Introduction to Automata, Languages and Computation
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Lecture – 50
Example of a Language Accepted by PDA

So, we are talking about the Language Accepted by the PDA just to recap. So, given a PDA what do you mean by the language accepted by the PDA.

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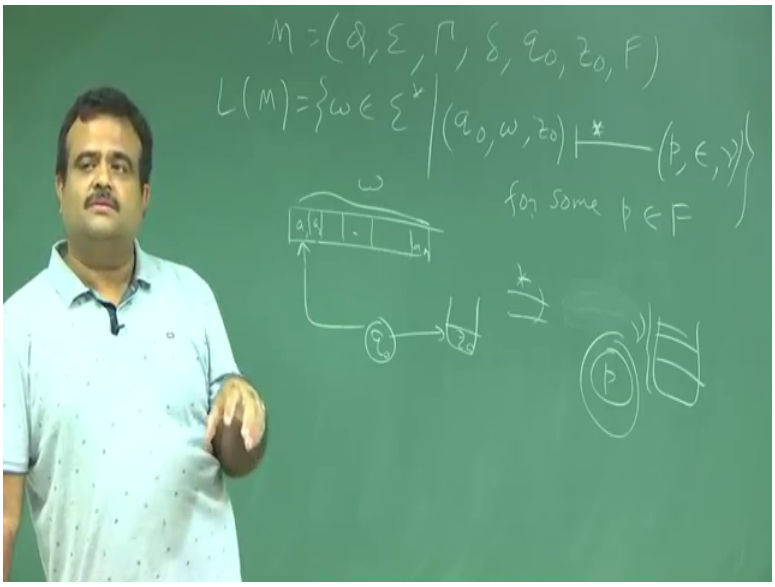

So, suppose we have given this PDA, this is the set of states this is the input alphabet this is our output a stack symbol all are finite. This is the transition rule q 0 z 0 F. So, z 0 is the current position of the stack. Now we have 2 types of acceptance by acceptance by this PDA 1 is the language accepted by the PDA by means of final states so; that means, if this is the set of all strings, for which the current this is the current idea of this situation.

So, we read we start with w. So, q 0 is the starting state and w is the current position of the tiff. So, this is w, w could be say a 1 a 2 like this and q 0 is the current which is pointing here and the stack symbol is z 0, if it is I mean z 0 this is the current id of this and if it is reaching to the sum of the id, we apply the delta rule on this. So, it will change it will go to the next input and it will change the this thing.

So, if eventually it is going to some p and if input alphabet is over if we read all the input and then we do not care about this output alphabet where p is some for some state p from the final state ok. So, eventually if it is going to. So, tiff is over there is no more nothing is there in the tiff and we are going to p and we do not care about this thing because this is accepted by the final state and if this is a final step then we saw we say the string is accepted. So, collection of such string we tell this is the language of this PDA by the means of final state.

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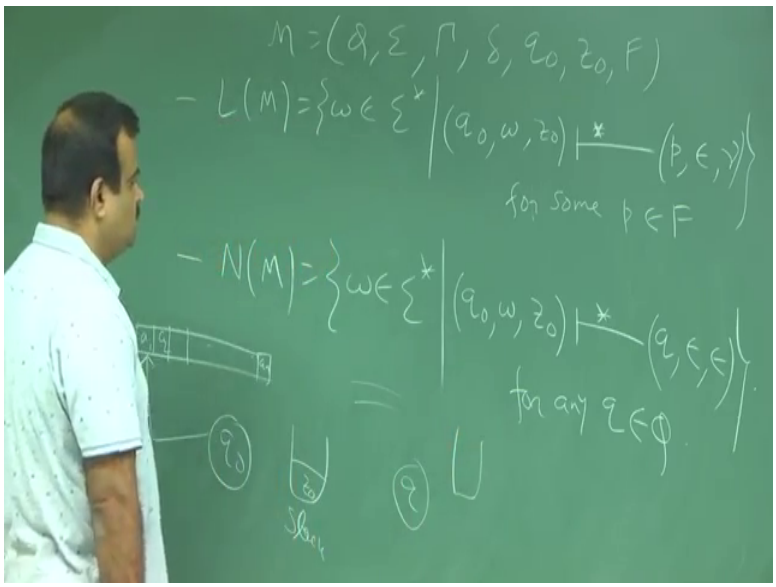

And similarly we have language accepted by the PDA by means of empty stack, this is denoted by n of this is set of all w such that we start with q 0 we take this z 0. If it is going to we did not care about this state it could be anything q epsilon, for any state for any q belongs to Q need not be a final state.

We do not care I mean would be a final state, but we did not we did not need that we only need the ultimately stack symbols would be empty. So, this is the current position of the input tiff a 1 a 2 a n and this is q 0 and our tiff header is this and this is z 0 stack and ultimately that input will be gone and we are at q and stack is nothing empty epsilon.

So, then that is called N of M. So, we will see how they are equivalent I mean given a they are basically the same I mean given a PDA which is accepting by which is accepting

a language by the empty stack, then we can construct easily a corresponding PDA which will accept by the acceptance state. So, we will see that.

So, before that will take some example where I will see it is accepting by the empty stack. Some of the string we will see those are accepting by the empty stack. So, we will take some example.

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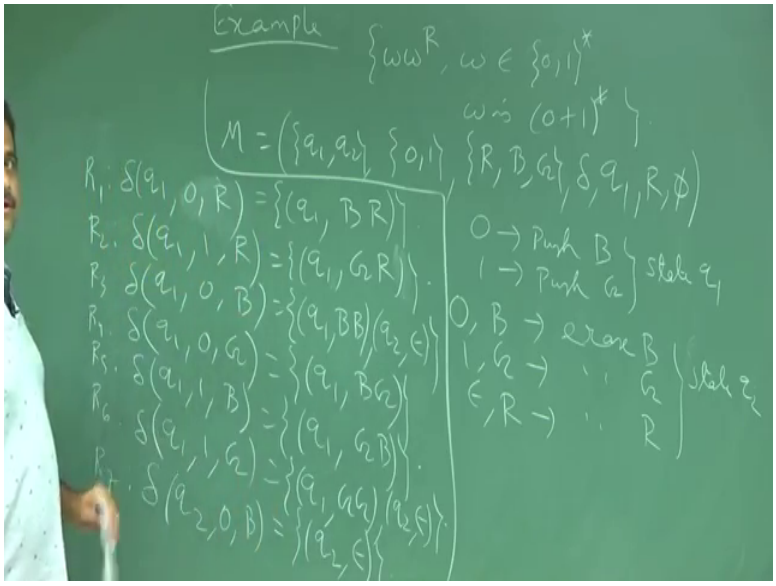

So, let us take some example. So, we want to construct a PDA that accept this w R. So, palindrome basically. So, a w is in 0, 1 star so; that means, w is w is 0 plus 1 star. So, we would like to construct a PDA which is going to accept this language I mean in terms of the empty stack ok. So, the idea is.

So, we take just 2 state because your states are not important if we are talking about the accepted by the empty stack and say this is our input alphabet. And the stack symbol are 3 R B G we will we will see and this is our transition rule and our q 1 is the starting state and R is the that z 0 that is the starting symbol in the stack and then we have no final state because we are talking about this in terms of accepted by the empty stack. So, we did not care about the final state ok.

The idea is if you see a 0 then we will push B into the stack and if we see a 1 we will push G into the stack ok and then we will be at state q 1 I mean if we see these are we have to remain a state q 1. And if we see a 0 and if we see a B in the stack we will erase

the B I mean we will go to we will replace this by epsilon we will see the rule erase B and we will change the state and 1 G will erase G and epsilon R will erase R and our state will be q 2 ok.

So, this is the rule we will follow. So, if you follow this rule we can write the delta. So, delta is like this. So, delta of q 1 0 R sorry R is the 0 comma R, R is the starting state I mean that is corresponding to that our z 0. So, then this is going to singleton set q 1 will be remain that q 1 and we will push B that is the idea B R will be remain same ok.

Delta of q 1 if you see a 1 R, if you see a 1 from q 1 we will push. So, we will be remain at q 1 that is the rule we are following and we will push G R. So, this is the this is again a singleton set and then delta of B this is non deterministic, but we have only one option over here that is fine.

So, delta of q 1; delta of q 1 0 B, if you see a B 0 B then we have option like this here we have 2 options if we can go to q 1. The 0 B means 0 if you see we push B that is the push B 1 option or we will go to q 2 with epsilon we will just erase this B. So, this is another option and delta of q 1 0 R 0 z will be this is also singleton set q 1 B G.

Let me write this then it will be more clear delta of q 1 0 B the delta of q 1 1 B this will be q 1 q 1 1 B this will be q 1 G B and delta of q 1 1 G. So, this is 1 rule 2 rule 3 rule 4 rule 5 we can write this R 1 R 2 R 3 R 4 R 5 R 6. So, R 6 is basically delta of q and. So, here we have 2 options like this. So, q 1 G G or q 2 epsilon this is non deterministic. So, we can have more than 1 option. R 7 is nothing, but delta of q 2 0 B; q 2 0 B it will be q 2 then epsilon it will be q 2 epsilon I will write it here ok.

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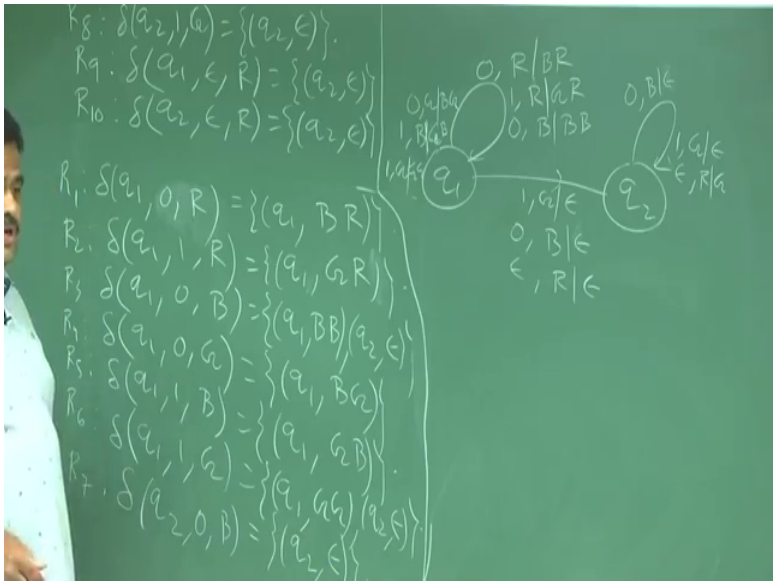

So, this is R7 R 8 I will write here. R 8 is delta of q 2 1 G this will be q 2 epsilon q 2 epsilon and R 9 and R 10 delta of q 1 epsilon we have epsilon move also. This is q 2 epsilon and the last one is delta of q 2 epsilon R this is nothing, but q 2 epsilon R ok. So, this is the transition rules and we have. So, this is the transition rules of a PDA. So, there are we are these are our this is our delta. So, now, if we draw this we can draw this. So, we have a q 1 2 state q 1 q 2 ok.

Now, from q 1 if we see a 0, so, let us write this move. So, from q 1 if we see a 0 and if the stack is R, it is going to q 1 and the stack will be B R. So, that we can write like this. So, q 1 if we see a 0 and if the stack is the top of the stack is R, then it is going to same state q 1 and the stack symbol will be replaced R by B R.

So, that is the way we usually like this. So, similarly we can write 1 if we see a 1 and if we see a R over here, then it will replace to the star it will be remain and q 1. So, you will be remain at q 1 and the stack symbol will be G R like this. So, let me complete this 0 if we see a B, it will be BB and if we see a 0 G, then it will be BG with the same state it will be remain and 1 B it will go to 1 B.

So, if we had q 1 if we see a 1 if we if the stack is B, it will go to remain at q 1, but the stack will be GB it will go to G B ok. And with 1 if we see a G it will go to with 1 if you see a G it will go to GG or it will go to with 1 it will go to there is 1 option here with 1 if

you see a G we go to q 2 and this 1 we are writing we go to q 2 and G will be erase like it will be epsilon.

So, if you have G it will be epsilon over here ok. So, this is just your trying to draw this ok. So, now, for it q 1 sorry with q 2. So, with q 1 you have another move over here with 0 and if we see a B, it will go to q 2 with epsilon and if we see epsilon R it will go to q 2 with epsilon. So, this one q 1 epsilon R it will go to q 2 with epsilon.

Now, here also you have the self loop. So, like this. So, if we see a 0 and B it will go to epsilon sorry and if we see a 1 G it will go to epsilon, similarly if you see epsilon it will go to R if you see R epsilon like this. So, this is the transition diagram of the PDA corresponding PDA.

Now, we want to see what type of language it is accepting? We want to see the string what type of string it is accepting? We want this to be this should be palindrome like w w R; R is the reverse of that. So, let us check whether where it is going. So, we can use that you can use the diagram for this transition id I mean description.

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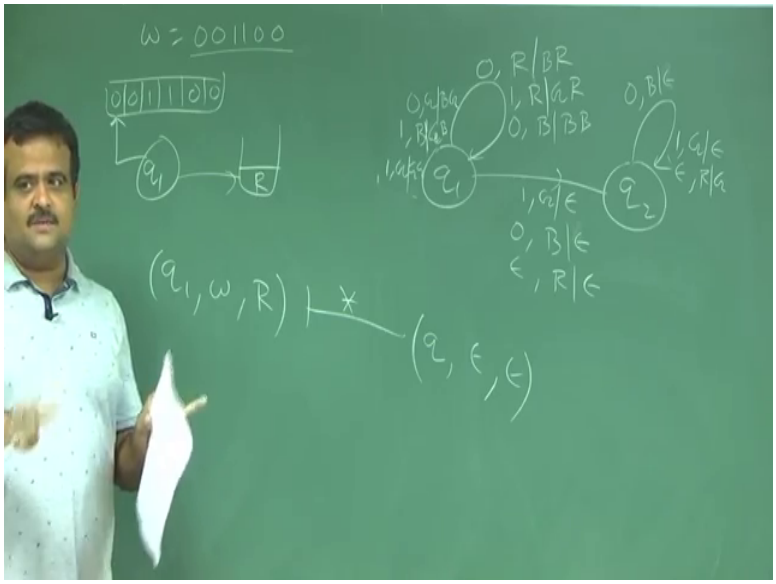

So, now we want to see we want to read a string like this. So, 0 0 1 1 0 0 this is a palindrome ok. Now what is the this we want to read we put these in a tiff and what is the current state of the we are at q 1, we put this in a tiff like this 0 0 1 1 0 0.

So, we start with this and what is the initial state? A initial stack symbol is R. So, we are going to read this and we change the id this the snapshot of the initial state of the initial photo of the initial snapshot of the PDA. And depending on the transition we will move we will go to some other id we will go to some other id other id like this.

So, eventually if we can using this eventually from q 1 w R, we want to see whether we can reach to this situation like we did not care about the states any state q; q is either q 1 or q 2 and this should be epsilon this should be epsilon. So, you want to see whether is there any path which can reach us to this situation by the move relation.

Move relation means id relation I mean we have the current instantaneous description, then it is going to the next inference in this description and like this. So, by that using the transition rules, we will see that ok. So, let us try to draw that.

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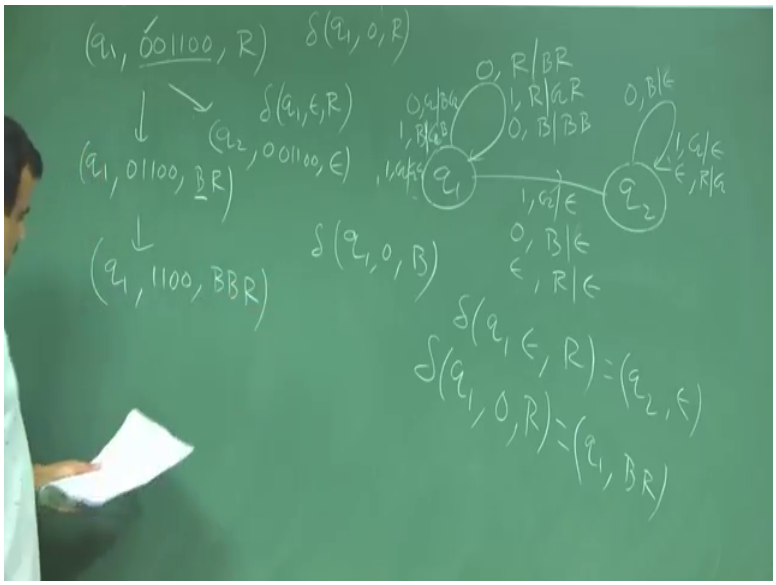

So, initially we are at this is the initial id q 1, this is w 0 0 1 1 0 0 and initially we our stack is R. So, where it is going it is going to? So, we are reading this. So, delta off this is now transition is q 1 0 R so, q 1 0 R. So, we are at q 1 we are reading 0, we are reading R. So, it will go to q 1 0 R. So, we will we take this rule q 1 0 R means. So, we will take this we are reading this symbol we can read epsilon also if you read epsilon, then it will be like delta of q 1 epsilon R. So, if you read epsilon.

So, q 1 epsilon R is nothing, but where it is going? Q 1 epsilon if which is. So, delta of q 1 epsilon R it is going to. So, it is giving us what it is giving us q 2 it is going and R is gone. So, it is going to this q 2 and this will be remain same because we did not explore any string other than epsilon, we are just only took the epsilon and then this will gain to you.

Once this stack symbol is empty we cannot move further. So, halt this is a halting situation halt. So, this branch cannot pass it further because once the stack symbol is empty, because for delta transition we need to have the stack symbol to be stack should have some symbol on the top. So, this situation is halt now we have to pass it this way. So, we can read this. So, if you read this we have to take the delta of q 1, this time we are not taking epsilon we are taking a valid input symbol which is 0 and R.

So, this is what this is nothing, but q 1 0 R is nothing, but q 1 and R is replaced by B R. So, this is going to be q 1 so, this is 1. So, 0 1 1 0 0 and R is replaced by B R ok. So, similarly we can try with now we can read 0 if we read 0, we are not considering epsilon because epsilon move will be getting halt in some to situation, but anyway I mean if you want to have a complete tree you should explore with the epsilon also ok.

So, now we are in a situation of delta of. So, we want to read 0, q 1 0 again and now our tiff here that is B. So, q 1 0 B; so, q 1 0 B; q 1 0 B has 2 option; one is q 1 we are at q 1 if you see a B it can go to B B. So, like this it will be remain that q 1 and 0 you have already read and B is going to BB B B R and another option is; another option is this one.

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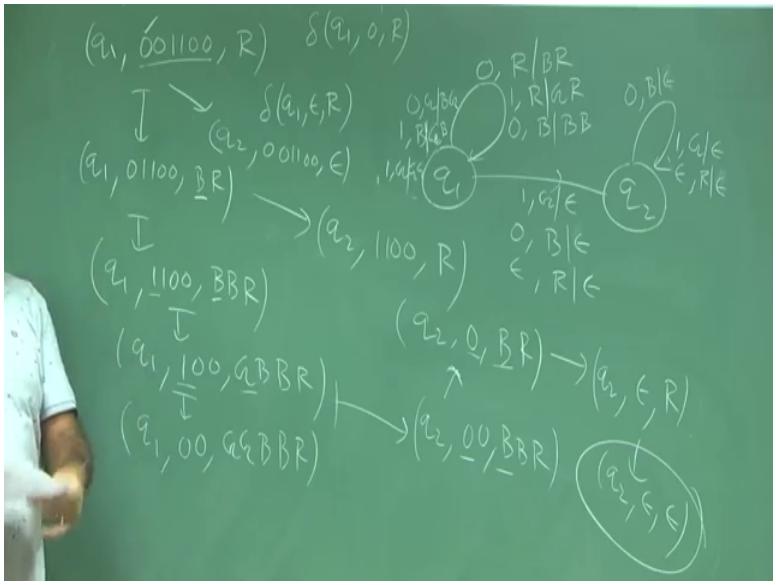

So, q 1 0 B it will B will be remove. So, this will be go to q 2 and 1 1 0 0 B is going to epsilon. So, this will be erase we have R ok. So, then q 2 then from here we can proceed, but we will see it will be halt after some state. So, even it will halt with the next symbol. So, anyway we will try to pass it with this. So, now, we will say this and B so, 1 q 1 1 B; so, q 1 1 B.

So, we are at q 1 we see a 1 B 1 B it will go to GB and it will be remain at q 1. So, q 1 1 0 0 B will be going to GB this B. So, B R will be remain same this is the stack this is the top of the we are always reading the top of the stack. So, like this and 1 0 0 we can now go to we can read this 1 with g. So, we are at q 1 with 1 G.

So, q 1 1 G; so, q 1 1 G have 2 options q 1 1 G can be epsilon also. So, this is going to q 1 again 0 0. So, G will be replaced by G G B BR this is one option another option is here if we take this one q 1 1 G. So, this will be q 2 and this is 0 0 and this G will be erase so, BBR ok.

Now, we got to explore this also, but let us try to explore this one. So, this will go to here this will go to now q 2 we will see a 0. So, q 2 if we see a 0 it will be with B. So, it will be q 2 and 0 B is erased B I then again this is the instantaneous prescription. So, this I should use this notation then this one, again we see a 0 now with B it can again go to q 2 then 0 is gone epsilon is here B is will be erased by epsilon.

So, this is R now q 2 0 q 2 epsilon R. So, if we see epsilon and R if you see epsilon and R it will be q 2 and R will be sorry this is epsilon this is not G. So, if we see epsilon and R, then it will be q 2 epsilon good. So, this branch is going to empty stack even with this if we if you work little more step also here it is going to empty stack.

So, finally, from the starting initial id we could able to reach to the this id I mean which is having the situation where empty string is empty and the stack is also a empty. And remember stack will hold if the stack is empty. So, now, it is halting and it is reaching to the we have already explore the we have already exhaust the input alphabets so; that means, this is going to. So, this is the initial id.

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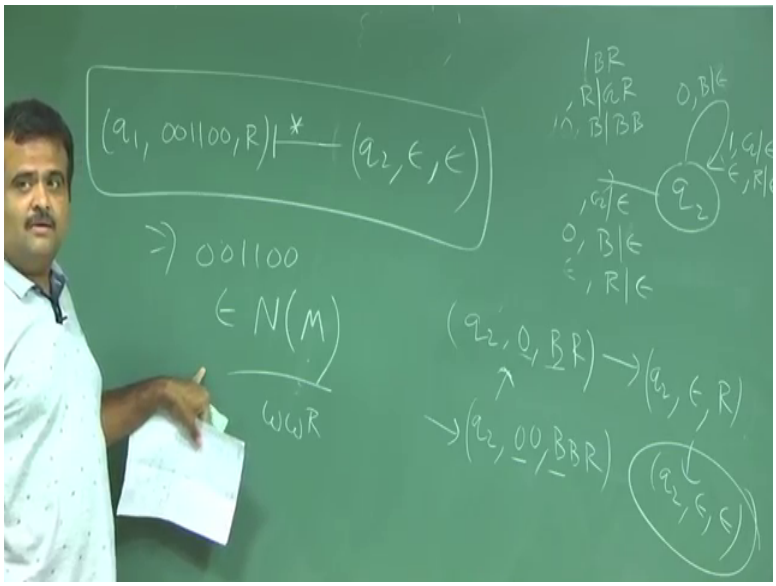

Initial idea is q 1 then the this w is 0 0 1 1 0 0 R, this is going to; this is going to the id like this q 2 epsilon. So, this implies 0 0 1 1 0 0 belongs to n of this is belongs to our epsilon. So, this belongs to n of this. So, this string is accepted by the PDA in the sense of empty stack ok. So, we can prove the all type of wwR, R will be accepted by this PDA in the sense of empty stack.

So, in the next class we will see this equivalency between the I mean this empty stack and the language accepted by empty stack and the language accepted by the final state, and we will see the these are basically the class of context free language.

Thank you.