Computer Vision
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Lecture – 15
Camera Geometry Part – V

So, we will continue our lecture on single view camera geometry. We are discussing about different kinds of homography that exists in a single view camera, when you have multiple view images of the same scene by single camera.

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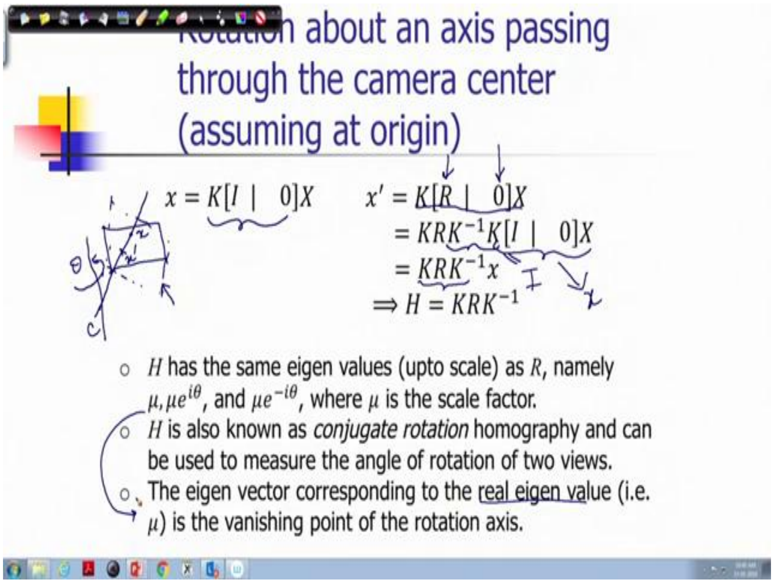

So, right now, what we are going to discuss suppose, the camera center is fixed, but you have rotated the image plane about its vertical axis. So, you consider this scenario that you have an image plane and you consider a center C and there is an image formed in this plane which is given by x.

Now, if I rotate this image plane say by an angle θ about an axis. So, in that case you can consider another image plane due to this rotation and you will get an intersection with that image plane with a point x' in that image plane of that rotated camera. So, now, we will see that there exist a homography between this two points. Already, we discussed this particular feature, when we discussed about the projective transformation. In terms of camera matrix parameters or camera matrices, we can relate this homography.

So, consider in this case, the first position as a reference position of the camera and its projection matrix is given in this ($K[I\,|\,0]$) form. So, we can note that this is almost a camera centric coordinate system. This is a camera centric coordinate system, but you have a calibration matrix. so, you have a calibration matrix's involved in this case. So, in that way I mean you can consider that ; it has those parameters of calibration matrix involved in this case.

$$x' = K[R\,|\,0]X$$

So, this is the projection matrix that we get and when you rotate this camera everything remains the same, so there is translation parameter; translation of origin, it is 0, because there is no translation of origin in that case and that is reflected by this, values 0 column vector, but it has the rotation matrix. This R is a matrix, l which denotes a transformation due to the rotation about this vertical axis or θ.

So, I can write this projection matrix of the second position as $K[R\,|\,0]$. So, this is the projection matrix. So, if I consider that I can simplify this by considering this so, you know that here I can equivalently right it as $KRK^{-1}K$, because $K^{-1}K$ is nothing, but it will give you the identity matrix. So, I can write this also and as we know that $K[I\,|\,0]$, this is the projection matrix of the first camera.

So, this is what is giving you the image point of the first camera. So, finally, x' is related with x with this kind of relation. So, it is a matrix multiplication KRK^{-1} that you need to multiply with x and that matrix itself is the homography. So, this is how the homography is established between these two views and with their corresponding image points of the same scene point. And there are certain interesting properties of this homography matrix, because this rotation matrix has a interesting property like first thing is that it has a same eigenvalue of this rotation matrix and rotation matrix eigenvalues are well defined. Because of this property of particular structure of the rotation matrix which rotates at an angle θ about an axis.

So, it should be μ, $\mu e^{i\theta}$ and $\mu e^{-i\theta}$. So, it is a complex quantity as we can see and μ is a scale factor. So, it is in eigenvalue or even I can write it as $1e^{i\theta}$ and $e^{-i\theta}$. So, it is a scale value.

Now, H is also known as conjugate rotation homography and can be used to measure the angle of rotation of two views. And the eigenvector corresponding to the real eigenvalue, which is μ it is shown here as a μ, which is a scalar, which is also indicating scale value on this set of eigenvalues. So, that eigenvector is the vanishing point of the rotation axis. So, that is another interesting information. So, we will let us see that how this homography could be used in relating different imaging.

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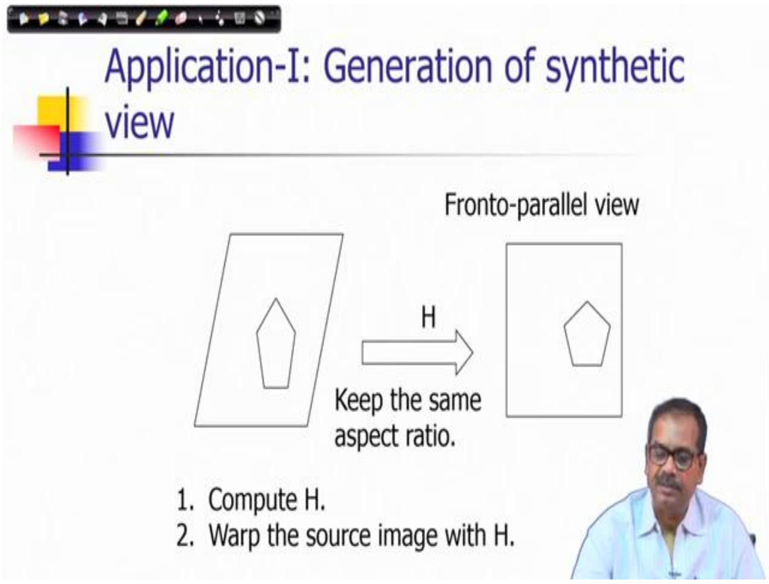

Some of the applications that we would like to discuss of projective transformation associated with this single view camera geometry, what we are discussing earlier. So, already we have considered the corresponding, affine rectification or stratification tasks that is involved using the homography with the images. Now, we can point out those applications once again here.

So, we can generate synthetic views given a view. So, as we did in the previous example of my lectures. So, you have an oblique plane where with respect to your reference view, if there is any oblique plane; that means, it is you consider our natural tendency is to look at fronto-parallel planes. And any oblique sensation of that kind of sensations will be coming with respect to that oblique fronto-parallel plane.

So, if you have a plane which makes an angle with respect to fronto-parallel plane and there is some object planner object or image on that plane. So, we can apply this homography to straighten out it, to make it on the fronto-parallel plane. So, you consider this particular example so in a fronto-parallel view we by keeping the same aspect ratio, we can redefine the image points and then we can compute the homograpghy. This example we have already discussed. So, we can compute and we can wrap the source image with H.

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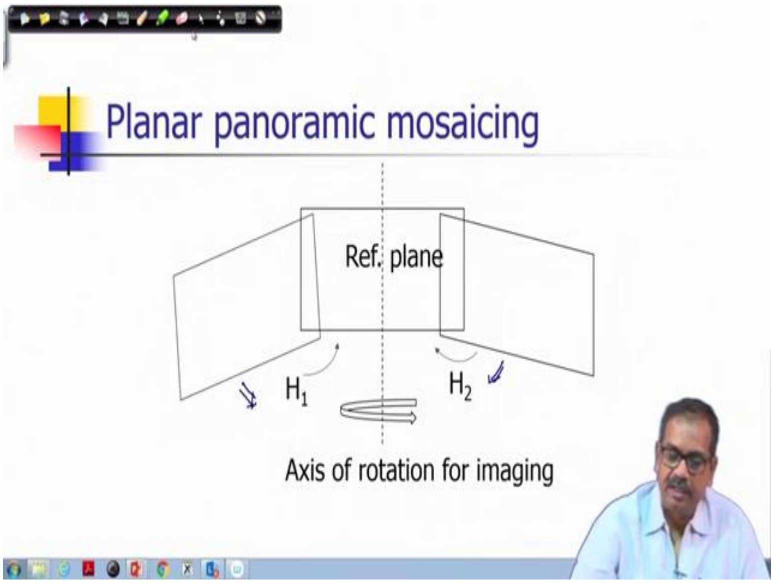

Another kind of application could be panoramic mosaicking of images. So, consider you have a wide view, but at a particular time you have only a limited view of taking images. So, you have a wide panoramic scene and with the single camera, you want to capture the whole scene, but your camera is restricted by only a small viewing angle.

So, you have an image plane, where only those points, which are intersecting with respect to the image plane, which is been sensed by the sensors of your camera those are only captured. So, you may in that case capture a series of images by rotating the camera. So, getting view, getting images from different views and then again perform the homography transformation with respect to a reference plane and put them under the same reference plane.

So, all those images; so they would look like as if images on the simple planar plane. So, that would that is the task of mosaicking. So, we discussed in my previous slide that how rotation of an axis lead to the homography. So, you consider here that this is your reference plane and these are the other views from where , you are looking at from your camera.

So, you apply homography from points on this view to this view say suppose, this homography is H_1 and again apply homography from point from this view to this view and then all the points are registered on the same coordinate system and you can get the larger image. So, that is how you can get a panoramic mosaicking, using this kind of computation.

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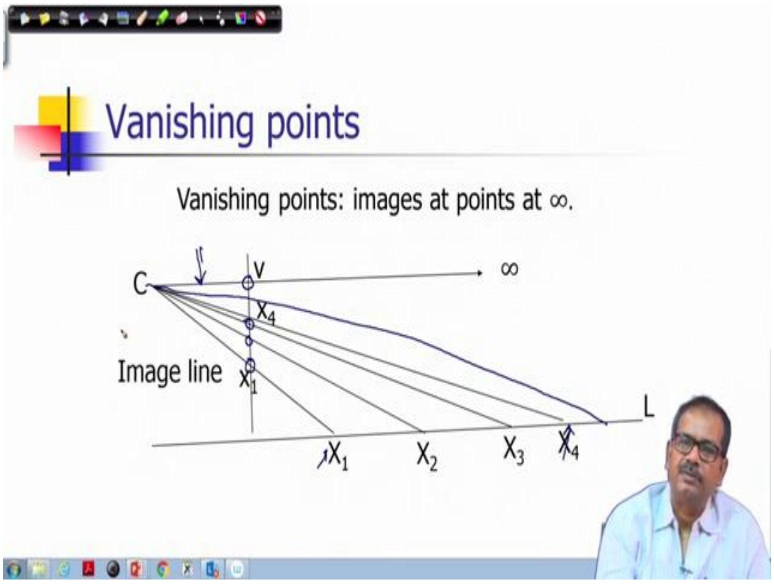

So, let us discuss the concepts of vanishing points in imaging also, I have referred to vanishing points in my lecture previously also. Vanishing points with respect to homography, vanishing points; that means, with respect to projective transformation, even for vanishing point with respect to the imaging from the camera transformation that also have the similar kind of features. So, vanishing points are nothing, but images of points which are at infinity.

You consider so, let me give you an analogy with respect to an one dimensional scene. So, an one dimensional scene can be thought about as an infinite line, any points on an infinite line. So, you are considering this as an infinite line and points are lying on it and we are projecting any point with the projection rule is that there is a center and take any point in this line and then you draw a ray from that line to that center and that would intersect on your image plane. So, in this case it is also image line. So, the point of intersection of this ray is giving you the image of this particular scene point. So, scene point as I mentioned is one dimensional space.

So, if I go on drawing this kind of imaging for all the points in this line. So, we will see its effect. Suppose, you take another point, once again this is a new image point then another point. So, these are images corresponding images that you are observing and if you go on doing these things. Finally, as you understand there is a limiting point beyond, which you will not get any intersection point, because once your projection ray becomes almost parallel then two parallel lines, they only intersect at point at the infinity.

So, what we can observe here that if you go on doing this finally, there exists a limiting point V which is defined geometrically in this way. Consider a line which is parallel to L and passing through that center C and then that line when it intersects image line at point V that is the vanishing point, because any other point you choose from this line, it will still intersect at a point which is not going beyond V in upwards in this case.

So, this is the interpretation of a vanishing point, when we restrict our scene as a one dimensional straight line and also our imaging plane imaging structure is also a line. So, we extend this idea for imaging of a three dimensional scene on a two dimensional plane and we will see that what kind of conditions we get as I mentioned this is the vanishing point and this is the summary that vanishing point of a line L is the intersecting point in the image plane parallel to L and passing through the camera center C.

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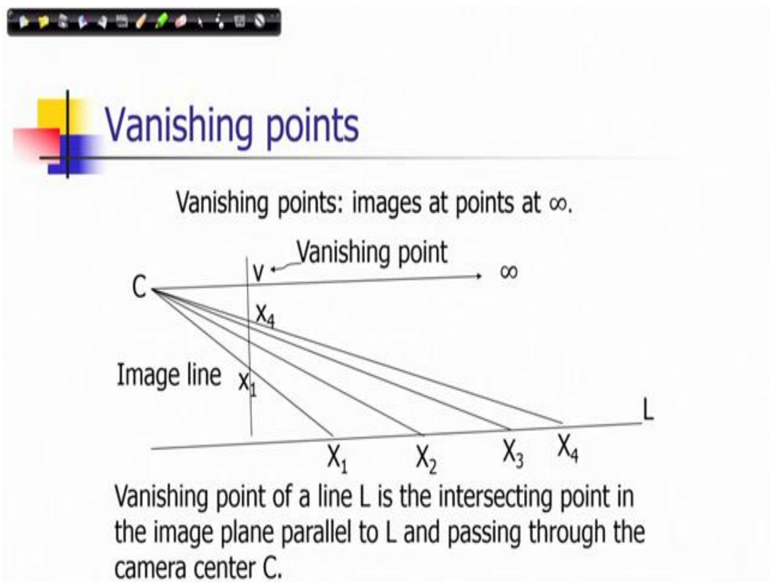

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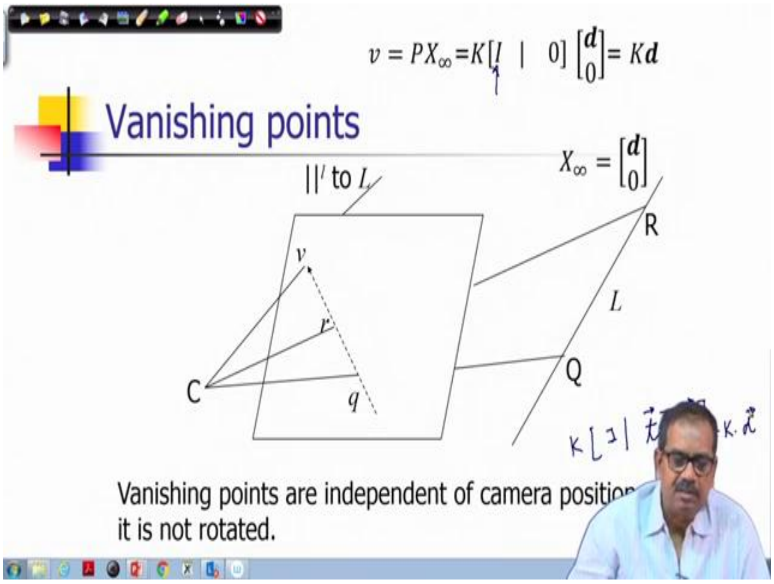

So, if I extend this concept as I was mentioning consider a three dimensional line in a three dimensional space this line is defined. Once again, your imaging geometry is defined by a center of camera or center of projection C and there is an imaging plane. And if I apply the similar projection constructs, so you will find all the projected point on the image plane, they also lie on the line. And finally, when the ray projected ray connecting through the camera center C, it becomes again parallel to the direction of line L then I mean there is that their intersection point would be a point at infinity, which means it that is the point at infinity in that direction L. And that is the, and their point of intersection and the point of intersection of that ray with the image plane acts like a vanishing point.

So, let me see the construct here in the similar fashion will go on constructing and finally, as I mentioned that the line parallel to L which is intersecting at the point V that becomes the vanishing point. You note v, r, q they all lie on a straight line, because it is an image of a straight line and that would be also a straight line.

$$v = PX_\infty = K[I \mid 0]\begin{bmatrix} d \\ 0 \end{bmatrix} = Kd$$

And so, even you can move as you go further and further still you will never cross v in that directions, in this particular line L in that direction. So, that is the vanishing point of the straight line L; so, this line is parallel to L. And as we already discussed that how a point at infinity in a particular direction d is denoted, it is $\begin{bmatrix} d \\ 0 \end{bmatrix}$. So, d is the direction of line L and if I apply the projection of that point, if I apply that mathematical model, where I am assuming that the camera model as a simple canonical form like $K[I\,|\,0]$ then into $\begin{bmatrix} d \\ 0 \end{bmatrix}$ that is a column vector, we will get Kd, that is the vanishing point.

So, in this case you know you note that the vanishing point is independent of the, translation of this particular or independent of the center or the position of the camera center. So, that is the camera position. So, if it is not rotated as you can see from the transformation matrix that here, we have taken only identity matrix, which means there is no rotation. It is a orientations of the axis remains the same, only you may translate the camera, but still if I apply that also.

$$K[I\,|\,t]\begin{bmatrix} d \\ 0 \end{bmatrix}$$

So, if I considered in that transformation, the new camera matrix after translation would be something like this K I say translation. So, we may have a vector t and then, if I consider the projection of vanishing point $\begin{bmatrix} d \\ 0 \end{bmatrix}$, so you understand d is a column vector and then what happens. So, t is also a column vector in my representation. So, this is also K into d. So, this how a vanishing point you know is computed and you could see that is the same point.

So, this is an important mathematical explanation that why we see the points at distance and there is almost they are always at the same position, because if you also move in with respect to our frame of reference they do not move, they also move along us. Like if you consider when you are moving and watching the moon at a distance, you will find in your frame of reference, the moon always remains at the same point, because you know if your motion does not involve any rotation, it is a simple translatory motion.

So, it will look, it will be dimmed like it is not. It is also, as if it is also moving with you, which means it remains at the same location with respect to you. So, this is one you know nice explanation of those phenomenas.

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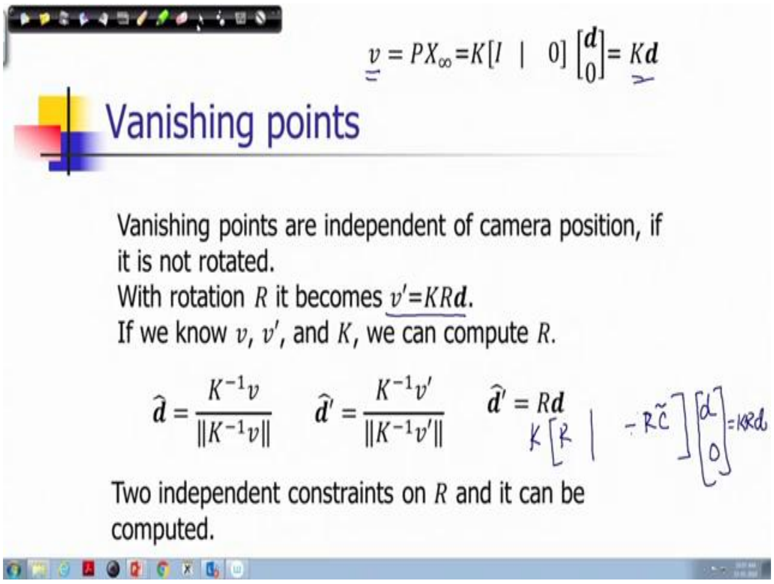

So, this is the follow up of that discussion that it is independent of camera position vanishing points if they are not. if the camera is not rotated. But you know when you have a rotated camera and then also you can get this expression, very simple expression by applying the same mathematical logic that if I rotate the camera center so your projection matrix can be written as say KR and then so, I think this was our $-R\tilde{C}$ that is the affect of moving the camera center at position $\tilde{C}$.

So, suppose this is the corresponding projection matrix after the rotation R and moving it at a camera center $\tilde{C}$ then if I apply the projection of vanishing points then I can simply write it as KRd. So, that is what is shown here, you can see this is the expression here.

So, the implication is that if I know the vanishing points, there are ways by which you can compute the vanishing points you may take straight lines, in parallel straight lines and take that images and find out their point of intersection, that would give you the vanishing points. And then if you know those vanishing points and also if you know the camera parameters like calibration matrix K then we can compute rotation. So, this is one of the interesting implication of this result.

$$\hat{d} = \frac{K^{-1}v}{\|K^{-1}v\|}$$

So, this is how even the direction of the corresponding line also can be computed from this relationship that is you can apply K inverse v normal to this. So, you can apply this particular relationship you can translate them in this form. You can also get the direction of the straight line in a three dimension, if you can get it is vanishing point knowing it is corresponding calibration parameter.

And also similarly, you can get the in the reference frame you can get another direction of the straight line with respect to the reference frame. So, from there you can get the rotation matrix. So, if the relationship between these two directions again, if they are related with the same transformation R and there are two independent constraints on R and it can be computed using this relationship.

So, this constraint let me explain that it is an orthonormal matrix that is the first constraint and there is a angle of rotation which is involved. So, you need to determine that. So, with this you can find out.

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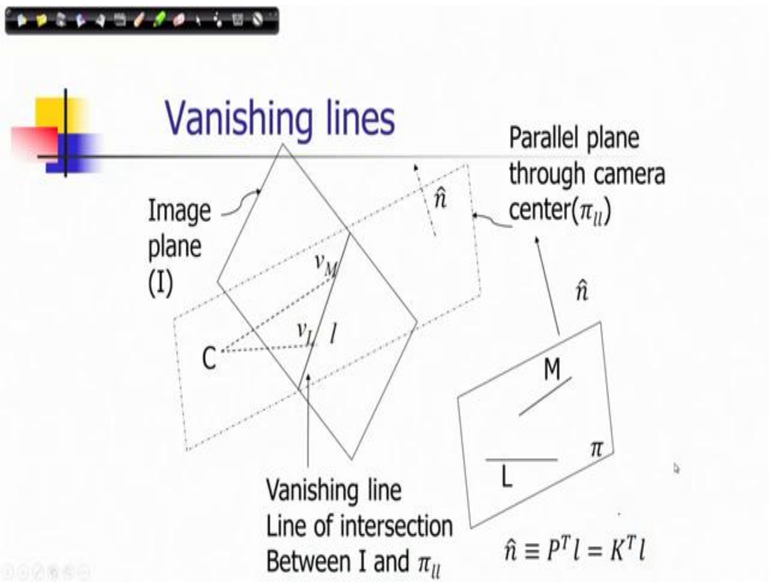

So, now let us consider another kind of geometric information or interpretations with respect to imaging. You considered a plane π and which is specified by its unit normal there is a direction that normal is known to us and of course, to specify a particular plane you need to know its point. So, in this case let us consider, we know a set of straight lines in this plane π , those are also specified. So, if I take the images of these straight lines, what will get.

So, if you consider a plane which is parallel to this plane; that means, whose normal of the plane is also the same in $\hat{n}$ as it is shown and so, that is the construct. it is a parallel plane through camera center and we can denote this plane as you know π_{II} as it shown here. So, what you are getting here, you are getting the vanishing points of the directions of M; that means, all parallel lines or parallel to that direction it has a vanishing point say at V_M and similarly V_L is the vanishing point related to the direction specified by L which is lying on this plane π. So, any line parallel to that it has its direction V_L.

So, if I connect these two points you will get a line on the image plane and infact that is the vanishing line the interpretation is that all the vanishing points of the lines lying on that plane π that would lie on that line. So, even any line or any parallel for any plane parallel to it, all those lines in those directions they have the same vanishing line. So, that is the interpretation. So, we can define then the vanishing line L in this way that it is a intersection of these two planes; that means, a plane π_{II}, which is parallel plane through camera center, parallel to the plane which we are mentioning here.

So, it is a plane which is parallel to this and which is passing through camera center and when it intersects with the image plane, then the intersecting line two planes they intersect at a line. So that intersecting line is a vanishing line that is the geometric interpretation of vanishing line and it is related with the plane. So, it is the vanishing line with respect to this plane. So, this is the interpretation.

So, from this we can compute also the normal of the plane because if you know the vanishing line, then I can compute the corresponding plane itself by using that same transformation $P^T l$ and would give me the normal to the direction of the plane for which this line is the vanishing line. And if it is a reference camera; that means, if the camera is represented by this projection matrix $K[I \mid 0]$, that is the projection matrix of this camera. So, if I multiply with l, it would be simply $K^T l$. So, for unidirectional you need to do the normalization of that vector.

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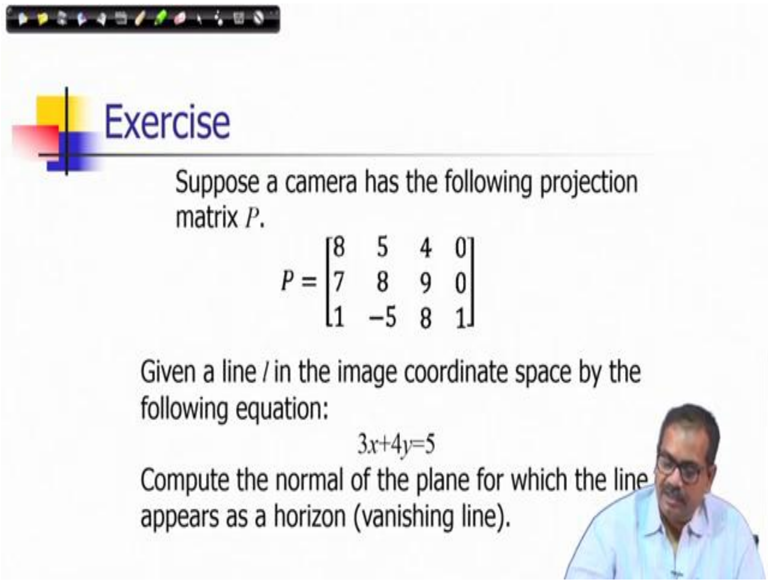

So, let us again workout an exercise for involving this particular computation. Suppose, you have a camera which has this following projection matrix that is P which is given here as you can see, and suppose you have a line in the image coordinates space by the equation $3x + 4y = 5$. So, we have to compute the normal of the plane for which the line appears as a horizon. So, you understand your horizon should be the vanishing line of that plane which is the parallel to our particular view.

I mean where this w which is having this particular vanishing line. So, you would like to get that plane and that is intersecting with the image plane with respect to this line. So, that is what we would like to find out.

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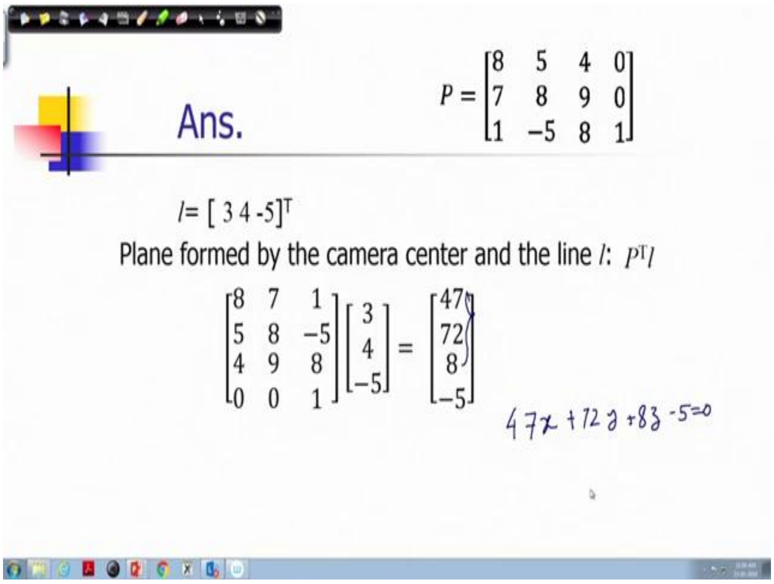

So, let us see the corresponding solution you have this projection matrix. So, the line is denoted by this column vector $\begin{bmatrix} 3 \\ 4 \\ -5 \end{bmatrix}$ and plane formed by the camera center and this line as you know it is $P^T l$. So, if I perform this computation you have to transpose the camera matrix and you know you should multiply with this l then you will get this kind of this is the equation. This is equation of the plane and if I would like to get the normal of this plane. So, I should restrict myself to the first three elements of that column vector, as you know that this equation of the plane is represented as $47x + 72y + 8z - 5 = 0$. So, this particular vector will provide you the normal. So, you have to normalize it and then you can get that unit vector. So, that is what will be doing.

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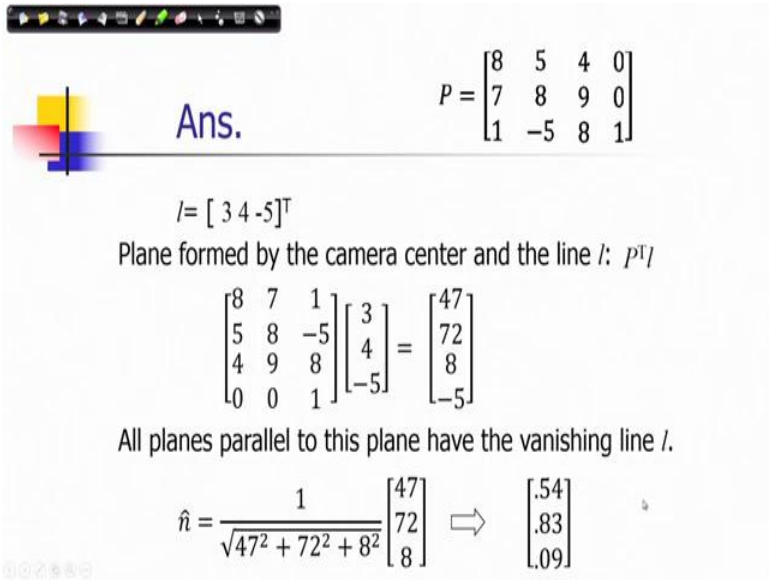

So, all planes parallel to this plane have the vanishing line l. So, that is the interpretation and the normal can be computed in this fashion. So, we can finally, we can compute the normal as we can see into this computation.

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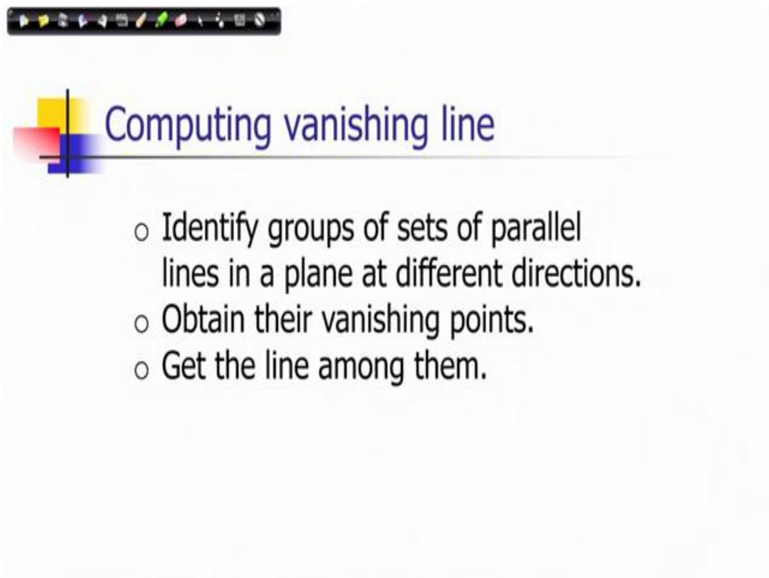

So, how do you compute a vanishing line in that case? You have to identify groups of sets of parallel lines in a plane at different directions. We can obtain their vanishing points and get the line among them. So, in this way you can get the vanishing line.

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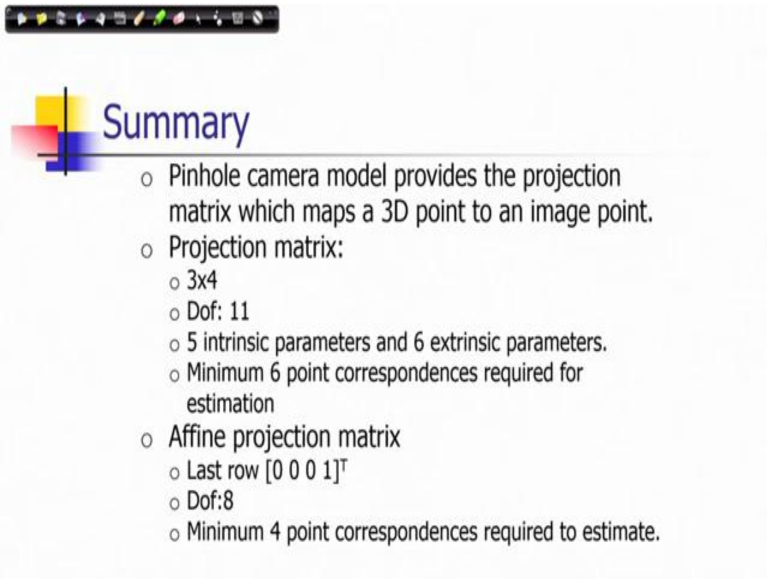

So, now I will be summarizing the content, summarizing those particular highlights which we discussed in this topic of single view camera geometry. First thing as you can see that pinhole camera model, it provides a projection matrix which maps a 3 dimensional point to an image point and projection matrix has some interest. I mean some known structure that is it is 3×4 matrix. It is a mapping from 3 dimensional world to a 2 dimensional plane and it has a degree of freedom 11; that means, there are 11 independent parameters.

So, as you can understand 3×4 means there at 12 elements in that matrix. So, since you can scale those elements still you get the same mapping. So, one of the element is a scale factor. So, rest of elements is the independent of that scale factor. Then out of this 11, there are 5 intrinsic parameters and 6 extrinsic parameters and you require minimum 6 point correspondences to estimate this projection matrix. There is another kind of projection matrix which we call affine projection matrix.

So, in this case instead of instead of a converging projection rays on a center of projection or camera center, we consider the images are formed by parallel rays and your center of camera lies at an infinity.

So, the structure of the projection matrix for this kind of affine projection has a unique distinction. It should have a colour it should have a row with [0 0 0 1] or any scale value in the place of 1 and then its degree of freedom is, because if I picks a scale value at 1, rest others are the independent parameters. So, if I take out these four elements from 12 it remains 8. So, it is degree of freedom is 8 or independent parameter is 8 and each point correspondence provides me 2 equations. So, , we require minimum 4 point correspondences to estimate this affine projection matrix.

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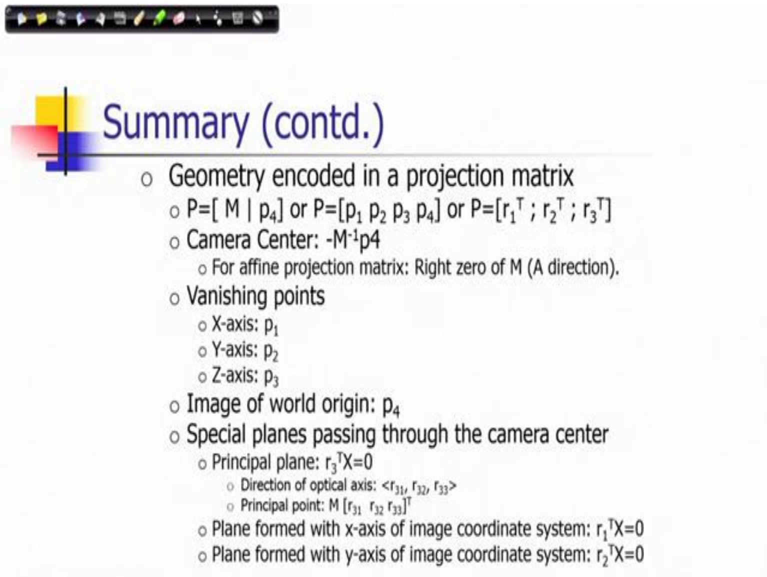

Then we discussed about the geometry which is encoded in a projection matrix like projection matrices can be represented in different forms to express these relations. As we can see that we can have a form of $[M \mid p_4]$ these are the notations we have used M is a 3×3 sub matrix p_4 is a column vector. It is a 4th column vector or projection matrix can be considered as a set of 4 column vectors p first second third fourth, they are denoted by p_1, p_2, p_3, p_4 or it can be considered as a stack of rows. Those are row vectors.

So, in this way projection matrix could be represented and some of the interesting information about the geometry, which are encoded, which can be retrieved from this values of projection matrix itself. Like camera center is given by $-M^{-1}p_4$ in its world coordinate itself and for affine projection matrix you have to take the right 0 vector of M and which is you know interpreted as a direction.

So, it would give you the direction of that parallel rays which is forming the images. Then we discussed about vanishing points in this case in the imaging. For example, X axis vanishing point of X axis is given by the first column vector p_1, then Y axis by p_2 and Z for Z axis it is vanishing point image of that vanishing point is at p_3 and image of world origin is at p_4. And there are so special planes, which we can recover from the projection matrices elements particularly from its rows.

So, some of the special planes like principal plane is given by this relation $r_3^T X = 0$, then from there since, it is a principal plane its normal will give you the principal axis. So, the first three elements of the vector r_3, it would give you the directions of the normal and then principal point is the intersection of the principal axis with image plane which is expressed by this relationship. M is a corresponding you know sub matrix or projection matrix 3×3 sub matrix. We need to multiply the directions of the optical axis with M and you will get the principal point. The plane formed with the x axis of image coordinate system and also the center of projection.

So, with the centre of projection and x axis this will make a plane 3 dimensional, in the 3 dimensional space and given by this relation $r_1^T X = 0$. And similarly, a plane formed by y axis of image coordinate system with again with the center of camera it is $r_2^T X = 0$.

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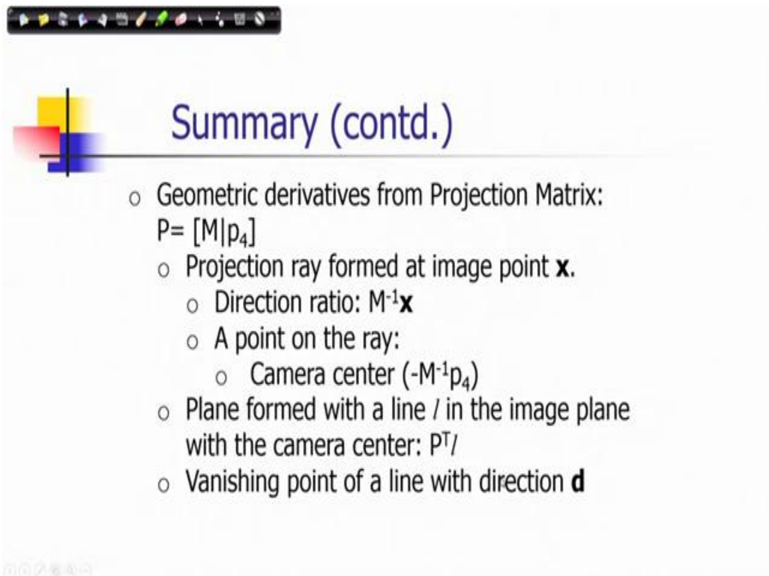

So, there are different other geometric derivatives from projection matrix. First thing is that you can form a projection ray at an image point, you can from its three dimensional equations of that particular line, it is direction ratio is given by this expression $M^{-1}x$. You note that x is a image which is expressed in the homogenous coordinate system of a 2 dimensional projective space.

So, this would give you a direction ratio in a 3 dimensional structure, 3 dimensional space and any point on the ray will define a line incidentally that one has to pass to the camera center, and you know that the camera center can be computed in this form you have $-M^{-1}p_4$, that would give you the camera center and this is how the projection ray is formed.

Similarly, you can also compute the plane formed with a line in the image plane with the camera center, it is given by $P^T l$ and vanishing point of a line with direction d is given by Md . So, with this you know let me conclude this particular topic here. I hope you have learnt some features of single view geometry; next, will be discussing about stereo geometry or two views camera geometry.

Thank you very much for your listening.