Introduction to Automata, Languages and Computation
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Lecture – 08
Language of NFA

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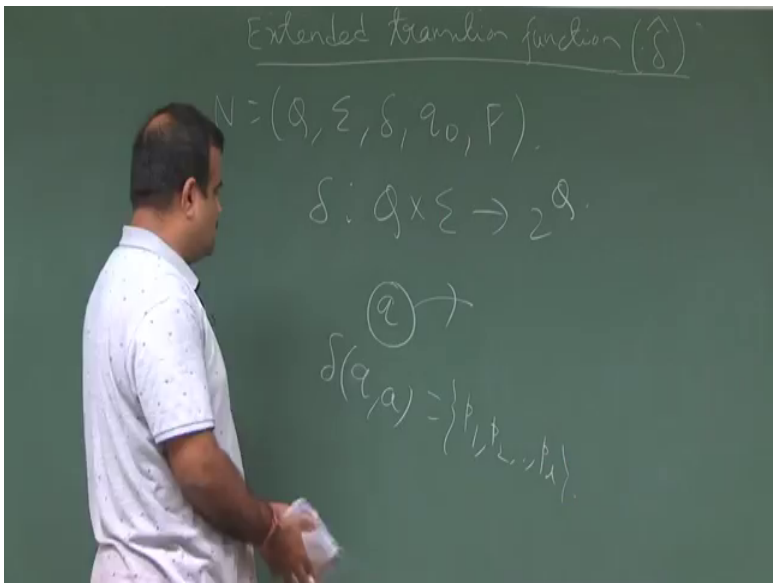

So, we are talking about NFA and the transition function of a NFA. So, now we are going to discuss the extended, how we can extend that transition function over a string instead of a alphabet. So, just to recap a NFA. So, this is the set of all possible state, this is the input alphabet, finite input alphabet, this is the starting state, this is the final state, this is the subset of Q. And this delta is a function here from cross 2 to the power Q. So, this is the power set of all possible states, so it is a subset.

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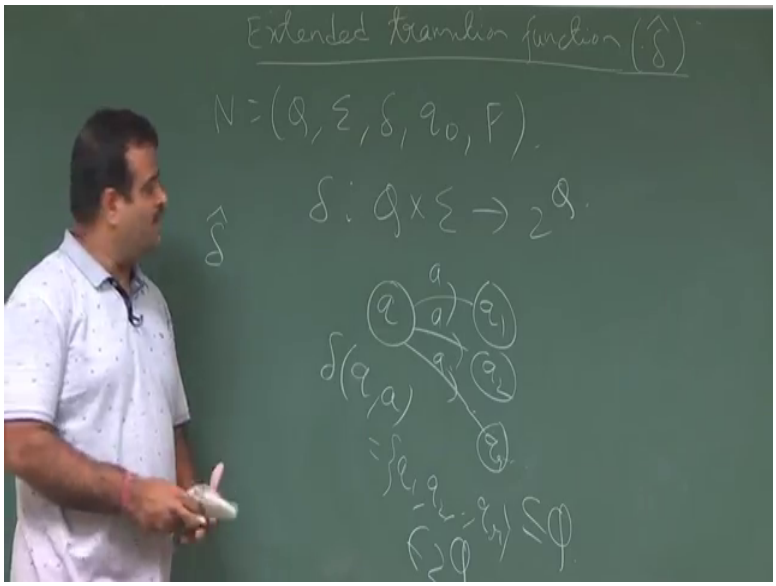

So, from a given state so if you are q, so delta of q comma a, if we take a input this is a set, so say p 1, p 2 some p l. Some set our q 1, q 2, q r. It can go to q 1, it can go to q 2 dot dot dot another dot dot dot q r. So, this is q 1, q 2, q r say, so this is a subset of Q, so that is why it is belongs to 2 to the power Q this belongs to 2 to the power Q. So, this is a non-deterministic move. So, we can go to any one of this part. So, this is on the input if you had a state q, if you take a input alphabet a, we can go to many states.

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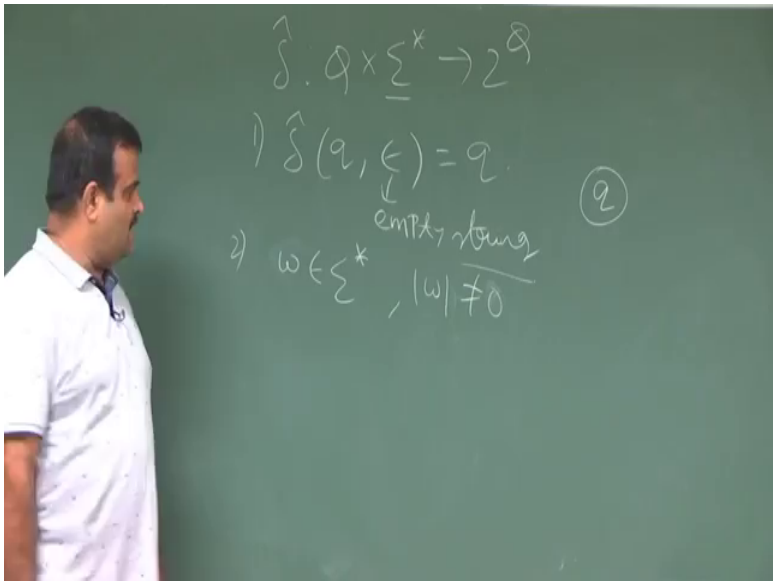

So, now we are going to extend this delta to delta hat, instead of taking a input if you take a string, how it will extend over a string. So, over a string as we said delta hat is a function, which is taking Q cross so this is also like this.

Now, here this is a string. Now, string could be epsilon is also a belongs to sigma star, so that is the base case, this is the convention, this is as q. So, this is the empty string empty string or this is also called null string. Not empty set, this is a string of length 0 no string I mean, so that means, if we had state q, if you are not taking any input from the input alphabet will be at remain that state q, so that is the convention. And the suppose it is suppose w is that w is a string and w is not 0, if w is 0, that is epsilon that is the first case if w is not 0, then we are going to define this by inductively.

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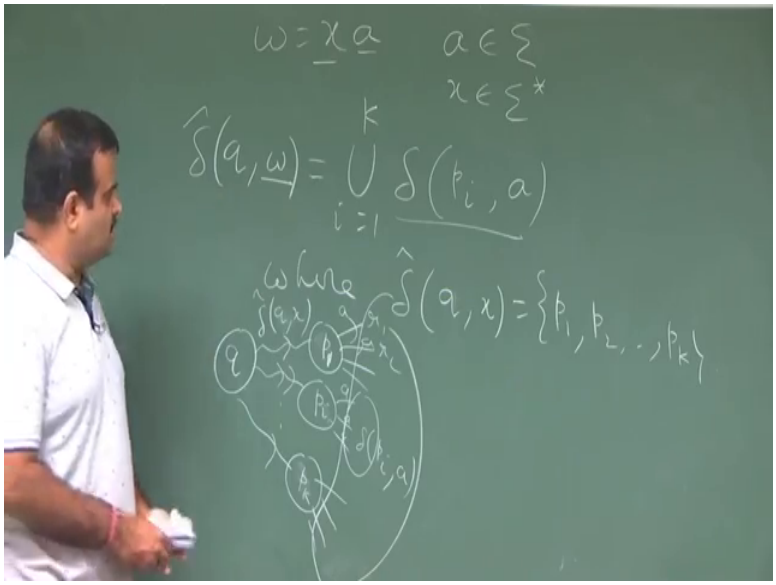

So, how to define that? So, we take a w, w is not 0, w is x a. Now, x could be 0 also I mean length of x x could be epsilon also. But, anyway so this is a is coming from this alphabet and x is coming from sigma star.

Now, we define w hat like this, w hat of q comma omega, omega is x a, this is nothing but union of delta of p i j a. What is p i j, I am explain, where this delta hat off q comma x is this set p i j p k say. This is the way constructive way we can. So, what is the meaning of this, suppose here at q, we want to read w, so first we read x. And suppose these are the possible state we can move for mixed. So, this is delta hat of q comma x, p 2 like this, p k p k. So, there are say delta hat of this.

Now, we can from here from p 1, we can have a move a so a so this can be some say r 1, r 2 this is this set. So, from p i if this is a p i, so this is this is with the a move, it is all going to delta of pi comma a. So, these are the so similarly so these are the all collection the union, this is basically delta hat of q comma w. So, will take an example, they need to be more clear, we will take an example yes.

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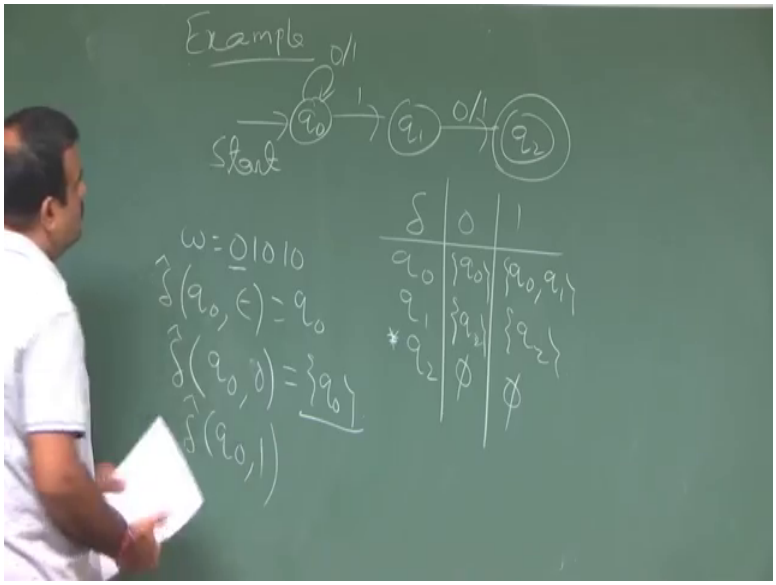

So, let us take a simple NFA, q 0, q 1 and q 2 ok, so this is the starting state q 0. And there are three state q 0, q 1, q 2. And from here if we have a 1 move, we can go to here or we can remain here, so this is 0 or 1 again. And from here, we can go to here with this 0, 1 move. And there is no move from q 2 ok. So, what is there, so this is the final step.

What is delta over here? So, delta we can simply write like this. So, we have two states three state q 0, q 1, q 2. From q 0 if we see a 0, we are going to q 0 only. And from q 0 if we see a 1, we can either go to q 1 or q 0 so, this is a two option. And from q 1, we for 0 or 1 both the case we are going to q 2. And from q 2, there is no move so, it is a empty set ok. And q 2 is the final step.

Now, you are going to see what are the string this NFA is going to accept. So, suppose we have a w like this say 0 1 0 1 0 0 1 0 1 0. So, you want to execute this, we want to get that delta hat of q 0 w, so this we want to see.

So, for that what we will do will first write delta hat of epsilon, this is q 0. And then we will write delta hat of q 0, 1. So, q 0, 1 is basically again q 0 so sorry 0, we see a 0 over here this 1. And then we are reading 1, so delta hat of here a q 0, then here a 1. So, this we can write in a in just a like this.

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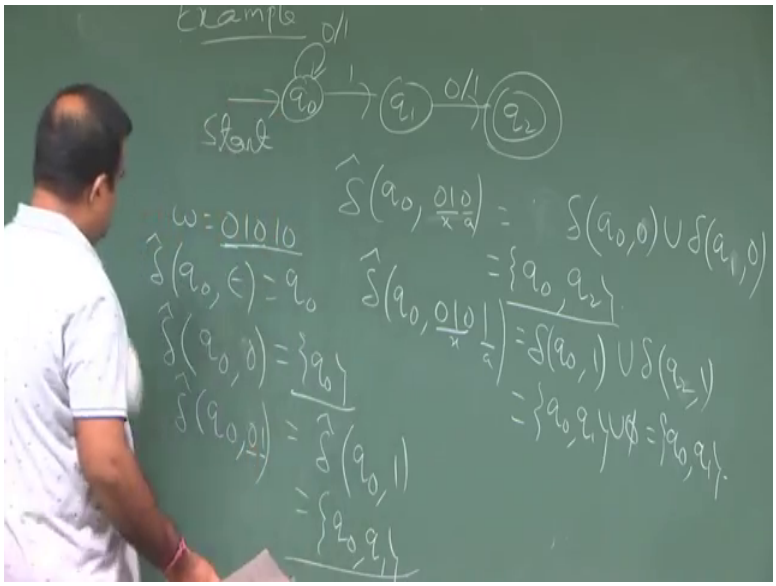

So, delta hat of q 0, 1 is basically this is 0 1, 0 1 means now we must read this is our x, this is our this. So, it is nothing but union of it is nothing but delta hat of q 0, 1 which is same as your from q 0, we can go to q 0 or q 1 like this ok.

Now, we are going to read delta hat of q 0 is three symbol 0 1 0. So, this is our w now this is our x. So, this is nothing but delta of; this is nothing but delta of I mean union of delta of that p 1, p 2, so this is our a. So, now this is going to this. So, this is delta of q 0, 1 I sorry q 0, 0, because this is the set. This is the set p 1, p 2 delta of delta of q 0, 0 union delta of q 1, 0 delta of q 0, 0, delta of q 1, 0 ok. So, now this is nothing but what from q 0, 0, we can go to q 0 only. And from q 0 q 1, 0 q 1, 0, we can go to q 2 so, this is the set for this q 0, q 2.

Now, we take up to this. So, delta hat of q 0, 0 1 0 1, so this is our x, this is our a now. So, this is nothing but now this up to this we know, these are the states. Now, this is our a so, this is nothing but delta of q 0, 1 that is the a union of delta of q 0 I sorry q 2 q 2, 1 q 0, 1, q 2, 1.

So, now this is again, we can write as q 0, 1 from q 0 with 1 we can go to either q 0. So, this is q 0, q 1 union of from q 2. There is no move, this is empty. So, this is same as q 0, q 1 this is same as a q 0, q 1.

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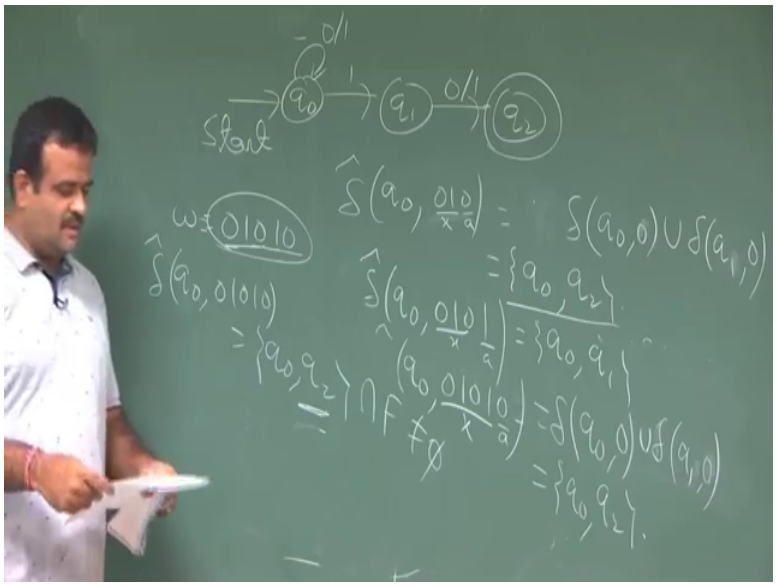

Now, we take this four, so that is the last one, so that let us write here, so this is nothing but q 0, q 1 q 0, q 1. Now, we take delta hat of q 0 of this 0 1 0 1 0. Now, this is our x up to this and this is our a. And we know delta hat of q 0 x is these two so, after so this is will be union of delta of q 0, 0 union of delta of q 0, 1 ok.

So, this will be what? This will be delta delta q 0, 1 q 0, 0 is q 0 and the q 0, 1 I sorry q q sorry q sorry this is 0, this is q 1, 0, because this is this is the last symbol. This is q 0 0 and q 1, 0. So, what is q 1, 0? Q 1, 0 will be q 2 q 1 0 will be q 2, so that is the final one so.

This is basically delta hat of q 0, 0 1 0 1 0 is nothing but q 0, q 2. And q 2 is the one of the final state that is only one final state over here. So, then this string is accepted by this DFA, because it is so this intersection of F is not empty, because it has a final state. So, if it is if any of the branches reaching to a final state, then we call this is a this is an acceptance state, I mean the language this is the accepted string by this NFA ok.

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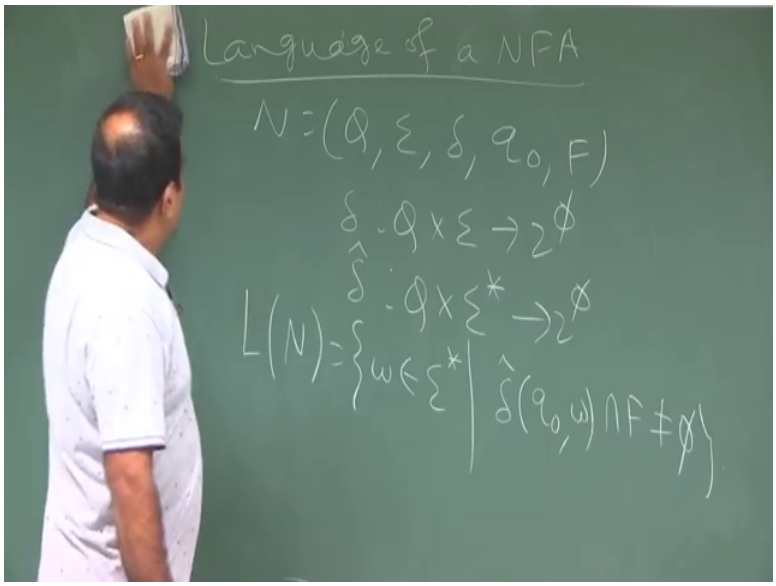

So, let us just define the language. So, this is clear now, we have many branches, we will we will draw the branch in a moment, but before that let us define the language accepted by NFA, language of the NFA. So, suppose you have a NFA Q, sigma, delta, q 0, F ok. Now, we know this is delta is a function from Q cross sigma 2 2 to the power this. And we have just now you have extended this to the sigma star.

Now, we consider all strings which are accepted by this NFA, so that collection is called language accepted by this NFA. So, this string is this is written as this, this is set of all possible strings such that delta hat of q 0 this. So, this is a subset, this subset should contain at least one final state. So, this intersection F is non-empty. So, it should contain at least one final state, then this is called so that string is the accepted by this NFA and the collection of such string is called the language of this NFA ok.

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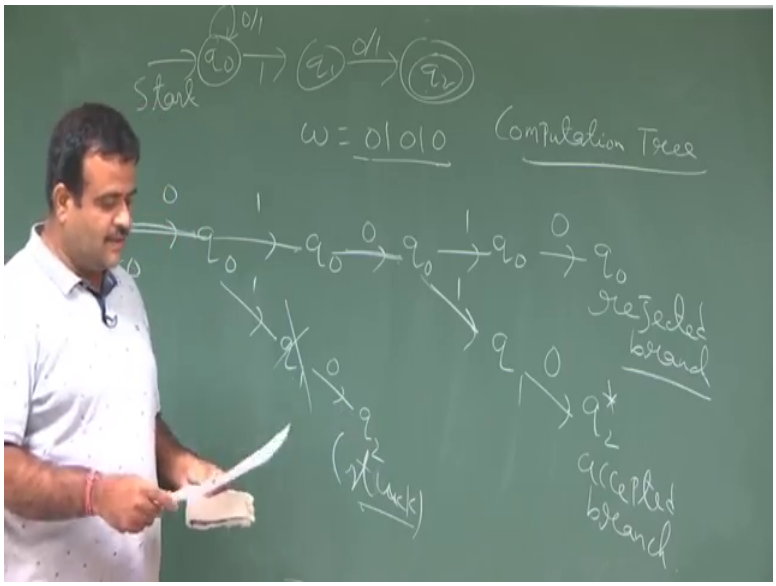

So, now we want to find the language of the NFA just the last example, we can continue. And I am going to find the language of that particular NFA. So, just to rewrite the NFA so, q 0 is the starting state, you have q 1 and we have a final state q 2. And we have a move like this 0, 1, and this is 1, this is 0 or 1 and there is no move like this.

And just now, we have you have executed this 0 1 0. So, this we have seen, but now we are going to draw the computation tree; computation tree. So, there will be many branches of doing I mean execute this. So, basically we start with here q 0. So, from q 0, we have a 0 input so from with the 0 input, we can either go to q 0 or we can go to; we can go to q sorry so let me write in the (Refer Time: 16:51) so q 0.

So, we can go to q 0 again with the input 0, there is only one option and from but from q 0 again, you have a 1 over here. So, for 1 we can go to again q 0 or we can go to q 1 ok. Now, so this is with 1, you can go one of this branch. Now, from here we are seeing a 0 so if we see a q 0, when we are q 0 again.

And but from q 1 if we take this branch; if we take this branch from q 1 if you see a 0, we can go to q 2. So, this is the we stuck here ok and we hold on reject. So, this because we have to have more string more in input to take, but this is stuck we cannot go anywhere here. So, this is stuck basically s t u c k, so this is stuck we are stuck. So, we hold and reject this branch so we just reject this branch. We will not go anymore on this

branch; there is no hope in this branch, because there is no further move from q 2 to any input.

So, we have taken 0 1 0, now we have to take 1. So, if you take 1, again you have two option, we can go to q 0 or we can go to q 1, so 1. Now 0, so if you take the 0, then we can go to q 0 again or we can go to q 2 0. So, this is the accepted branch.

So, if the if we can have a one branch, which is going to a accepted state. This is exceptional state, because this branch has no because we have already this is rejected branch, because this is not leading us to a accepted state. The accepted state is q 2 first this branch so, we start from q 0, q 0 again, q 0, q 0, then q 1, and q 2. So, this branch is going to a accepted state.

So, from a given string if we if there is a branch, there may be many other branch like this branch is stuck, we cannot go anywhere now hold. This branch we are completely executing this, but it is not ending to a accepted state. But, this branch if there is all if there is at least one branch, which is leading us to your acceptance state, then that string is called accepted that string is accepted ok.

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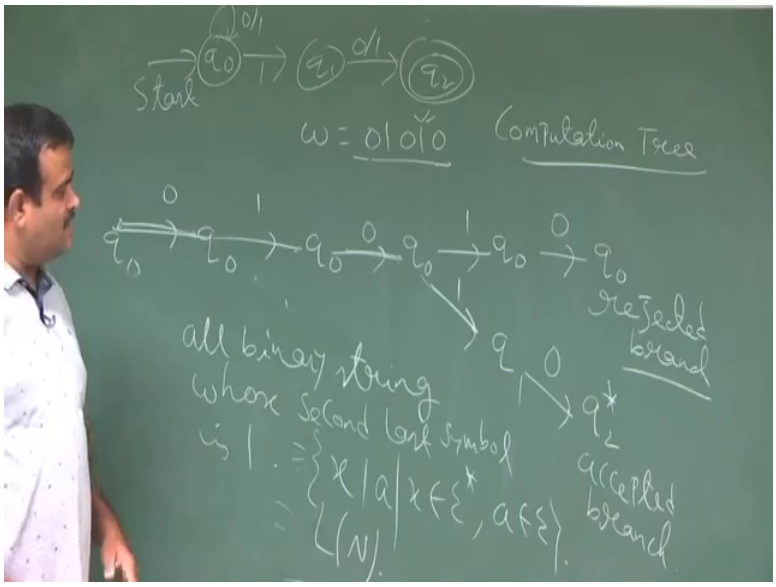

In fact, this the language of this is this one the it is accepting all the binary string binary string whose second last symbol is 1; second last symbol is 1; second last symbol is 1, this one ok, so that means, it is like this even if we have 1 1, then also we go to so after

this say suppose if it is instead of 0, if it is 1. If it is 1, then also we can go to q 2 so that is fine.

So, this is basically this language is can be written as x 1 a. So, x is belongs to sigma star, a is belongs to sigma. So, I think this is the N L of this is the language of this NFA ok. But, this is the way we are to so if a string has to be accept, then we should have a branch which is leading to as a accepted state, so that is the accepted branch. So, if w will be rejected, if there is no branch, if every branch is leading to a rejected state or hall or stock, so no branch if there is no branch, which is going to a accepted state, then that w will be rejected.

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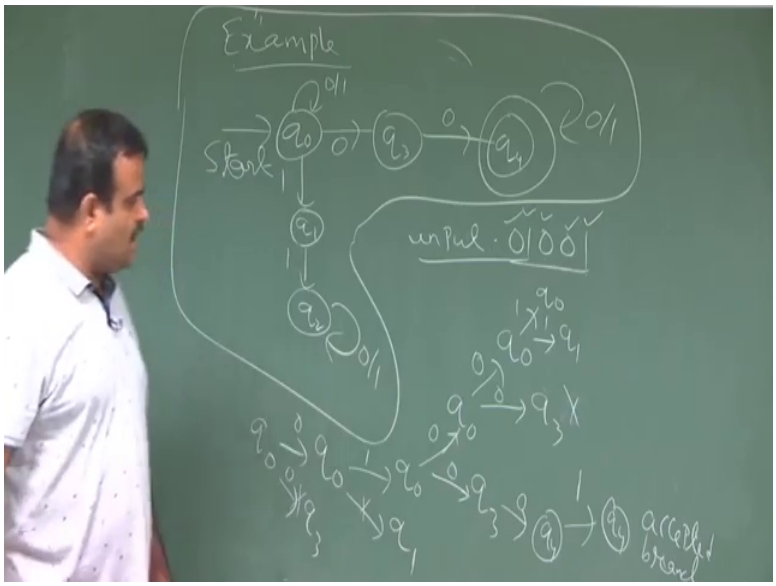

So, we will take some more example; so we take some more example. So, some more example of NFA and their the compression (Refer Time: 22:43). Like, suppose we have this NFA you have a starting state q 0, and then from q 0 we can go to q 1, this one I think we have seen this picture, this is one move, one move and from here we can go to 0 or 1. So, from here we can go to the q 3 and from here we can go to q 4. So, this is 0 1, this is 0, this is 0 and from here we can go to just 0 and 1 ok.

So, now we want to see, what type of string, if this is accepting. So, suppose we have a input like this say 0 1 0 0 1, you have a input like this. And we want to see whether it can I mean it can be accepted or rejected this string. This is string w, we want to check whether this w is accepted or rejected.

So, we start with so this is this is the so we start with q 0 and we are going to read this. So, if you start with q 0, if this is 0, so it can go to again q 0 or it can go to q 3. So, you have to mention the final state, this is the final state ok. It can go to q 3. So, once it go to q 3 and we are having so this is with 0 move.

And next input is 1, but there is no move from the q 3 with 1. So, we stuck here this branch but, from q 0, again we see a 1 for 1, we can go to again q 0 or we can go to q 1 1. So, if you go to q 1, then again we see a 0, there is no 0 move from q 1, so it is stuck. So, this all are stuck, I mean the halt on this branch ok.

So, now we can from here what we can go, we can we see so this is 0 1 again 0. So, from q 0, if we see 0 again, then we can go to again q 0 or we can go to q, so this is 0. You can go here or you can go to q 3 ok. So, now this is these are done.

Now, again we see a 0. So, if we see a 0 from q 3, we can go to q 4 q 4 with a 0. And from q 1, we are seeing a 0, then we can again go to q 0 or we can go to q 3 again, this is the 0 move ok. Now, from again the last symbol is input symbol is 1. Now, with q 0, if we see a 1, then we can go to either this q 0 again or we can go to q 1, this is both with 1. And from q 3, we see a 1 so there is no move. So, this is stuck this is stuck.

And from q 4, from q 4 if you see a 1, then we can again go to the q 4, because this is the thing. So, this is the accepted branch. So, this is 1; this is the accepted branch ok so, this is the accepted branch. So, this means this is this string is accepted by this, because at least we have a one branch starting from q 0, it can lead us to the this the final state ok. So, we will discuss more on this.

Thank you.