular constrict could be written in this form. Now, here your projection matrices are given in this; so it is related with A; A is K prime R; so this is A. And your e prime is given by this know e prime is the image of these coordinate which is 0; centre is at since it is 0 center is at 0 0 at the origin. So, if I take the image here and we will get the corresponding; so it is t.

So, the epipole is image is at t right. So, this e prime we will find out e prime will be t and let us see what is K inverse? K inverse because this is I 0; so K inverse will come here. So, we can make it in a canonical form; if I transform the image coordinates by k inverse.

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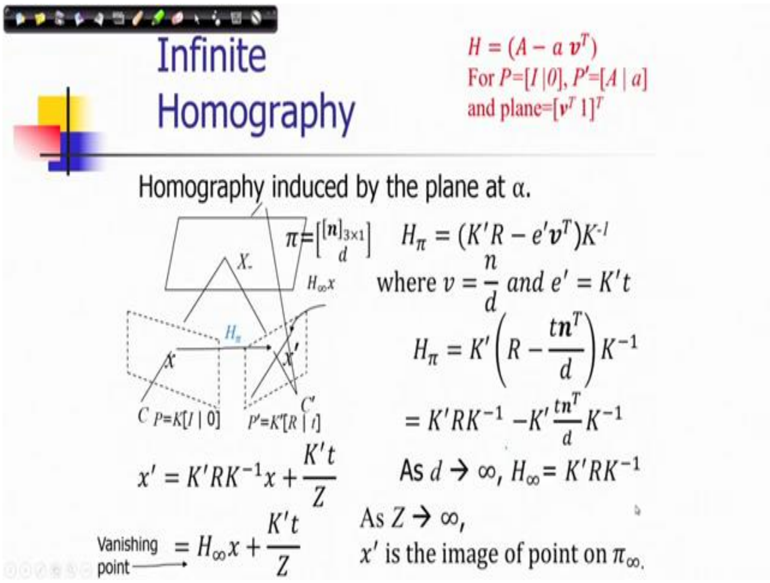

So, these are the things; so v should be equal to n d and e prime is K prime t; e prime is K prime t from here; so that is what is e prime. And then H pi following this you know formulation can be written in this form. So, you note that v is given this n d; so it is t n transpose d. So, finally, it is K prime R K inverse minus this. So, now as d tends to infinity H infinity is K prime R K inverse.

This is a formu[la]- which we know discussed earlier also that is the homography at infinity can be expressed in terms of these camera parameters. And we discussed also earlier how a in general configuration how the corresponding pairs of points are related with this particular parameter; so this is what we get using infinite one. So, as Z tends to infinity; x prime is image of point on pi infinity. So, you this is; this shows you that that is a vanishing point over theepipolar line; as you are you are projecting the points at infinities.

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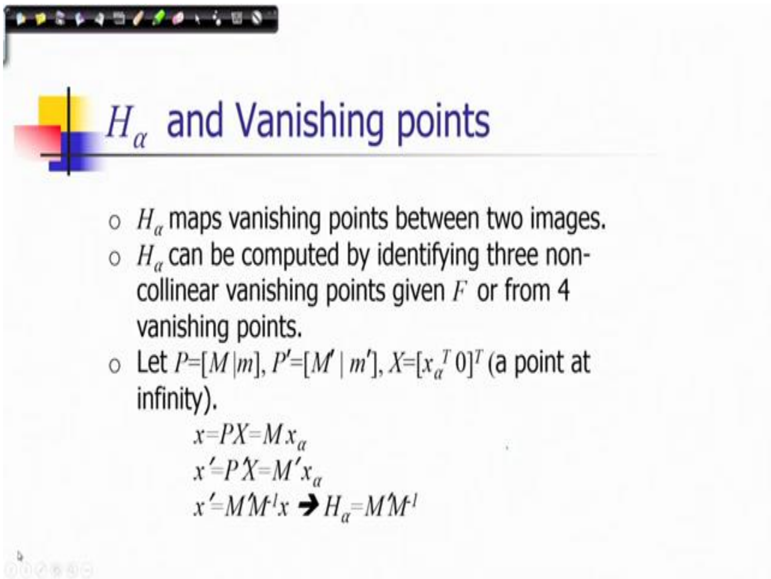

So, this is a summary between the line the homography of you know; homography at infinity at vanishing points H infinity. It maps vanishing points between two images and it can be computed by identifying three noncollinear vanishing points given F or from 4 vanishing points. So, we discussed earlier also that given three points and fundamental map fix; you can compute the homography.

So, if you have vanishing points ah and corresponding vanishing points; you can compute H infinity in that case or you can get 4 vanishing pair of vanishing points; you get again compute H infinity. And if your camera mattresses are given in this form P and P prime and any point at infinity is given in this form; we discussed earlier also this shows at H infinity is M prime M inverse.

This relationship we discussed while solving a problem itself that given a general camera representation, how homography at infinity or plane in this tomography when the plane at infinity is inducing plane is expressed in this from or H infinity is M prime M inverse. So, with this let me stop this lecture at this point we will be continue our discussion on stereo geometry in also subsequent lectures.

Thank you very much.

Keywords: Plane induced homography, line reconstruction, epipolar constraints, infinite homography