Introduction to Automata, Languages and Computation

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Lecture - 59
Turning Machine

So, we will introduce the Turing Machine; this is the end of this course. So, let us define first the what do you mean by Turing Machine. So, we have seen the 2-way DFA. So, there we have a option.

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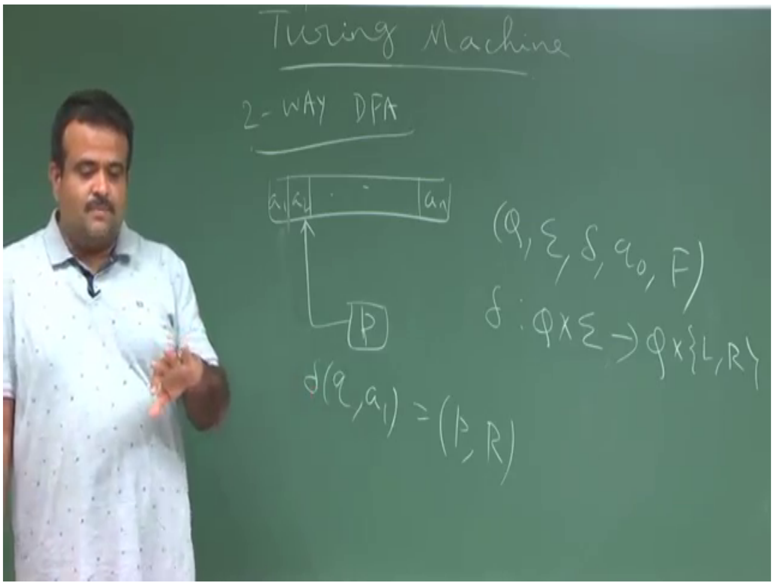

So, it is of a 2-way DFA. If we recall the 2-way DFA, so we have a input a 1, a 2, a n, and we have a state q which is taking this input. So, depending on the input so delta that time so it is a Q, sigma; sigma is the input alphabet and we have a delta, q 0, F. And Q was here, the Q cross sigma to a Q cross L or R that means our (Refer Time: 01:27) either will go to the left position or right position.

So, we start with so if it is this, this is the situation, if the delta of q, a 1. So, this is the heading of the tape. So, if it is say p, R, that means so this state will change to p and it will tape rotor will move to the right of this; so this will go here. So, this way it is a it can I mean it can go from left position or right position the tape header, so that is the way we define that 2-way DFA.

So, this is the Turing machine is kind of 2-way DFA, but in the DFA we do not have any options to change this symbol.

This is only the read permission is only we can read the symbol, we can read the input; but here Turing machine we can write something there. So, we have the I mean this system can read and write both the capacity, so that will define. So, it can change the symbol, it can write the symbol into the tape. So, Turing machine is given a tape.

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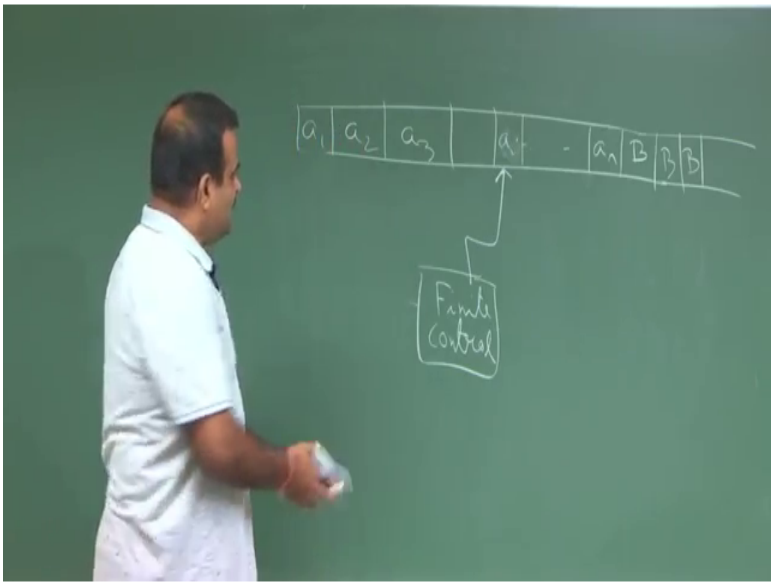

So, it could be one sided I mean it is a; it could be both sided, but we are defining one version of the Turing machine. So, we are given a tape which is initialized by a input of size n, a n and this tape is infinite in one end. So, and once the input is ending we will put a special character, which is called blank. So, blank means no symbol, blank is a special character which is also a symbol, but which is not a part of the input alphabet.

So, these are all filled by blank, this is a; this end it is infinite, but this end we can start with we can bound this; but there is a another version, they are both the ends are infinite. So, we will take this version and we will define the Turing machine. And we have a finite control ever over here that means, there we have a many states are there which is changing and it is pointing to some part of the I mean at some point of time it is starting with this, but at some point in times you say this is pointing to a i ok.

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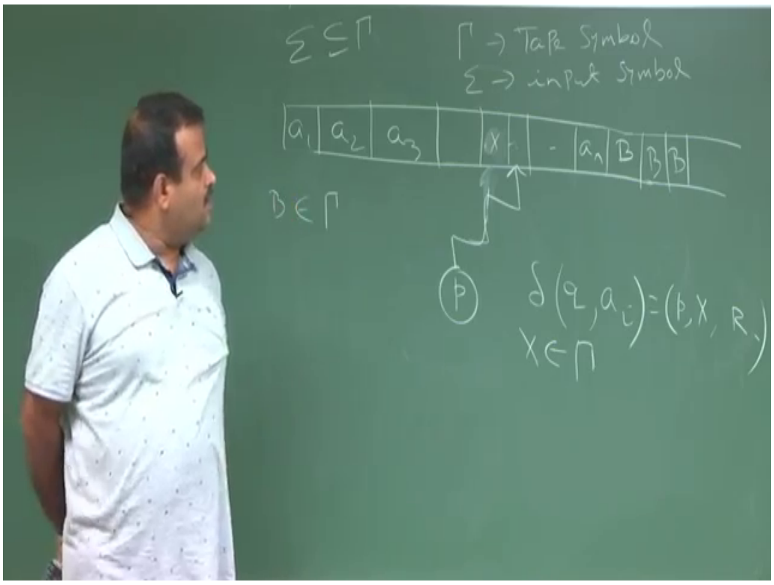

And suppose our state is at say q i or say q, q is the state where I mean, this is the situation this is a snapshot of this machine. So, we take a snapshot photo of this machine. So, this is the position our tape is pointing out to a i and our state current state is at q. Now, we are following that transitions rules, now the transitions rules over here is delta of so we are at current state q and we are taking a i; so this will be 3 tupple.

So, we will go to the new state p and we will change this symbol, we can keep the same symbol or we can have a different symbols and that symbol may not be input symbol, we can have another symbol including B also. So, we can have some new symbol, so will formally defines that so that is the tape symbol we called.

An input symbol is part of the tape symbol; because ultimately we are going to accept a language, which is coming from input symbol sigma. So, there are two concept; one is tape symbol and another one is input symbol. And this input symbol is subset of tape symbol, but not all the tape symbol are input symbol. And this blank symbol is a special symbol which is coming which is also a part of the input tape symbol, which is not in input symbol ok. Blank is not, we are not going to input the blank string blank ok. So, this is the situation.

Now, this is going to some symbol x and the position of the head is either going to left or right. So, it could be left or right; so if it is left, then this will next to this; so this will if it is say right; if it is right, then what we are doing? So it will change to p state. And this will be replaced by X and our head will be point to tape head will point to the right of that. So, this is the next ID of this next ID of this machine, Turing machine.

So, we can have move like this we change the state as well as we replace the symbol. We are replacing the symbol a i by X; X could be a i itself. So, X could be anything, it could be a input symbol, it could be a basically it is a tape symbol it is coming from tape symbol, but that could be a input symbol or it could be a blank or it could be a other than blank non-input symbols. So, this type of machine is called Turing machine. So, this is one version where this side is we start from this side and it is that side it is infinite.

Now, we will formally define the Turing machine, it is a basically a tupple. So, let us formally define what do you mean by Turing machine ok.

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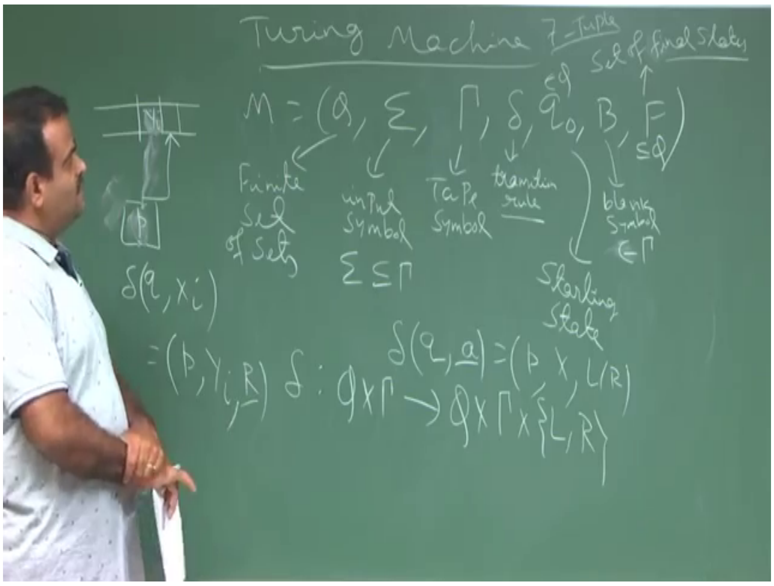

So, it is a 7 tupple, Q it is a finite state machine, Q is the set of all finite set of states. Sigma this is the input symbol and this is the tape symbol, but sigma is part of this ok. Here we do not have tape here, everything we are doing on the I mean here we do not have stack here, like PDA we had stack, but everything is we are doing on the tape.

So, we are reading on the tape, we are writing on the tape. So, every time we are reading and we are writing, we may write the same symbol, but we are write doing the writing operation. So, these has a this automata has a mechanism to write on the tape. So, this is, this b also belongs to this; so delta will defined q 0, B, F.

Now, this we know this is the finite set of states, where you can move another one. And this is the input symbols, set of finite set of input symbol and this is a subset of the tape symbols. So, tape symbol is a collection of all symbol including B. So, this is a tape symbol, this is also finite, all are finite and these we have to define this is the transition rules, transitions function or transition rules. And this is the starting state we know, we will define transition rules this is the starting state ok.

And B; B is the blank symbol that means, once we take a input into the tape then the after input is over, I mean after the last symbol of the input we start the blank symbol. So, but this is this B is the special symbol which is called blank symbol b l a n k blank symbol, which is also coming from this set of all symbols. And this is the set of final state, because here also you are going to accept some string of alphabet. So, this is set of final states.

And this delta is a transitions rule. So, delta here is taking two values; one is we are at state q, the current state the our system is now and the tape the symbol in the tape, I mean symbol in the tape, it could be a input symbol, it could be a take I mean, it could be any symbol. I mean it could be a blank also, it could be a input, it could be input symbol, if it could be a it must be a tape symbol, but it can be a it would be a non-input symbol also.

So, this is going to some other state p may be same state and this will be replaced by this symbol will be replaced by some new symbol say X and either the tape position will go to left or right. So, this we can define as delta is taking 2 input Q comma tape symbol and it is going to where 3 input is there, it is changing the I mean it is changing the state, it could be same state again.

And a new symbol will be written in the place of that symbol a and it is the tape header will be move to the left or right. So, this is the function of delta, this is the transition function of delta for Turing machine. And here q 0 is belongs to q and F is a subset of Q, this we all know, these are always symbols we are familiar with that.

The only thing given a tape symbol, suppose this is a X i, I mean given any symbols or delta of suppose our system is at q. So, we are reading this, so delta of q we are add X i. So, this if it is like say p and if X i is replaced by Y i and if it is R that means, our system is so this will be replaced by Y i. And this will be pointing to the next, because this is the this is telling us to the move to the header to the right. So, this is the pointing to the right and this will be going to the next, I mean next state is the p. So, this is the instantaneous description of the Turing machine.

So, we will formally define this, but this is the way we defined this is the 7 tupple. Q is the set of all finite number of states, this is sigma input symbol, which is a subset of tape symbol and tape symbol consists of the blank symbol also. And we have transitions rules which is a functional form which is taking current state.

And a current symbol of the tape, where it is pointing and it will change the state and it will write the symbol, I mean it will replace the symbol and the tape header will either move to the left or right, so that is the way ok. Now, let us talk about instantaneous description of a Turing machine ok.

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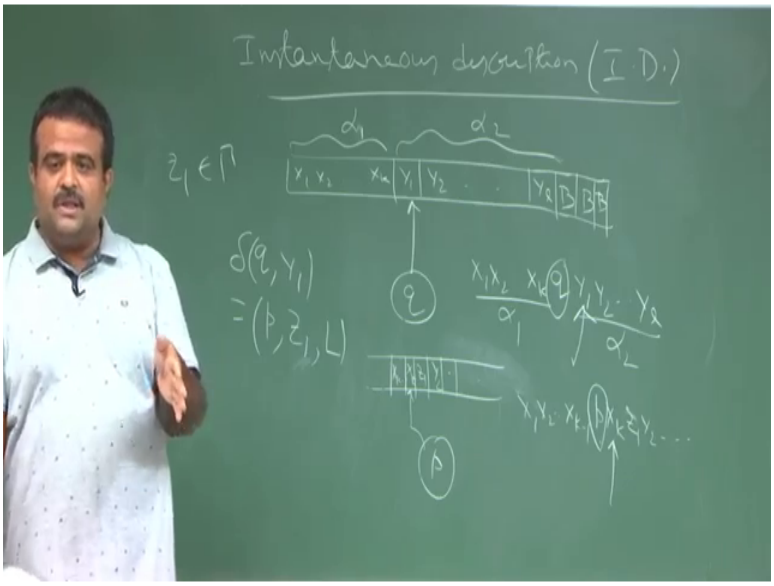

So, let us talk about instantaneous description of a Turing machine. Instantaneous ID we know this is the snapshot of the system, this is snapshot of the system. So, suppose we are having this suppose this is the situation of the tape, as you know our tape is bounded by this side. There is other version of the Turing machine, where this side it is also open infinite.

So, there I mean but we are restricting our Turing machine to this side will disclose I mean. So, suppose we have the situation like this is say we have some alpha, and this is the alpha 1, we have already read and this is say alpha 2, we are going to read up to this say. And our so this here we are pointing ok. So, here we are pointing and we had a alpha say this is say X 1, X 2, some X k and this is say Y 1, Y 2, Y l and then we have blanks no symbol means we put it, we refer this there is a blank, we have the blank symbols ok.

Now, suppose this is a at some point of time this is the snapshot of the machine that is the we take a photo this is the situation, we are at state q and our tape it is pointing here and with this. So, we write this in this way, so X 1 X 2 X k, then we put a q. Then we start with the X l, this is the way we this is very similar to what we have seen in the 2-way DFA or even in the, basically 2-way DFA we have seen this type of symbol.

So, we are just so wherever we have finished. So, this is a alpha, this is alpha 1, this is alpha 2; so alpha 1 q alpha 2. So, wherever we are putting q, this is just a representation; wherever we are putting q, q is the state, it is not a tape symbols or anything, but this is just a representation to know that who is the next symbol to be read. I mean where is the tape head is pointing, so tape head is pointing, so q is see the this is our state. So, tape is head is pointing here, so that is the idea. So, tape head is pointing here. So, this is the current snapshot of the system of the machine.

Now, if this is the current snapshot, if we have a transition rule like this say. So, now we are reading Y 1, suppose delta of q and say Y 1. If it is going to some p and this is say Z 1 some new symbols, we do not know what is Z 1 is, but Z 1 is coming from set of symbols. And if it is L say, this could be either L or R; the it can be go right or left, left or right like this.

So, then this will be situation will be like this. So, this is X k, now this was Y 1; Y 1 is replaced by Z 1 and this will be remain same, Y 2 like this and our state is replace state is moved to q 2 p. And where the tape head is pointing, tape head will be pointing so it is left. So, it will it was pointing here, now it go to the left. So, it is now pointing here, so this is q X k; so this is the next ID.

So, next ID means this is like X 1 X 2, then we have so now we are pointing this. So, you have to put the state before this. So, here this is the X k minus 1. So, X k minus 1, then we put this p, then X k, then Z 1, then Y 2 like this ok. So, this is the meaning of this is we are just wherever we are putting the state, our tape head is pointing; tape head is pointing to the next symbol after the state. So, we are putting the state there and next symbol is X k. So, exactly the tape head is pointing to the X k, so that is the idea of this representation.

So, we put the state in such a way that after the state symbol wherever the input wherever the tape symbol is will be pointing that ok. So, this is the way we defined this. So, now we can have breakers, we can have this is called move relations. So, we will formally define the recursive way to define the move relation, then we will see how we can a according mean by acceptance of a language.

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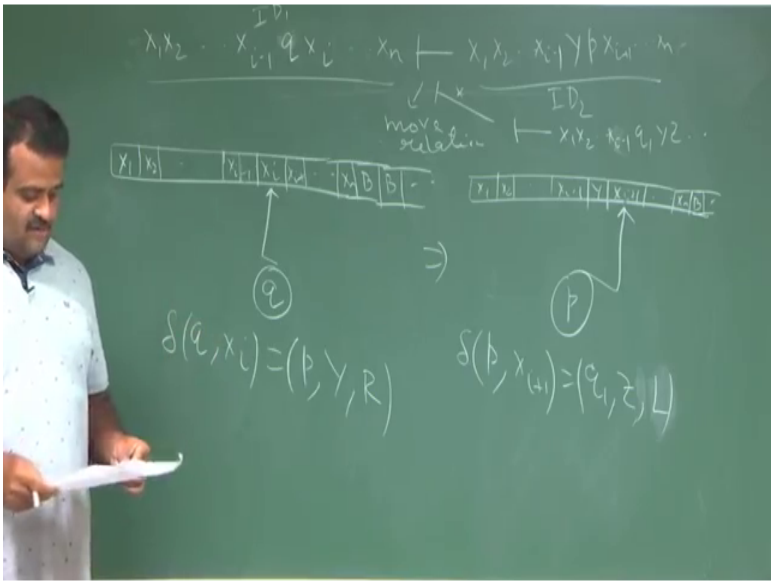

So, yes so, we take this suppose this is the situation X 1 X 2 X i minus 1 and this is the q X i then X n, this is the situation of the tape. So, we have X 1 X 2 X i minus 1 X i X i plus 1 dot dot dot X n. So, suppose we are and after that we have all the blank symbols.

And now our state is q and it is pointing to here, so that is why we represent this by this is a snapshot and this is the corresponding ID, this is the ID 1, this is the corresponding ID and we have blanks over there. Now, if we have a move like this if the delta of q comma X i, if it is say p, Y, R that means, we are reading this X i we are at state q if the state will change to so this will change to where, so this is the symbols. So, we have X 1 X 2 everything is fine, X i minus 1, only this X i will be replaced by Y. So, this is Y, this is X i plus 1 dot dot dot X n and then we have the Bs over here.

And now our state is changed to p, our state is changed to p. And where we are now, where is the tape head will point? So, it is the right move, the head will move to the right. So, right means it will go here. So, it will point to the this location. Now, what is the corresponding ID for that this is the ID. So, these ID is going to so we have X 1 X 2, then X i minus 1, then we have Y i sorry, Y then this is p and this is X i plus 1 dot dot dot X n like this ok. So, this is the situation we have.

Now, so this is called move relation; so from one ID to another ID, so this is say ID 2. Now, from here if again we have the relation like this. So, delta of say p our current state is p and we are reading X i plus 1, if this is going to some q 1 and if it is Z, then it will go to this ideal go to, where? X 1 X 2 and sorry if it is say R again ok; let us take a L again.

So, if it is going to L, then it will go to like X 1 X 2, then this is going to left and this will be replaced by Z. So, it will point here and this will be replaced by Z. So, X 1 X 2, then Y will be there and it will point to here. So, X i minus 1, then q 1 and then Y and this is replaced by Z and then this. So, this is the move relation.

So, then from here to here we can say we have star relation to go from here to here ok. So, this is a reflexive relation of the move relation. So, using this move relation we can define hard what do you mean by language accepted by a Turing machine, so that we will discuss in the next class.

Thank you.