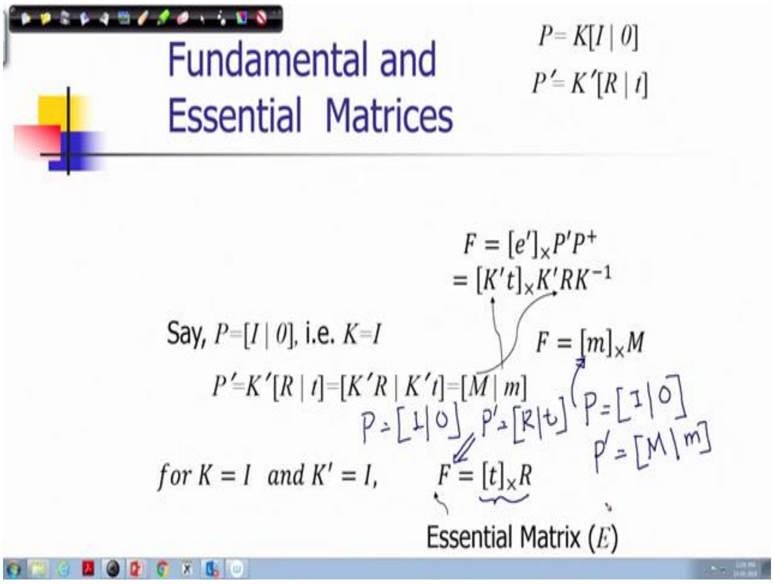
Computer Vision
Prof. Jayanta Mukhopadhyay
Department of Computer Science and Engineering
Indian Institute of Technology, Kharagpur

Lecture – 17
Stereo Geometry Part – II

We will continue our discussion on epipolar at geometry and we are discussing the definition we discussed about fundamental matrix and how this matrix is related with the parameters of projection matrices. First thing is that a fundamental matrix is a matrix if we multiply any image point it will give you the corresponding epipolar line in the second image plane.

(Refer Slide Time: 00:50)



And this can be; this can be computed from the camera parameters itself and those relations are shown here which we discussed earlier also, it could be in various forms. Here you can consider that you have the reference camera is in the form of say standard canonical representation say [I|0]. And the other camera this is given in this representation, with respect to this we can compute the fundamental matrix from this configuration you can compute it by this expression.

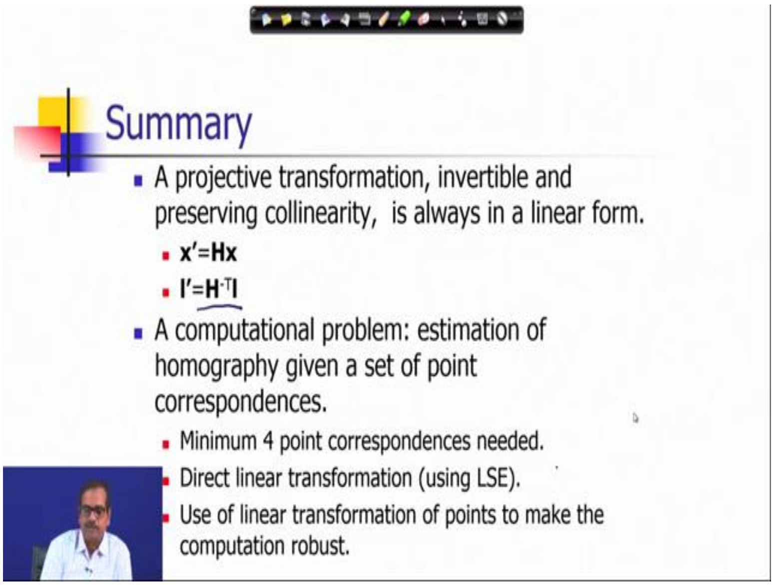
So, this is one kind of relationship that we discussed and we also observed that a fundamental matrix can be simplified into a form of an essential matrix if I use the calibrated camera which means I know the calibration matrices. And then I can make necessary transformations on the image coordinates so, that effectively calibration matrix becomes an identity matrix. So, in that form its fundamental matrix its reduces to this particular structure when you are defining the camera mattresses in this form. So, P is given as [I|0] and P' is given as rotation matrix and the corresponding, translation matrix translation of the origin.

So, they are given in this form. So, then the fundamental matrix can be obtained by performing this operation. So, it is a cross product of translation vector with respect to the rotation matrix. So, this is a relationship and we have also defined how this notation, how you can convert a cross product operations into this matrix multiplication from.

$$F = [t]_X R$$

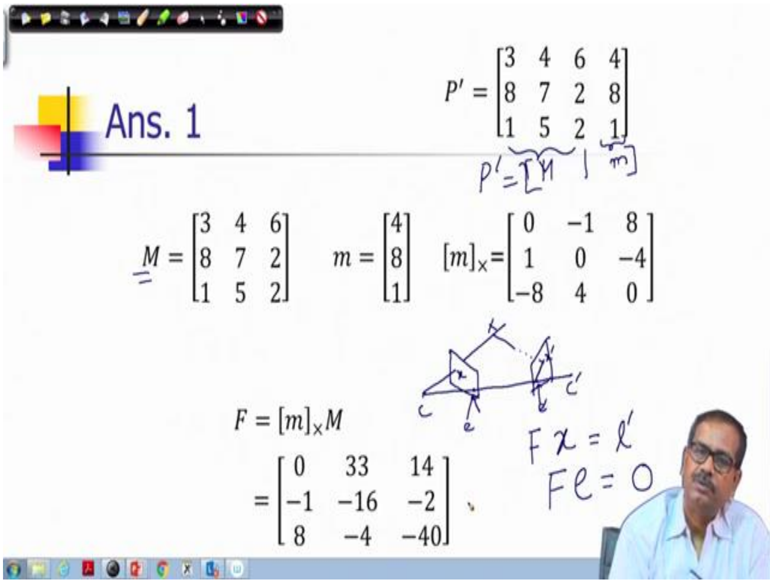
And then this fundamental matrix is called essential matrix and we denote in this particular structure by another notation E in our discussion. So, this is what we discussed in the last lecture, now I will consider solving a problem just to illustrate how these concepts could be used to retrieve various information.

(Refer Slide Time: 03:50)



So, take this problem, here in this case it is a problem of simply computing the fundamental matrix. So, you look at the problem once again that reference matrix P is given as [I|0] which is in the canonical form itself whereas, P' it is in the non-canonical form representation; that means, it is not calibrated. Its calibration parameters are not known to us rather I know all the elements of the projection matrix that is what is P'. So, with this configuration we would like to compute the fundamental matrix of the system. So, let us see how this configuration we can do.

(Refer Slide Time: 04:34)



So, this is given that P' as I mentioned P is also given here. So, here we will take this 3 X 3 sub matrix which is denoted here M and then we will also consider the column vector which is denoted the m. So, these are the notations we have used earlier. So, we represent this P' into this form into this notation with the sub matrices. Then we will apply the corresponding know relations between M and m which will are configuration involving M and m which will give us fundamental matrix.

So, we first will represent to the scheme symmetric matrix we will get the scheme symmetric matrix. So, which is performing the equivalent cross product operation with the vectors. So, from [4 8 1] we can get this scheme symmetric matrix and then fundamental matrix is given by these computation. So, if I multiply this matrix with M so, this m; m cross let me call this matrix as simply $[m]_X$ matrix. So, this $[m]_X$ matrix if I multiply with M, then we will get a 3 X 3 matrix.

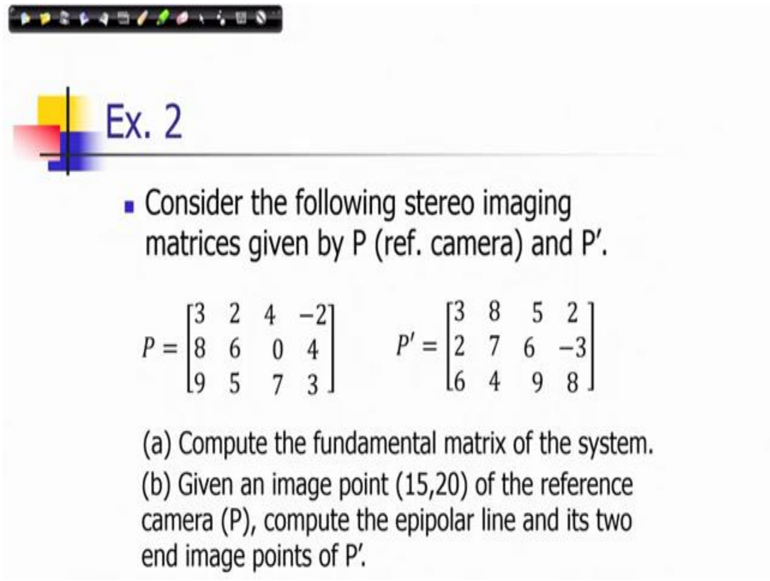
So, this is what the fundamental matrix is. So, as you understand if the camera matrices in these particular forms then it is very easy to compute the fundamental matrix. We will also discuss that how a general forms of camera matrices can be also used for deriving this fundamental matrix. One thing you should note one of the property of the fundamental matrix that is there which I need to discuss here, that in a fundamental matrix if I perform as I mentioned if I convert consider any image point x. And if I multiplying with F then you get the corresponding epipolar line in the right plane, in the right image plane.

And what is an epipolar line? Epipolar line is a line found by the epipole in this configuration. So, you have this configuration. So, this is center C, this is center C' and this is the baseline. So, these are the epipoles, this considered these are the intersecting points. So, this is e' and a point x and its corresponding image point say this is x this is x '. So, epipolar line is formed by these two points; one is the epipoles in the image plane right image plane with or we call it right epipole or e' and the image point. Now, you just consider this image point incidentally which is the left epipolar e.

Now, its corresponding image point itself is e', then what would be epipolar line, how do you form this epipolar line? So, as we have seen earlier also that actually this is the case of singularity. You cannot form a line by just by connecting the same at the same point itself, you cannot define it on a line just using a single point. So, that is why this mathematically this is geometric constraint geometric constraint that we understand. But, mathematically this constraint is expressed in this form if I multiply this fundamental matrix with this epipole in the image point what I will get?

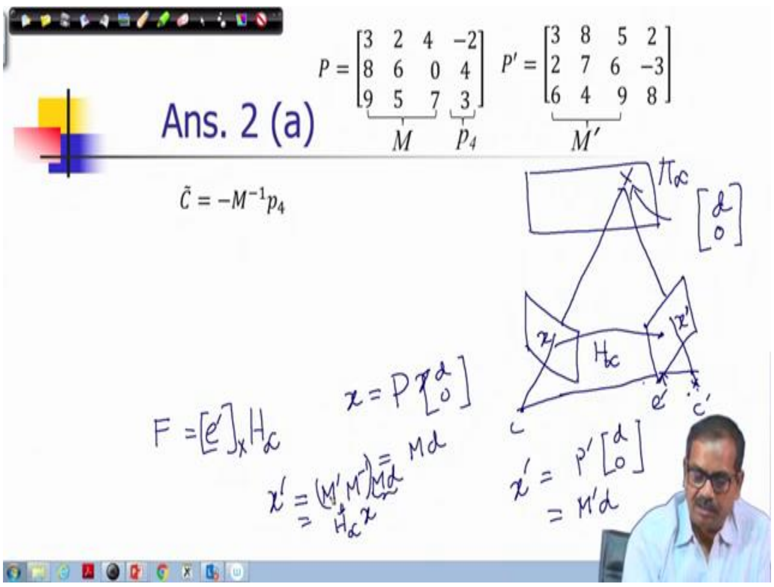
I will get a 0; that means, a 0 column vector. So, this is how this constraint is expressed and which means that this fundamental matrix has a 0 vector and from the linear algebra it says that; that means, this fundamental matrix is a rank deficient fundamental matrix. So, if I take the determinant of this matrix you should get 0 so, that you can also check with this result.

(Refer Slide Time: 09:19)



Now, we will proceed with the next example and in this case we will consider a more general scenario of projection matrices where, we would like to compute fundamental matrix epipolar lines. And also I think you can compute also the for epipolar line you need to compute a epipoles definitely and there are interesting concepts of end image points of P prime. So, let us consider let us discuss this solution.

(Refer Slide Time: 09:57)



So, if you would like to compute the fundamental matrix that is between P and P’. So, P once again is no expressed as in this form [M p4], similarly this matrix is M’. So, first we need to compute the homography at infinity what we discussed earlier; that means, we need to find out the homography between the corresponding points, between a point and the corresponding point of the same point which is lying at plane at infinity.

So, let us discuss these issue. So, we have a same point suppose this is plane at infinity. Now, for this convenience I am just showing it in the diagram as you understand plane at infinity is not realizable physically, but mathematically there exist a plane. And how a point in a plane at infinity will be represented? You consider any particular direction and then you use the scale factor 0. So, this is how the point obtain at infinities you know represented. So, this is my same point say X and in a stereo setup I would be considering the corresponding images of this point.

So, this is an image point x and this is so, this is my camera center. So, this image is x' and we are considering homography between these two and which we will be expressing as H infinity. So, how they are related with this let us see. So,

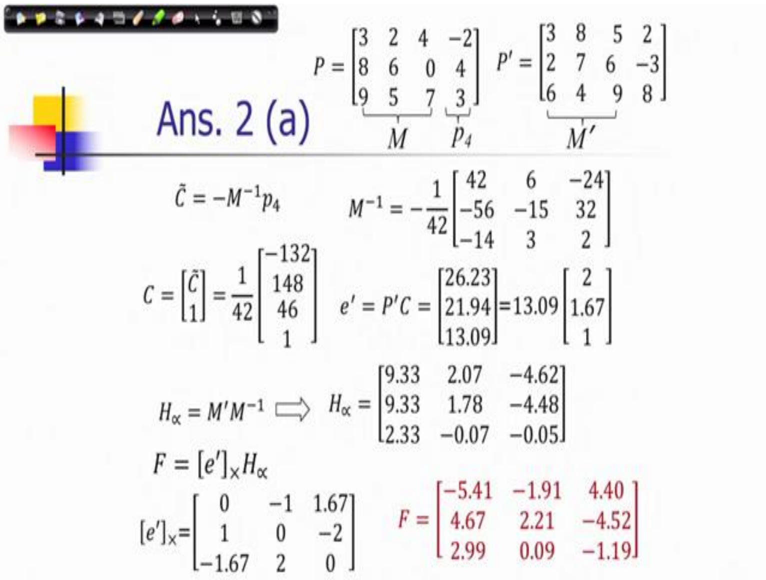
$$x = P\begin{bmatrix} d \\ 0 \end{bmatrix}, x' = P'\begin{bmatrix} d \\ 0 \end{bmatrix}$$

So, $x' = (M'M^{-1})Md$

You note that $M^{-1}M$ makes an identity matrix so, this will give you this. So, this can be considered as a 3 X 3 matrix H infinity and Md is x. So, you find there is a homography between these corresponding points. So, this is what we will be using here in this case because we know that fundamental matrix can be considered if I know the right epipole here.

See if the right epipole is e' then fundamental matrix is given by e'XH infinity that is what we discussed previously also. And then we have used the expansion of H infinity to get all those forms, but we will be using simply we can compute H infinity. So, in this problem what we will need to do, first we will be computing e'. So, e' is nothing, but image of the camera center of the first camera. So, we have to compute the first camera and then take the image which will give me e'. And, then we need to compute H infinity from M and M' by performing this operation which is $M'M^{-1}$ and then by performing this we can compute the fundamental matrix. So, let us carry on this computation as we discussed.

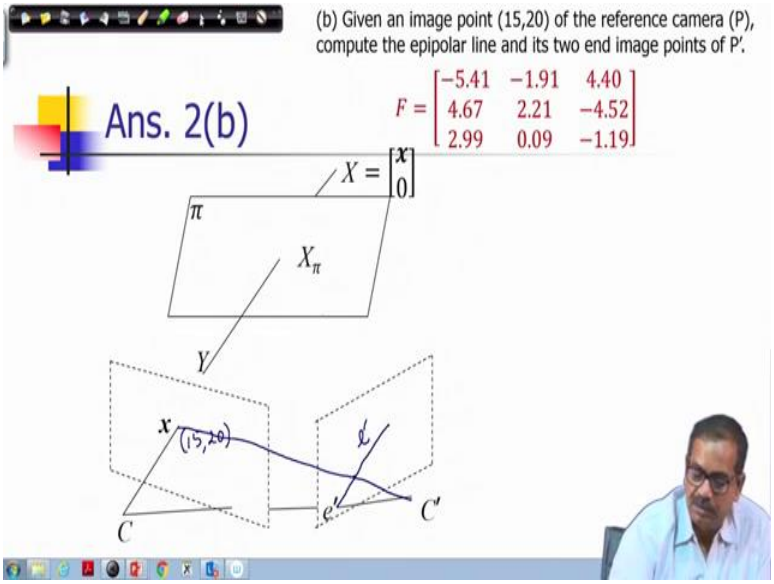
(Refer Slide Time: 14:49)



So, we have computed the camera center in this form. So, we need to perform the M^{-1} operation and then now you can compute the camera center $\tilde{C} = -M^{-1}p_4$ which is given in this form in the projective space. And then compute the right epipole as P'C which is given in this form. This is the image of the camera center and you which is giving you the right epipole and this is right epipole. So now, you have to perform the $[e']_X$ you have and you should compute the homography at infinity as we discussed which is $M'M^{-1}$.

And if I perform those operations we will get this homography matrix. So, fundamental matrix is $[e']XH_\infty$ and which will give you this particular form. So, $[e']_X$ is this one so, if you find out this is e' I can get the $[e']_X$ I can represent. So, I am ignoring this scale factor here, scale value is not important here and then you get the answer as in this form. So, you note that fundamental matrix is also an element of a projective space. So, if I scale these values it will also denote the same fundamental matrix.

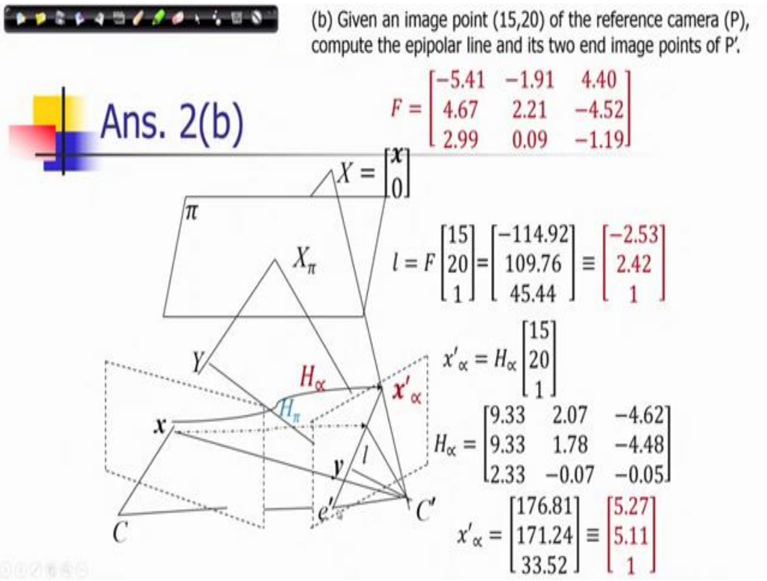
(Refer Slide Time: 16:10)



So, the next problem is that if I give you an image point which is given as say [15 20] in the reference camera. So, in this case suppose this point is denoted by the coordinates of 15 20 then I need to compute the epipolar line. So, which means if I join this, so, I need to compute the epipolar line so, this line.

So, what I need to compute? I have to compute the corresponding, so this is a thing what I need to do. So, what I will do? I will multiply this point with F then I will get this epipolar line that is the relation that is how fundamental matrix is related with a image point to its epipolar line.

(Refer Slide Time: 17:13)

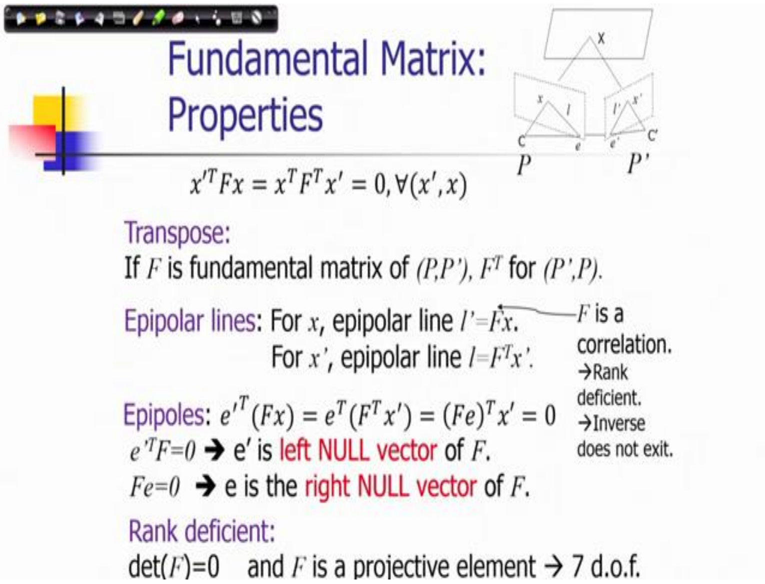


So, this is what we need to so, it is just showing that now all the points along these rays which is lying there. And also as we have discussed that there would be homography between these points, but finally, the point which is lying at infinity that is the limiting point. So, you see that is that concept of vanishing point is also here in this kind of epipolar line constraint. No other image point will exist, it will not there will not be any intersection beyond this point. So, you have an epipolar line, you have a very finite representation of epipolar line.

At one end there would be right epipole and the other end there will be the image point corresponding to the homography of plane at infinity or corresponding point of the same point which is lying at the plane at infinity. So, this is what we sometimes represent in this notation also and this is this diagram is explaining this part. So, this is how the epipolar line is computed, I multiplied the point [15 20]. The representation is [15 20 1] in the homogeneous coordinate space and if I multiply it with F. So, I will get the epipolar line in this form which I can again reduce in into this form by taking the third dimension scale equal to 1.

So, which means the equation of this line will be $-2.53x+2.42y+1=0$ and to get the x infinity because, in this case problem also you wanted to know the end points of the epipolar line. We can compute or we have already computed homography at infinity simply you can multiply with this point and applying this we can get this is a point. So, we know epipolar epipoles of in the right image plane and also the corresponding end point of that epipolar line in this way.

(Refer Slide Time: 19:24)



So, now let me summarize the properties of fundamental matrix. So, we have discussed and we have also shown how you can compute fundamental matrix given the projection matrices. How you can compute the epipoles, again given the projection matrices and also we have discussed how the homographies are particularly homography at infinity is inducing induced in this case. They are they can be they are related how the how it is related also with the elements of projection matrices.

So, now let us consider the properties of fundamental matrix, we will be summarizing some of them we have already highlighted. So, the very basic property that if you get two corresponding image points of the same scene point then they are related by this particular relationship $x'^T Fx=0$. When we considered the reference image point is x and the second image point is x'.

If I consider the other way then again you can use we can also convert this relation as $x^T Fx'$ which means x' is the reference image point. So, you note that the matrices F and F transpose they are related in this fashion. So, F is the fundamental matrix of this configuration and F transpose also is a fundamental matrix of the stereo configuration, when we change the plane of reference or reference camera, image camera.

And this is true for any pair of corresponding image points that is the property first property. So, next is this transposition what you have observed that if F is the fundamental matrix of camera setup P, P'. So, we will be denoting a stereo setup by a pair of camera matrices. The first one in that couplet the first one we will denote, the camera matrix of the reference camera.

So, P, P' if F is the fundamental matrix for that setup and for P' P then F^T will be its fundamental matrix. Then the properties with epipolar lines, that if we have an image point x then epipolar line in the second camera is expressed as $l'=F x$. Similarly, if you have an image point in the other plane x' its epipolar line in the reference plane would be $l=F^Tx'$.

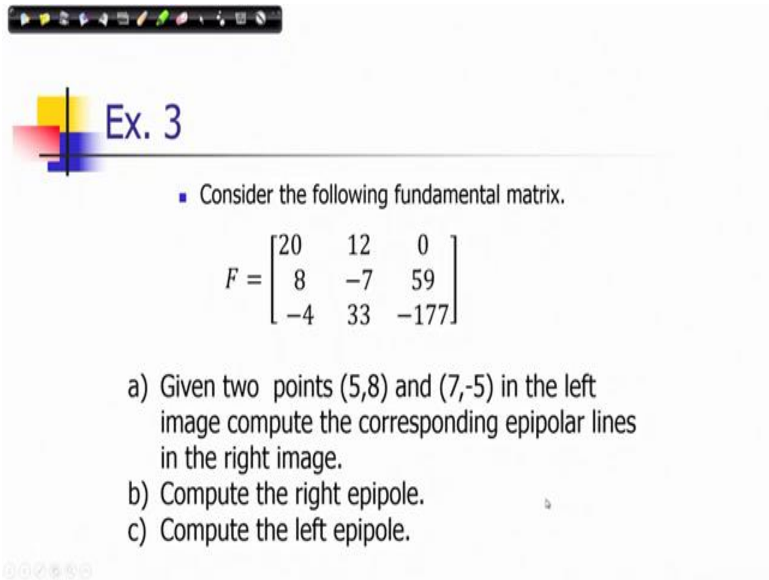
And this is what I mentioned earlier that, if this relation is called correlation because it is converting a point into a line, its transforming this kind of transformation is known as correlation. F is also rank deficient we discussed this while discussing about the solution of a problem. And so, which means that its inverse does not exist and how then this epipoles are related with x that we discussed. In fact, epipole also they are lying on the epipolar lines always they will lie on an epipolar line. And they are the intersections of all the epipolar lines which is given by this e prime transpose F x that should be equal to 0 or e transpose so, these are the conditions.

So, this should be equal to 0 or $e^TF^Tx'=0$, $Fe^Tx'=0$. So, it is just denoting the point contentment relationship of epipoles on their respective epipolar lines. The other thing what we I have discussed just a we made a very brief mention about that, that F e equals 0 which means e is the right null vector of F. Similarly, e prime transpose F is also equal to 0 and e' is the left null vector of F in the same way. So, these are some important properties of fundamental matrix and this is another property that determinant of F is 0.

Because, it is rank deficient we have mentioned we have discussed that and particularly it is interesting to note that how many independent parameters are there. So, one thing I already mentioned that there is a scale factor associated with F which means if I scale the elements still it will give me the same, it will express all those relationships; you can note from the relationship itself. So, for example, you take the first fundamental relationship between the corresponding points say $x'^TF x = 0$, if I multiply F with an scalar value k still this relationship holds.

So, you can check with any other relation that would be the case and then this so, this is one particular constraint that the scale is there is a scale factor. So, which means F has a 9 elements and it reduces one particular parameter by that constraint. The other thing is that its a rank deficient, its determinant is 0 that is the second constraint. So, there will be 7 independent parameters out of those 9 elements so, its degree of freedom is 7.

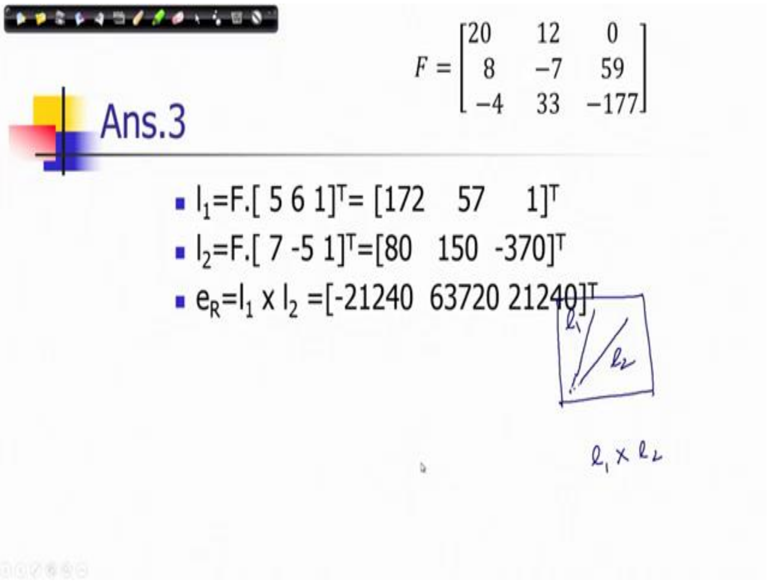
(Refer Slide Time: 25:15)



So, we will be using this property to discuss about know to you know solve a problem here. Suppose, you are given only fundamental matrix, in our previous problems we have given you the projection matrices and from there you could very easily compute the epipoles. Because, by applying that property that epipoles are images of camera centers and given projection matrices you know how to compute camera centers. But, now if I just give you simply fundamental matrix; can you compute its epipoles?

So, this is what we will we would like to say, first thing is that in this we are also given an additional problem that is to compute the epipolar line. So, the first problem is about computation of epipolar lines. Suppose you have two image points 5 8 and 7 - 5 in the left image and you have to compute the corresponding epipolar lines in the right image. And then you compute the right epipole and compute the left epipoles. So, as I suggested previously also you should stop, you should give a pause at that in my video and then again you should work out on this problem. So, let me discuss the solution of this problem.

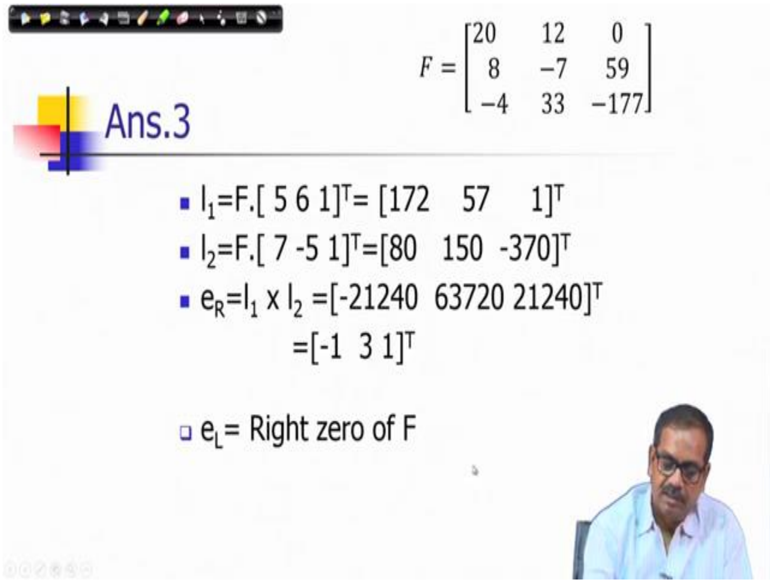
(Refer Slide Time: 26:28)



So, consider this is the fundamental matrix and for the image point [5 6] its epipolar line is given in this form. If I multiply with F, I will get its epipolar line l1 which is given by this vector, similarly I can get the epipolar line for the other image points [7 - 5 1]. So, you have two epipolar line so, that is giving me the answer for the first part. But, one interesting thing I should note here you should note here that suppose this is your image plane where epipolar lines are computed. That means, is second cameras, image plane of a second camera; say this is your l1 and say this is your l2.

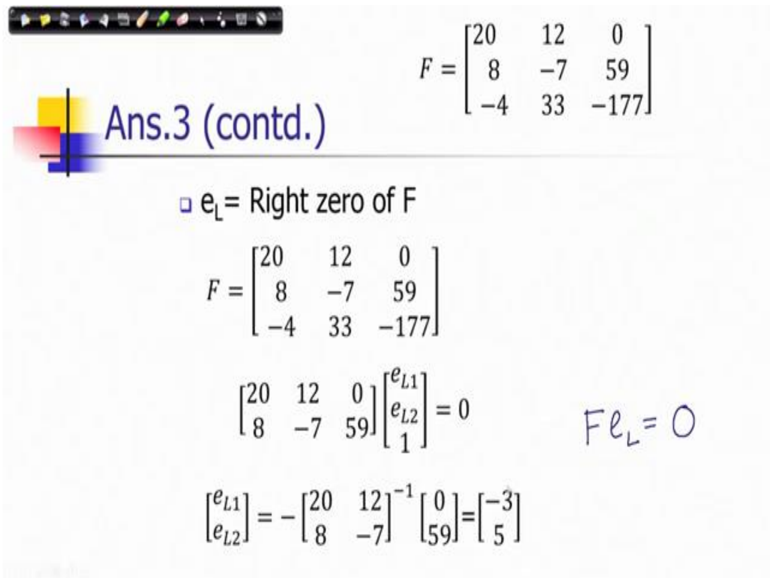
So, you know that all epipolar lines they intersect at epipoles. So, simply by finding out the intersection of l 1 and l 2 I can get the right epipole. So, what I need to do? I need to perform only l 1 X l 2. So, that is what we can do here. So, if I perform that l 1 X l 2 I will get a epipolar I will get the epipole. I mean it is a big numbers, but you can always not choose the scale to make it a smaller value, make all those values small; I mean which is shown here as [- 1 3 1] that is the value.

(Refer Slide Time: 28:04)



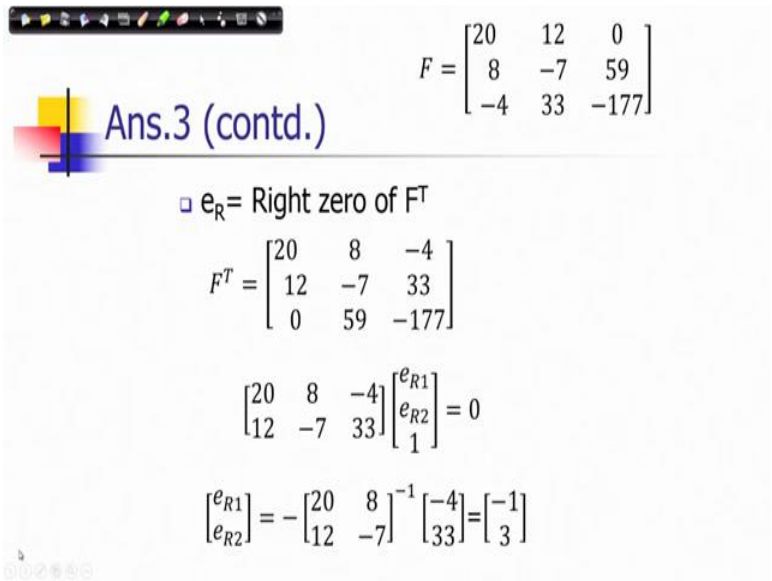
So, the coordinate of epipole in the right plane is [-1 3] in the conventional or two-dimensional real geometry. To compute the left epipole I could have taken note to such any two arbitrary image points in the right image plane and find out they are epipolar lines and apply the same technique. But, instead I will be discussing about right zero by finding right zero of F because that would also give me the left epipole which means I have to compute F.

(Refer Slide Time: 28:44)



The right zero means that if I multiply the left epipole with F, it should give me a 0 vector which is a know 3 vector form. So, this is how this competition is shown here. Since, it is a ranked efficient matrix one of the row I can ignore, there are only two independent rows. And then I will convert this equation, I will assume that the coordinate is given of the right zero is given in this form [eL1 eL2 1]. So, I have to take the inverse of this part minus of this I can reduce this part, use the sub matrix operations and you can show that this is nothing, but computation of this particular configuration. So, you will get the right zero of F as [-3 5]; I mean it is giving you the coordinates already in the image plane, image coordinate plane, non-homogeneous coordinate system because scale factor you have already assumed as one.

(Refer Slide Time: 30:05)



I will show you also that actually the right epipole what you have computed as an intersection of two epipolar lines we can compute it also a right zero of F^T. So, you take the F^T so, just transpose the matrix F and once again apply the similar technique; that means, ignore the third row and convert the equations, you form that these two equations or set of equations in this form. And then you solve for the corresponding right epipoles and you will get again you see that you are getting the same answer [-1 3]. So, with this let me stop this lecture at this point. We will continue this discussion in my next lecture.

Thank you for listening this lecture.

Keywords: Fundamental matrix, essential matrix, right epipole, left epipole