Introduction to Automata, Languages and Computation
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Lecture - 60
Language Accepted by a Turning Machine

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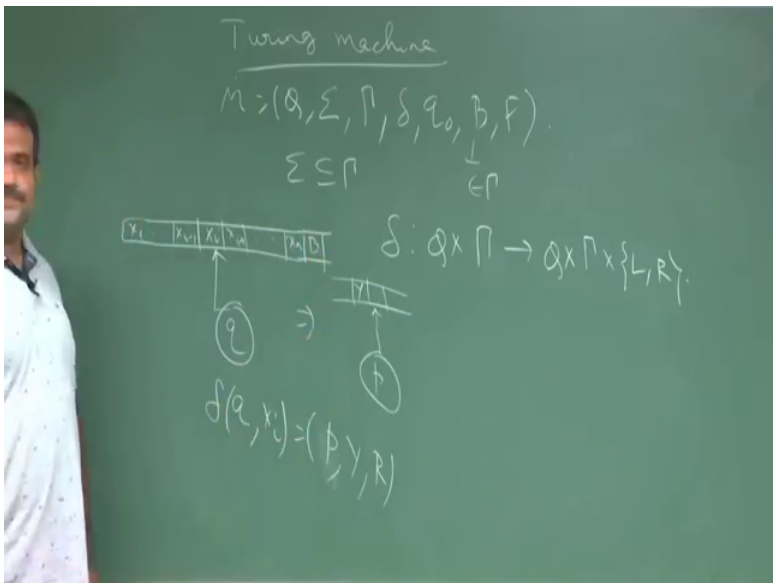

So, we are talking about Turning Machine. So, just to recap, it is a seven tuple; Q this is the finite number of states, input alphabet, tape symbol, and we have delta, transition rule q 0, and you have a blank symbol, and set of final states. So, here we have a concept of final state.

This blank symbol is belongs to this and input symbol is subset of the tape symbol ok. And delta is a rule form Q cross, we take a input, we take any symbol, it could be a tape symbol, I mean in general, it could be blank also, you may have rule for blank input. This is going to Q and it will change the symbol this machine has a capacity to write. So, it is reading a symbol, it will write a symbol. And then it will the decide to go the tape head left or right.

So, here we have a tape head. So, this is a x i and suppose we are at q, this is pointing here. So, if the delta of q comma x i, if it is Y comma sorry if it is p, then the symbol will be say Y, and then if it is right. Then this will next will be so it is this will replace to Y and next this will be pointing and our next state is p like this so, it will go like this ok.

And using this, we have instantaneous description. So, suppose this is say X 1, X 2, X i minus 1 and then X i plus 1 X n. Then after X n, we have B and so it will be what? So this will be like this, let me just erase this.

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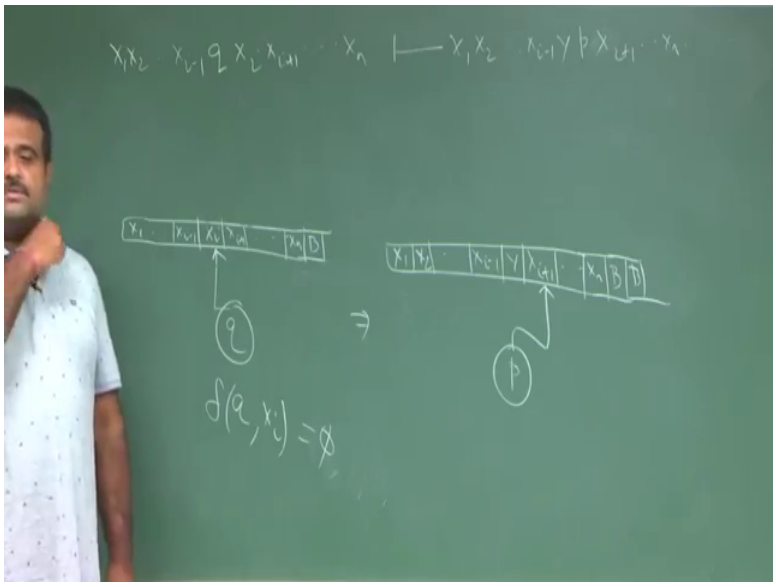

So, this is going to where this is going to so our state tape is position is like this, so you have X 1, X 2, and this is X i minus 1, and this X i is changed to Y, X n and this is B, blank symbol blank symbol, all the this has this has unlimited cell, each cell can contain a symbol.

And now our (Refer Time: 03:30) our state is moved to P and then this is the tape is pointing to the right, so right means, it was pointing here, so it now it will point to here. And this we have defined by the in the last class we have discussed the ID, the instantaneous description. So, ID of this is we are writing this X 1, X 2 and we are putting the state symbol in such a way that after the state symbol thus input symbol, I mean this tape symbol will be pointing like. So, here so it is pointing to X i, so we put the state here. So, X i will be here, this is the indication where is our tape is pointing.

So, after the state symbol, the next symbol; the next tape symbol will be the pointing to the tape, so this is X i plus 1 like this X n like this. So, this is going to this symbol, this ID. So this ID is X 1, X 2 dot dot dot X i minus 1 Y, now it is pointing to here. So, we will put the p over here, then X i plus 1 dot dot dot X n ok. Now, it may halt like I mean, when it will halt? Suppose we do not have any move for this, suppose this is empty, you

do not have no move for this, then it will halt, the system will halt. And it will halt in other situation that is the accepted state.

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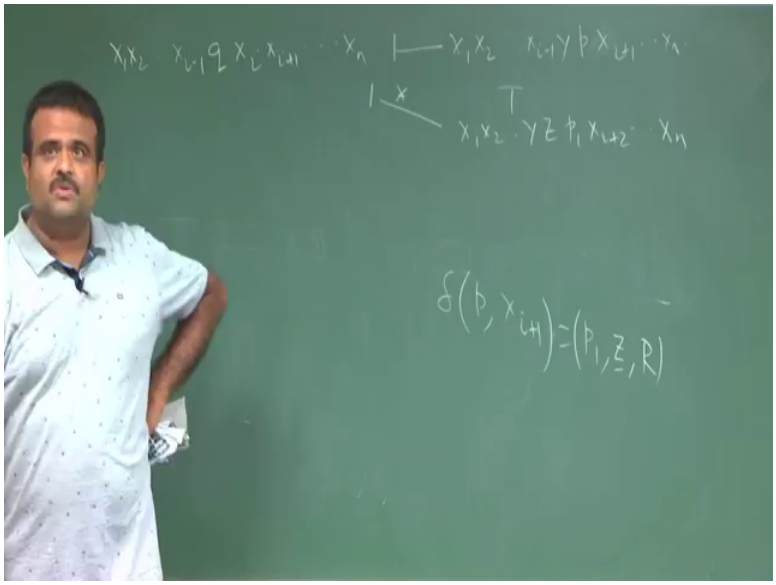

So, if the during the transition, if we can encounter with some final state or acceptance state, then that is called that will be the language accepted that is the string accepted by the this turing machine. That means now if it is going to the next symbol like this, if we have a rule like this delta of say, now we are reading, now we are at state p, and we are reading X i minus X i plus 1.

And if it is going to some state say p 1 and if it is z, and if still it is R, then it will go to next position X 1, X 2. So, this is pointing now, so it will replace this by z and it will go to the right, so this will be z and then it will go to the right, so p 1 X i plus 2, this is like this. Now, these from here we go to this then this, so from here to here, we have a more than one time move relation we can go to there. So, this is the way, we can define the move relation. Now, based on this moved relation, we can define the language accepted by this to a turing machine.

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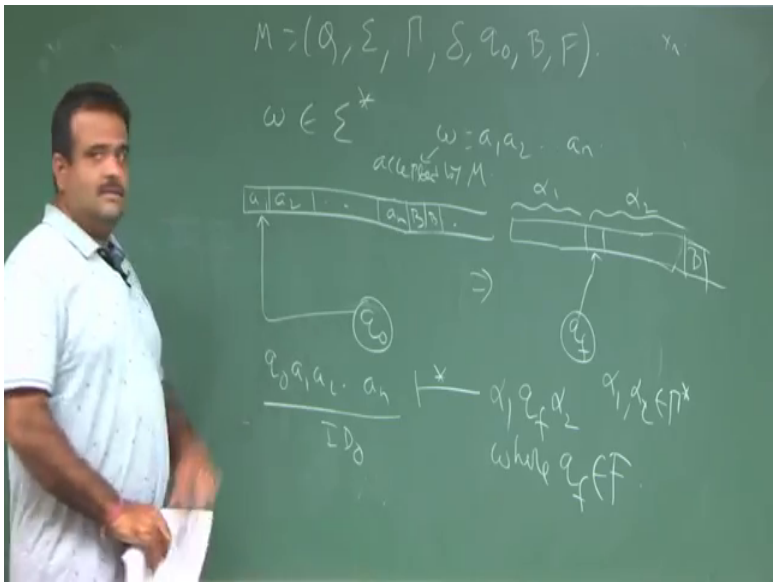

So, given a turing machine, we have this q 0 is the starting state B, F; F is the set of all final state. So, we say a w is the string of alphabets, only the input alphabets. Say w is a 1, a 2, say a n, this is the alphabet string. Now, we say that this will be accepted by this turing machine. If we start with this in the tape a n sorry tape is not bounded and we have had q 0, and this is pointing here.

This is the first I mean the initial snapshot of the machine, so that means, what is the ID? ID is q 0, the way we defined ID, and then we have whole w. So, a 1, a 2 up to a n, this is the initial ID, this we denoted by ID 0. Now, if this initial ID, will move to the some situation like this some alpha 1, then q f, alpha 2. Here we do not care about alpha 1, alpha 2; alpha 1, alpha 2 are just a string of tape symbol, mixture of input symbol blanks all these things. Only thing, we do care about the states, so where q f is a final state.

If q f is a final state, then this string will be called will be accepted by this, I mean this is the acceptance string, this string is accepted by the turning machine, and our machine will hold there, once we reach to a final state. So, there are two ways, we can machine will hold; one is there is no move, there is no transition for a given state, and the given input. And another way, if we reach to a final state, so there may be so like if it is going to situation like this, we have some we are at say q f we reach and our tape is like this, no there is no end over here. So, it is pointing here and this is say alpha 1 and this is alpha 2 up to this.

And we have after that we have blank symbol when this is pointing here. And alpha 1, alpha 2 are coming from then we say this w; w is accepted by the turing machine M. And collection of such w is called language of the turing machine. So, we will define the language of the turing machine, but that is the idea. So, we just in the middle of the executions, once we reach to a final state, then we halt, machine will halt and that string is the acceptance string.

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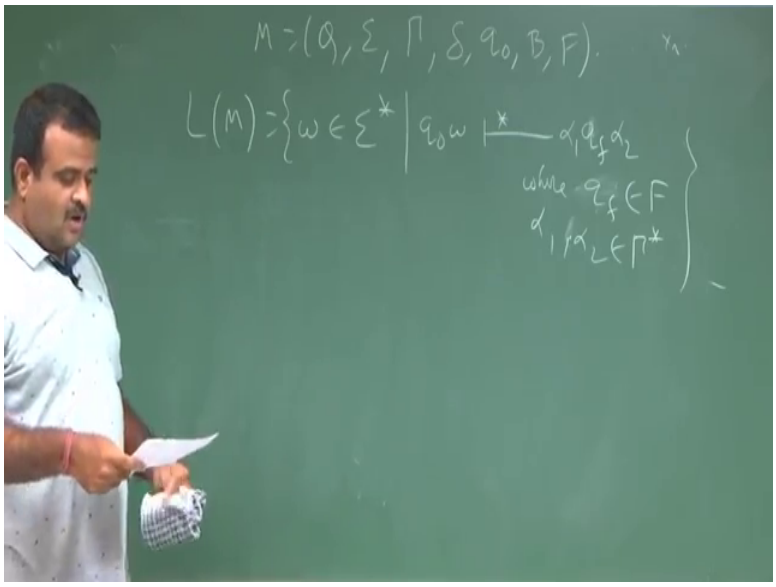

So, we define the language of M like this. So, this will the set of all w. We start with this is the initial idea of the machine and if we reach to a I mean recursively with the many moves of this delta. If it is reached to a situation alpha 1 p or we referred as q f alpha 2 where q f is the final state; and alpha 1 alpha 2 are the any string of tape symbols, then we say this is the accepted by the; this is the language of the turing machine ok. Now, we will take an example, then it will be more clear.

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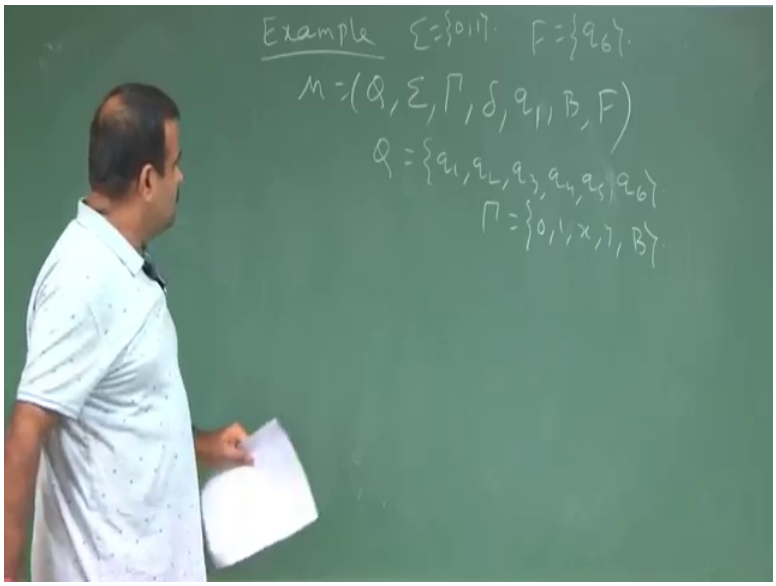

Let us take an example ok. So, yeah so let me define some turing machine M, which is Q, sigma, delta and then q 0, B and F ok. Now, Q we have say so this is q 1, here and Q is a q 1, q 2; q 1 is the starting state here, q 3, q 4, q 5, there are 5 states and q 6 sorry. There are 6 states and q 1 is the finals sorry starting state and q 6 is the final state; so F is q 6.

And input alphabet is 0 1. And F symbol is 0 1 along with the non I mean along with the blank and two other symbols x and y we will take ok. So, this is the way we defined this. Now, we need to have the transition functions delta; so we write the delta in a table. So, let me erase this so, we write the delta.

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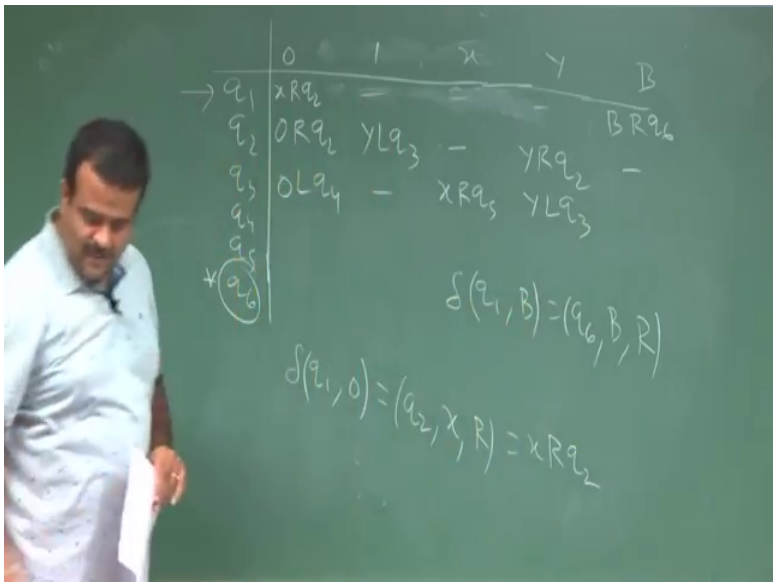

So, we have the state q 1, q 2, q 3, q 4, q 5, q 6 ok. Now, so this is the, starting state and this is the final state, we put the star over here. Now, what is the input symbol? 0 1, this is the input symbol. And what is the other tape symbol we have taken x, y, and beta is the blank symbol we are using ok. So, delta of we are at say q 1, and we see a 0 in that tape. So, this is we are telling this is we are going to q 2 and s 0 is replaced by x, and we move to the right.

So, in some book, this is written as in this form x R q 2 I mean the other way round I mean this is the symbol you are going to write, we are going to replace the tape symbol and this is the position of the tape they provide and this is the new state. So, this is the another way to write there. So, let us write that x R q 2. And we have no move for this, no move for this, no move for this and for these we have a move like this delta of if we are at q 1, and if you see a blank, then it will going to say q 6 and then we go to the we replace x by blank sorry we replace this is blank again and this is R.

So, using this notation this is nothing but B R q 6 like this. So, if let us fill up this, so with q 2 we have 0 R q 2 and then we need some more space over here. So, this is 0, 1, x, Y B. So, this is B R q 6 (Refer Time: 16:36) ok. Now, with q 2, this will and this is with 1, we are going to replace 0 by right 0 by Y, and we are moving left and we are reaching to q 3. And with Y, this has no move, with Y you have a move Y R and then we have a q 2 and we have no move for this ok. So, this is from q 3 let me complete this 0 L q 4, no

move for this. And with x we have x R q 5; and with Y, Y L q 3 ok. This is given I mean this delta transition is given. And we will see for this turing machine what are the string it is going to accept.

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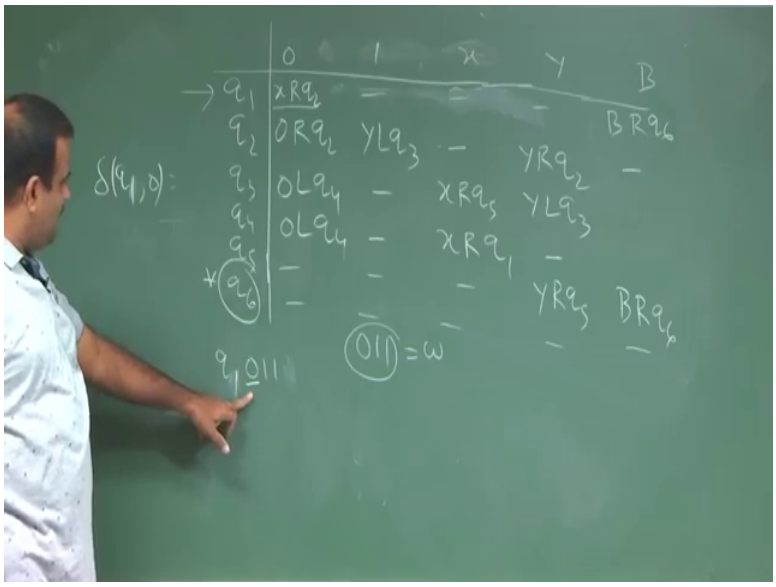

And for q 4, we have 0 L q 4 and this is no move, x R q 1 like this. And from q 5, no move, no move, no move, so we have YR q 5 and with blank we have blank R q 6 and we have no move for. So, this is the transition rules given by given. So, this is our; this is an example of a turning machine. Now, we want to see what type of; what are the strings it is accepting. So, this we can write in graphical way also. We want to see what are the string is it is accepting.

So, we want to check first few and we will say we whether it is accepting the string like 0 1 1 this is our w. So, how to check that? So, we start with q 0 and this is the initial id 0 1 1, sorry this is the initial id. Now, we read 0. Now, we have to check delta of q 0 0. So, delta of q 0 0 is this one I sorry q 1 is the starting state. So, q 1, so delta of q 1 0 is this one. This means we are replacing the symbol by x and we are going right and our state is new state is q 2.

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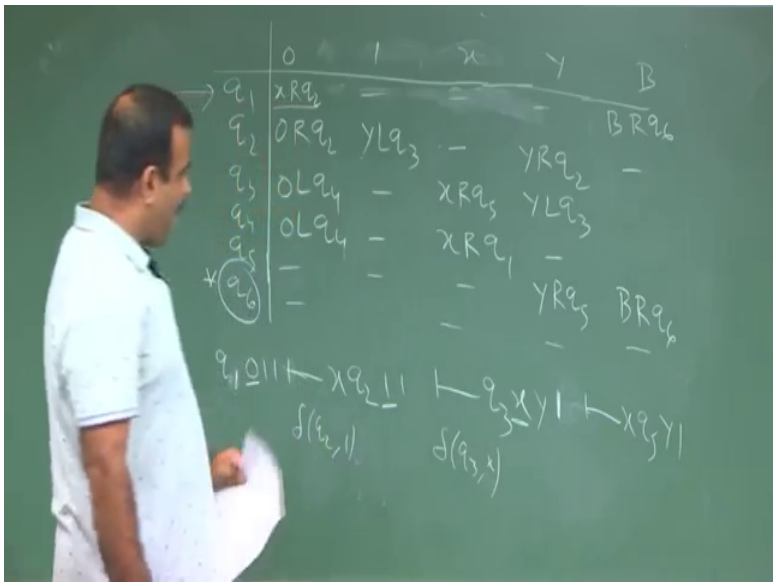

So, this will go to we are replacing; we are going right and replacing this by x, x, and this is q 2 1 1. Again this will go where, so this will again take this is the now we need to check delta of q 2 1. So, delta of q 2 1, so delta of q 2 1 is this one. So, Y will be replaced by I mean 1 will be replaced by Y, but here we are again going to the left. So, that means, next input symbol will be x and this will be replaced Y and this is going to q 3. So, this will be q 3 and x will be the next input symbol and this 1 is going to replace by Y, this is the way.

Now, in the next one sorry we have so this 1 is replace, we have another 1 over here. So, now this is the situation now where it is going now? Now this is the symbol it is pointing, so delta of q 3 x, so, delta of q 3 x. Again, it is replacing x by x, and it is going to the right and it is changing the state to q 5. So, this is basically x q 5 Y 1 ok.

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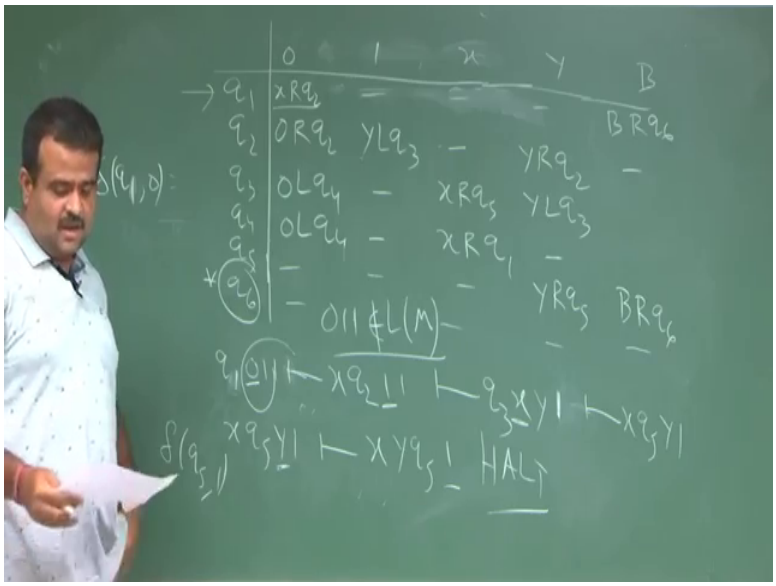

So, let me write here this one, so x q 5 Y 1. Now, this is it is reading this q 5 then Y and q 5 Y move is like this q 5 Y, yeah we have a move like this, Y is replaced by Y, and it is going to the right. So, it will go here and state is again q 5. So, it is x, Y is replaced by Y, then it is going right; so it will see 1 there. So, q 5 will be written here and 1, 1 is the next input symbol to be read.

Now, we should have a rule for delta of q 5, 1. But there is no rule for delta of q 5, 1 so, our system will halt. Halt without reaching to the there is no way I mean in the middle we are reaching to a final state. So, it is no further movement is there. So, it is halting and this string is not accepted. So, this 0 1 1 is not belongs to L of M, because it is halting without reaching to a without giving as a final state ok. So, let us take one example where it is accepting string. So, let me erase this.

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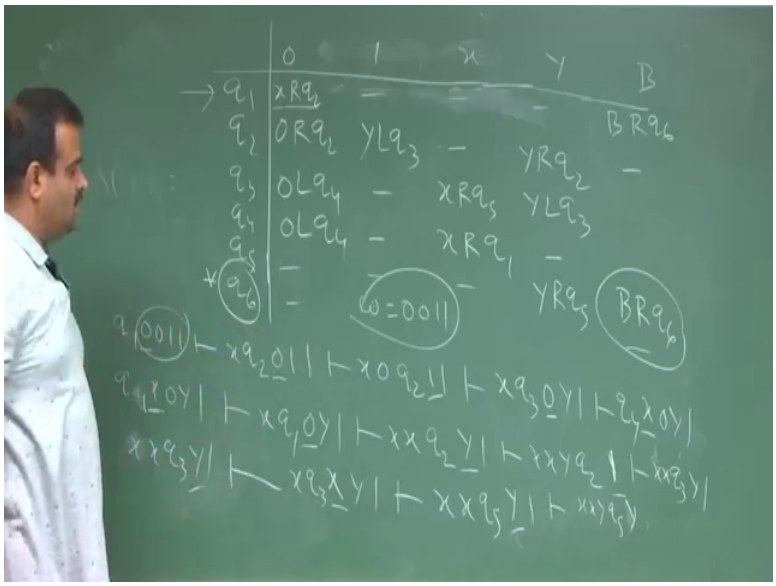

So, let us take another string say yeah, so let us take another string 0 0 1 1. So, we start with q 1 is the starting state, and then this is the now we read this one, so q 1 with 0. So, q 1 is 0 is this move, so, it will go to the right. And this is this will replaced by 0 1 replaced by x and q 1 will go to q 2 so, x q 2 0 1 1 ok.

Now, again q 2 is 0, q 2 is 0 again it will go to the right and 0 will be replaced by 0 so x 0; q 2 will be replaced by I mean this will be q 2 again 1 1. Now, this one is the next symbol. Now, q 2 1, q 2 1 we 1 is replaced by y, and now it is going to the left and it is state will change to q 3. So, it is going to the left means next symbol it will read this one. So, this will go to x, then q 3 0 will be the next symbol to be read and the 1 is replaced by Y 1 ok.

So, now this is the next input. So, q 3 0 is L 0 symbol is same, but only thing the state is changing to q 4 and it is going left. So, it is come back like this; so it is q 4, and then x 0 Y 1 like this. So, let us start again here q 4 x 0 Y 1 so, this is the next input symbol to be read. Now, if we do this it will go so with q 4 with x, q 4 with x where we are going x R q 1. So, R means it will go right so, x will remain x and we are reaching to a q 1 0 Y 1; so this is the next symbol.

Now, q 1 is 0, again it will go to the next one and 0 will be replaced by x. So, this is x, x then q 2 and next symbol is Y 1. Now, this is the next 1 Y. Now, q 2 y again it will go to

the Y will be replaced by Y and it will go to the right. So, this will be this is going to x x Y q 2 1. So, this one is now going to we are going to read this one.

So, let me just rub this, yeah we can use this. So, now, the x so now let me check this. So, q 2 1is again it is going to the left. So, this is going to the, so this is going to the left. So, that q 2 1, 1 will be replaced by Y that is ok, but it is going to the left means it is q 3 it is going. So, x x, so now, this will be pointing q 3 Y 1 this is the next symbol. So, x x q 3 Y 1, this is the next symbol.

Now, we are seeing a Y at that point of q 3. So, q 3 Y again it is left is the going, but then this will be remain Y, but this is again q 3. So, it is x q 3 x Y 1, now this is pointing out. Now, q 3 x; q 3 x is again going now it is going to right. So, and the x is remain x, but the state is changing to q 5. So, x this x is remain x, q 5 Y 1. Now, q 5 yis again going to the right; so it is Y is replaced by Y; and it is remain at q 5, so x x Y q 5 1.

Now, q 5 1 sorry x x q 5 Y no this will be replaced by somewhere I missed this, this will be replaced by Y at some state we made some mistake; so you can verify at home. This will be replaced by Y. So, once this is replaced by Y then again will go to the right and we will see a blank there. When we see a blank there with q 5 will go to q 6 and that will be the acceptance state.

Once we reach to the acceptance state, we stop. So, this string is accepted you can check that I mean we might have made some mistake over here. So, you can just follow these transitions, and we can verify this, and it is going to the final state q 6. So, this string is going to the acceptance. So, this will be given in the lecture note.

Thank you very much.