Introduction to Automata, Languages and Computation
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Lecture – 47
Greibach Normal Form (GNF)

So, we are talking about normal form in the context free grammar. So, we have seen one normal form which is CNF. Now, we will talk about GNF which is giving normal form this is of the form suppose G is the grammar.

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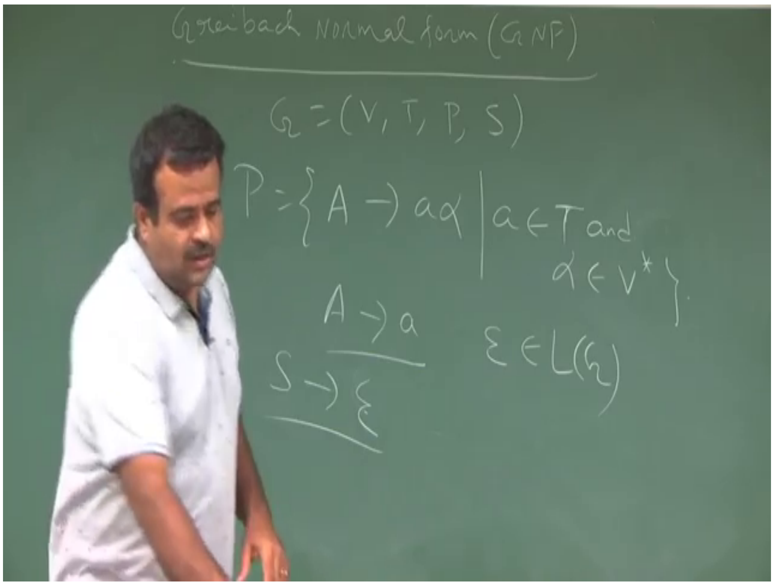

And if the all the rules are of the form like this A is going to a alpha, where a is a terminal and alpha and alpha is coming from string of variables. A is a terminal alpha is coming from string of variables. So, this is alpha could be epsilon also because this is a string. So, in that case it will consist of all the rules like this. And also if this is if epsilon belongs to this language generated by this, then we have a rule S is going to epsilon so, that will be there.

So, if our grammar is a grammar if the productions are of this form, then the corresponding grammar is called in the normal form and this is called a Greibach normal form.

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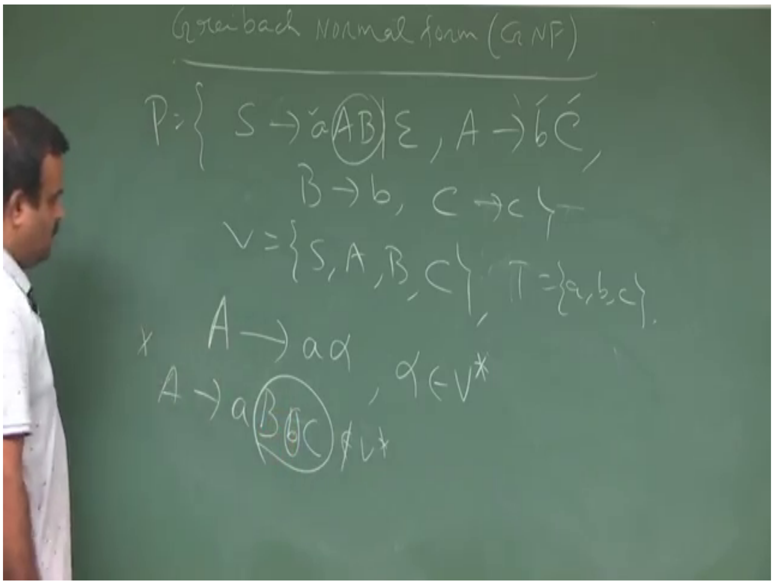

The quick example you can take, suppose we have grammar like this rules are this a A B or epsilon and A is going to b C capital C and B is going to b and C is going to small c. So, here the V is S A B C and the terminals is small a b c and S is the starting variable and this are the this is our P; P consists of this once for actions, this is in giving normal form. Because, every rules are of this form is A variable any variable going to some a alpha where alpha is belongs to and here since this way you want to generate the epsilon, then we have to take the rule S is going to epsilon.

So, this is of this form, this is terminal this is variable, this is terminal and this is variable like this. So, this part we have all the variables. If, we have a rule like this, A is going to say a, B b c. So, this is not Greibach normal form, because this part is not belongs to this term, because we have A terminal over here. So, this is not in Greibach form GNF ok.

So, now in general not every grammar is in this form. So, we have to reduce the grammar in this form, because this will help us to prove the we will see the pushdown automata, which is accept the grammar in this form. So, we will see that part later on ok. So, we want to convert every grammar in GNF. So, for that, similarly we can first eliminate the null production and then the unit production and then we can convert in CNF first, then from that we can convert to GNF or directly we can do that ok.

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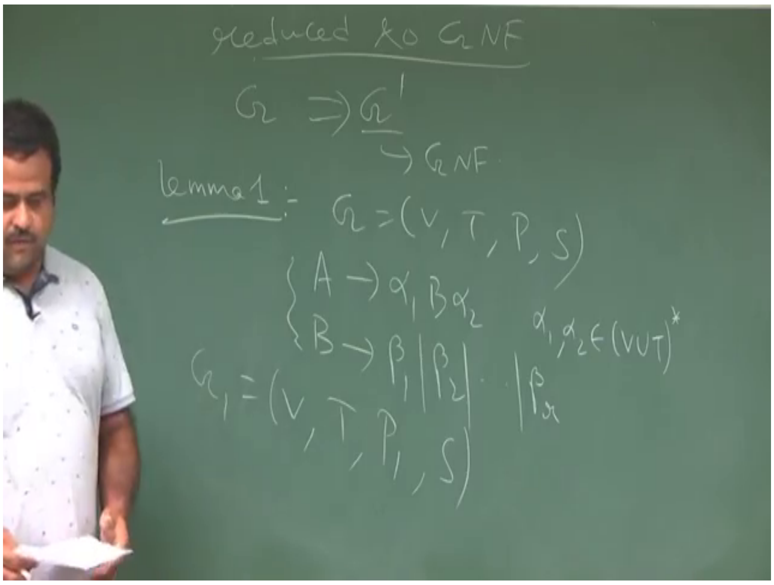

So, for that we will be using 2 lemmas, 2 you will be using 2 lemmas to convert reduce to GNF; given a grammar G you want to reduce this grammar to some G prime, which is in the form of GNF. So, for this we need to use 2 lemma; lemma 1.

So, lemma 1 is telling suppose you have a grammar G, which is V T P S. And suppose we have A rule like this alpha, B alpha 2 ok. B is a variable alpha 1 alpha 2 could be anything like it could be a string of variable and this could be string of alpha 1 and alpha 2 could be anything like alpha 1 alpha 2 are coming from ok.

Suppose, this is A production in P. And we have a B production, this is A production in P and we have A B production in P like this B is going to some beta 1 or beta 2 dot dot dot beta r. Now, we want to suppose these two are not containing any beta or maybe we do not bother about that, but this B sorry this B we want to remove, we want to eliminate this B.

So, for that this two production I mean set of productions we can say this two rules set of rules, we can replace this grammar we have many other rules. So, this grammar we are going to replace by G 1, which consists of V T P 1 and S 1. So, P 1 all the rules will be same of G only thing these two rules we are going to; these two rules we are going to replace by a new rules, like we want to eliminate this B.

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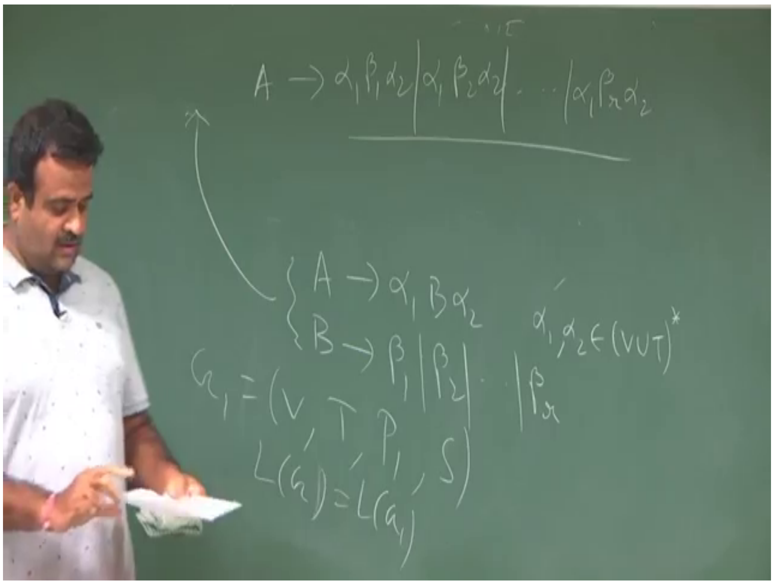

Like, A is going to so, we are going to add this, A is going to B is going to this so, if you put B over here. So, A will be going to alpha 1 beta 1 alpha 2 or alpha 1 beta 2 alpha 2 dot dot dot alpha 1 beta r alpha 2.

This is the so, these two products and we are going to replace by this productions. And we can easily verify that these two grammar will be same as you generate the same grammar, because see we are just going to replace this B by this Bs rules, these are the base rule Bs production and this is As production.

We do not want B to be here. So, that is why we are going to replace all the Bs production into the As production. So, this is referred as lemma 1 this process. And we can prove that in the lecture note that what the details prove will be given these two grammar are same ok.

So, we want to just eliminate this B variable by the all the B s production, this is called a this process is called lemma 1. And then we have another process which is called lemma 2, which will be also useful for the converting the grammar to a GNF.

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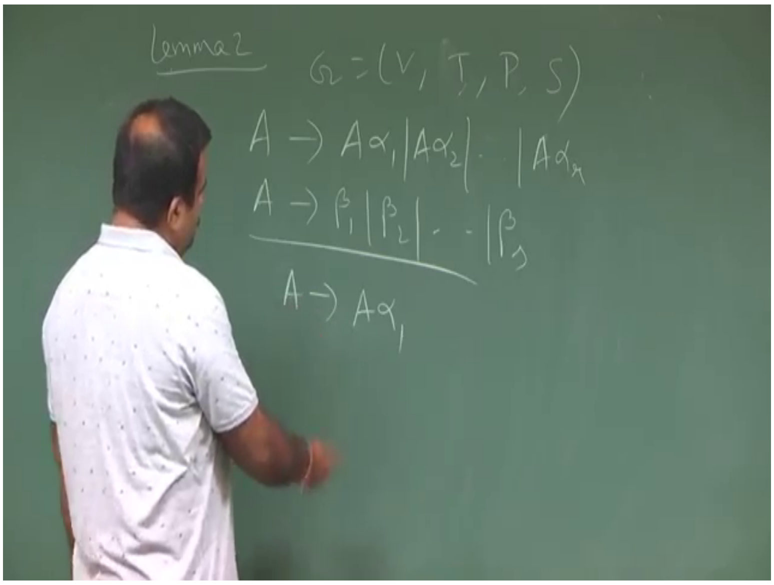

Lemma 2 is telling, suppose you have a grammar G sorry V T P S. And suppose we have this productions A is going to A alpha 1, A alpha 2, dot dot dot A alpha r. And also we have A is going to beta 1, beta 2, beta s ok and this betas are not starting with A.

So, betas are not starting with A. So, these are all strings string offs variables and terminals. This A is a variable, we know this type of so, A is going to; A is going to this type of rules are not allowed in the GNF, because GNF A must start with A variable for I sorry terminal followed by the variable. So, we have to; we have to change this, we have to remove this. So, this lemma 2 will be is the process to change this. So, how to change this?

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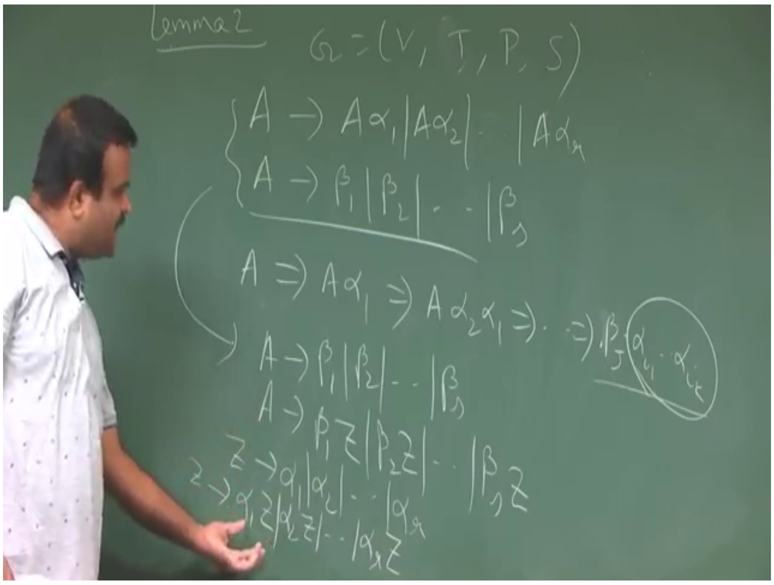

So, we can see A is going to this then we can just have A so, if A is derived A from A we can derive say A alpha 1. Then, again we can apply any one of this rule say A alpha 2, alpha 1. Then, we can derive any of this rule like A or finally, this A we can replace one of these betas say beta J and followed by some alpha 1 alpha like this. So, everything will be converted this ok.

So, now here these betas is not having any A, that is our goal we want to remove this A from the rule. So, for that what we can do we can bring a new variable called Z and we can introduce the Z production like this. So, we will keep so, this 2 will be replaced by we will so, this one this one can be written as like this. A is going to this will be remain same this we have no issue, because they are not these betas are not involved so, any A starting A. And A is going to beta 1 Z, because this we are going to represent by Z, which can be generated we have A Z production beta to Z dot dot dot beta s Z.

This is the A production and we have A Z production like this. So, Z is going to alpha 1, alpha 2, because Z is role of the Z is to generate the string of alphas. So, alpha 1 alpha 2 alpha r and to generate the string of alpha we need to have this one, another rule Z is going to alpha 1 Z, alpha 2 Z, alpha r Z; alpha r Z, so, this is the rule.

So, this can be derived by this way, because A is going to alpha 1 Z. So, then we can apply this by getting that. So, eventually this lemma 2 is telling, eventually this grammar will be replaced to this grammar where so, this is the given grammar V T P S.

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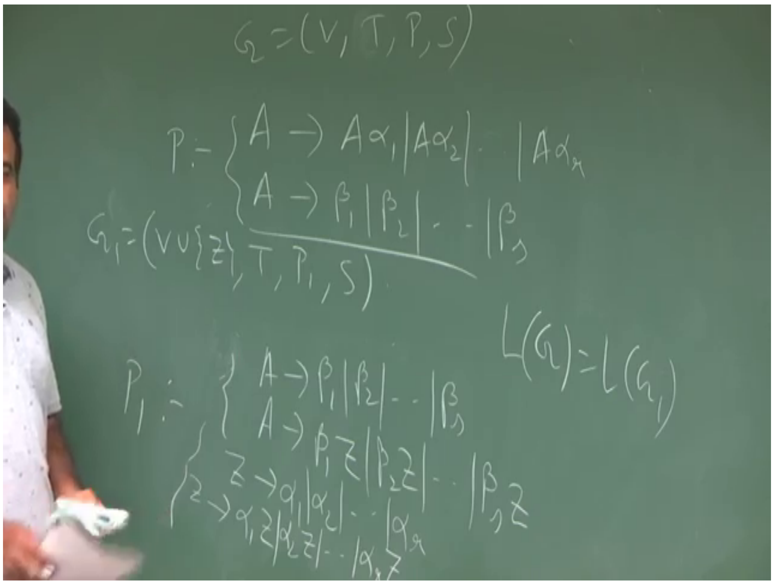

And we have these rules these rules we are going to replace by this rule with the new grammar G prime or G 1, where we are introducing a new variable Z and remaining are same. And in P 1 this was in P with some other rules. In P 1 this type of rules A production will be replaced by this type of A production along with this type of Z production, so, this will be in P 1. So, other will be in the P other will be in P 1. So, then we can show that the grammar generated by G is same as grammar generated by G 1 ok. So, is this clear?

So, this is basically we want to get rid of the starting A, because our GNF is not allowing any starting A. So, for that we need to take help of another variable Z for which this has no issue so, this will be remain same and this will be at. So, there will be each of these will be ending with some beta. So, beta and Z will be we will use Z to have a string of alphas.

So, that is why this rule. So, this A production and this Z production will be added here in P 1. Along with the also only for this we replace this by this A production and Z production. So, this is called lemma 2 ok. Using lemma 1 lemma 2 we can reduce A grammar to GNF so, that we will see. So, the proof is there in the lecture note, the rough idea of the prove I have given.

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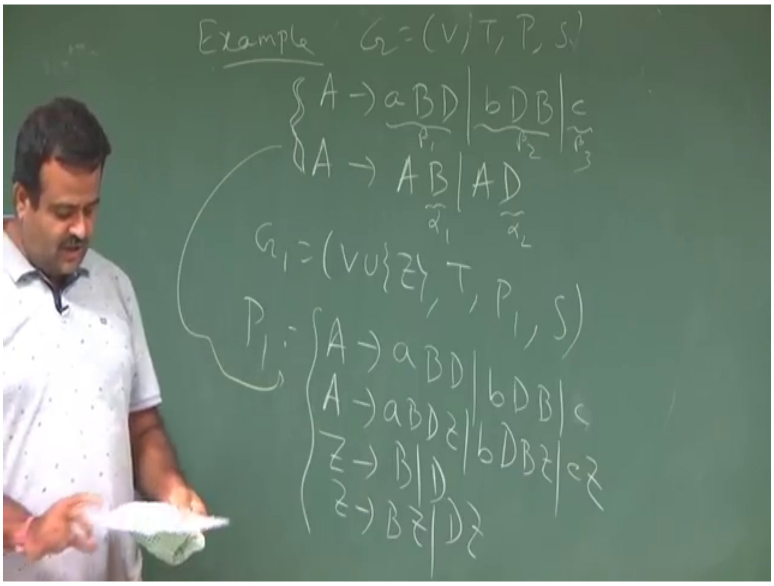

So, let us take an example of this lemma 2. So, suppose we have this grammar A is going to a B and D B and c small c. And we have another rules with A which is having A involve AD. Now, if you compare this is our beta 1, this is our beta 2, this is our beta 3, and this is A; so, this is our alpha 1 this is our alpha 2. Now, you so, this is our G. So, G is consist of V T P S, where B is A a B this and we have a so, we are just talking about all these productions where this is going.

So, this will be replaced by G 1, which is V union from Z P 1 S. And P 1 will be all the rules, which does not involve this and these two will be replace here by this. So, A will be going to we have no problem with this a B D and c small c and we have a A production like this alpha 1 z. So, alpha 1 is this b beta so, sorry beta 1 Z so, beta 1 is this. So, a B D Z, then b so, b D B Z and c Z, this is the A production and the corresponding Z production is so, Z is going to alpha 1, alpha 2 and their recurrence.

So, alpha 1 alpha 2 or their recurrence alpha 1 Z and alpha 2 Z ok, so, this is the rules and we can show that these two will generate. So, these two production will be replaced by these four productions, two is A production, and another modified A production and this is Z production, this A production was remain same at this we are changing we are introducing A new variable Z ok. With the help of this two lemma we can show we can reduce a grammar to the GNF from. So, that algorithm we will know, but basically we will be using lemma 1 and lemma 2 ok.

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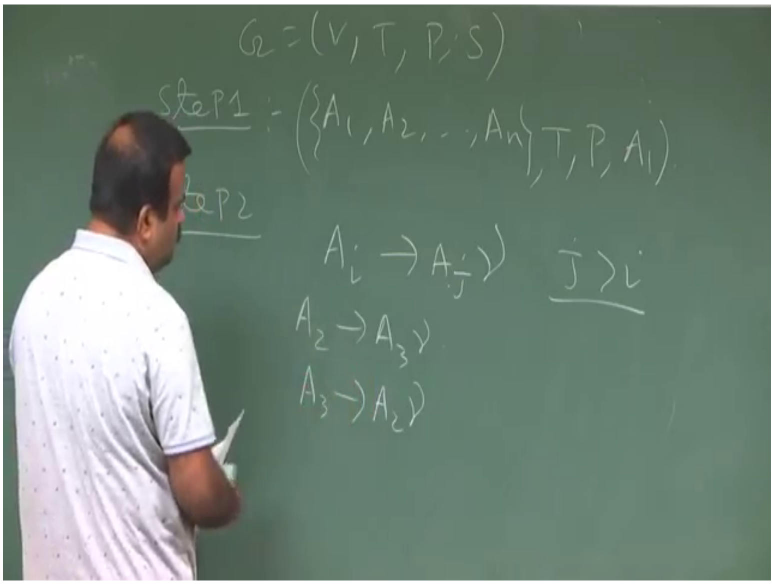

So, for this suppose we have given a grammar G. Now, step 1; step 1 first what we do we rename the; we rename the variables by A 1, A 2, A n. We start with this S, S we rename A 1 and then if we have a rule like this A B. So, S we are renaming A 1 and A can rename A 2, A 3 like this. So, we have basically the variable set A 1 A 2 A n. And A 1 is basically S. So, that we are doing so, this is our V, this is our grammar and T is remain same we are not disturbing T and A 1 is the variable ok, we are renaming the variable.

Now, in the step 2, we want to do what we want to do we want to modify the production such that, A i will be A j gamma, if J is greater than i. So, this will be there. So, we want to have a modified rules or modified products and such that, if A i going to A J gamma then this J must be greater; that means, A 2 is must going to some A 3 gamma something like that. But, A 3 should not go to A 2 gamma, because this 2 is not greater than 3. So, for that we need to do something we need to apply the lemma 1 or lemma 2 like this, so, we will do like this ok.

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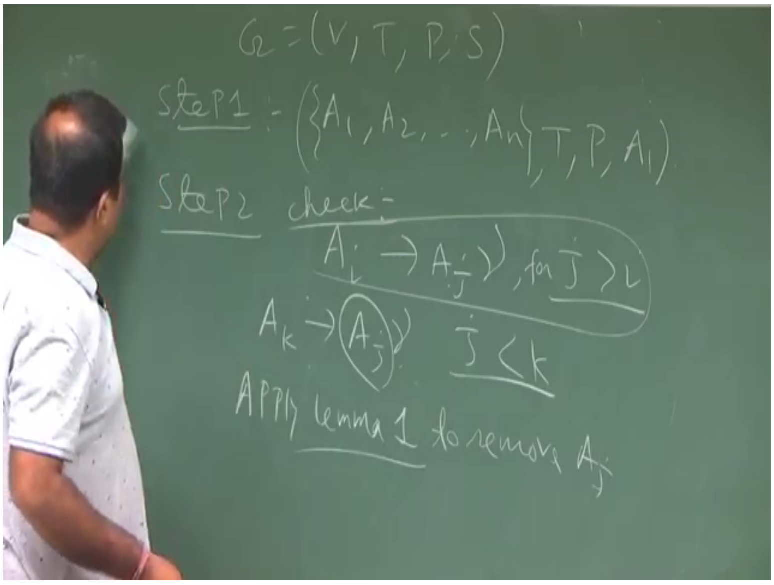

Now, if we if so, first we check whether we check this we check whether all the productions are like this. If all the productions with starting with the variable are like this, if not this is for, if not if there is a production like this such that, A k is going to A j gamma where gamma is like where j is less than k..

If, there is a production which is violating this, then you have to do something on this production what we will do we want to change this. So, how to change this we will apply lemma 1? So, just apply lemma 1 to remove this to remove A j.

And then we will see whether we are getting or not. So, will repeatedly apply this ok, until we got this type of change so, this is step 2. Now, in step 3 we will apply lemma 2 there so, for that we check this so, in step 3 what we do, so in step 3 suppose we have a variable like this, A k is going to A k gamma ok.

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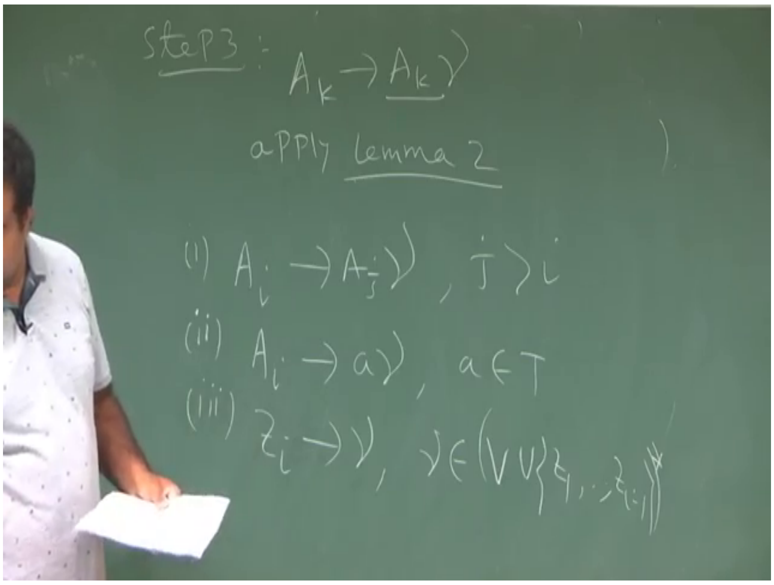

So, this is we have a A k production A k is the variable. Now, we know this type of things is not assigned not allowed in GNF. So, what we have to do we have to apply lemma 2. So, you have to introduce the new variable Z, if you remember and then we can apply the lemma 2 to get this. So, this is the step 3.

Now, repeatedly applying these, what we get. So, after step 3 we will remove all this we will take an example. We will be having this type of form A i is going to A j gamma, where j is greater than i. And A i is going to a gamma, where a is belongs to T and we have introduced the Z variable for applying lemma 2. So, this is Z is going to this, where Z is coming from V and we have introduced all the Z variables like this maybe Z i 1 star ok.

So, up to step 3 we will be having will reach to this situation. Now, we have to check the we have to apply step 4 and step 5 to finalize this so, step 5 is sorry step 4 ok.

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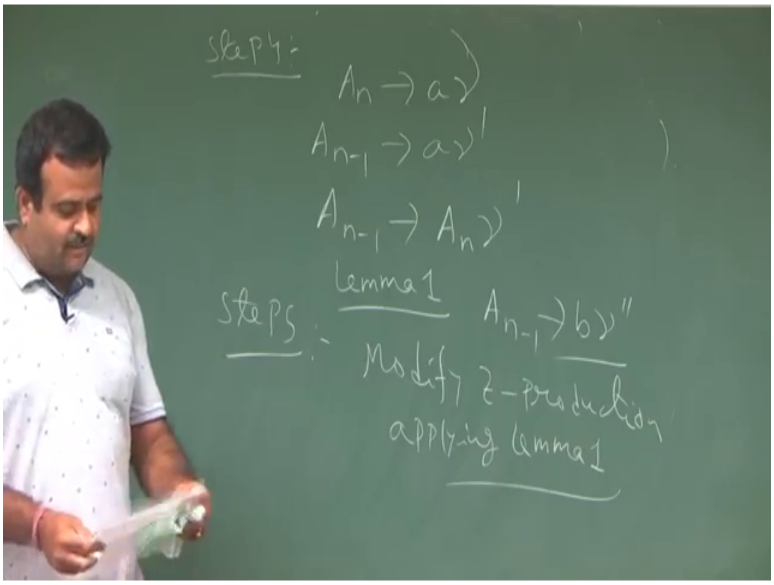

Step 4 is so, we have the all the production like this. So, A n is going to a gamma. So, that is fine, but we have A minus 1 production they are we may have like this. So, A minus 1 is going to a gamma prime we will not do anything, but what we do A n minus 1 is going to say, A n gamma prime something like that. So, these we have to change because this is not allowing GNF.

So, what do you do we apply the lemma 1. If, you apply lemma 1 this we can apply here and we have to we can change this to A n is going to A n minus is going to some b like this ok. So, this may be in GNF. And we will repeat this process up to A 1; we will repeat this process up to A 1. So, this is step 4 and step 5 is we do the same thing with the Z production.

So, we modify the Z production by applying the lemma 1 applying lemma 1. So, these are the 5 step and this we will do in the step 5. So, we will take an example then it will be more clear. So, there are basically 5 step, but basically we are applying lemma 1 and lemma 2 repeatedly ok.

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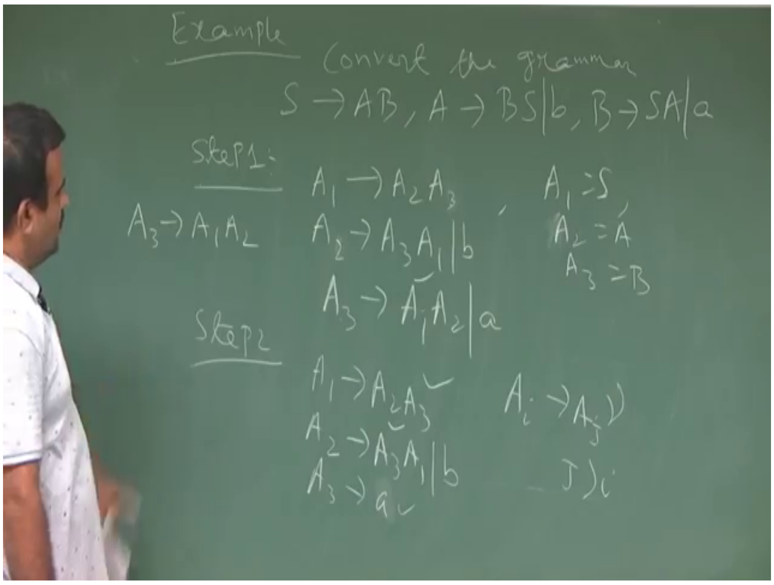

Let us take an example; suppose we have a we want to convert this grammar, convert the grammar to the GNF, what is the grammar S is going to A B and A is going to B S or b, and B is going to S A or a. This is the; this is the given grammar and this is we can easily check this is not in GNF form ok.

So, we can faster I do we can eliminate the null production and we can eliminate the unit production, then we can convert this in first CNA, but we can directly convert into the GNF by applying this for 5 step ok. So, what we do step 1; step 1 is we can eliminate the null production, we can eliminate the unit production, we can convert into CNF, then we can rename the variable. So, this we are going to rename S 1 S is S 1 is going to sorry S is A 1 A 1 A 2.

So, A 1 is S and A 2 is A, A 3 is B like this. So, and A 1 is going to so, A 1 is going to this and A 2 A sorry A 1 A 2 A 3. And now A 2 so, this A 2 is going to A 3 A 1 or b, and A 3 is going to A 1 A 2 a this is step 1, we rename the variable by A 1 A 2 A 3.

Now, we check step 2; step 2 is we check all the productions whether A i is going to A j gamma where J is greater than i. So, this is ok. So, we can add so, this A 1 productions A 1 is going to A 2 A 3 these are in required form and A 2 production A 2 is going A 3 A 1 this is also in required form along with the b. And A 3 productions this A 3 production which is going to a is in required form, but A 3 productions this is not in this form. So, this is not in this form.

So, A 3 which is going to A 1 A 2 is not in this form, because this is one this is not greater than 3. So, we have to apply lemma 1 here so, we do that. So, let us apply lemma 1 here for this for other it is ok. So, we just let me just erase this.

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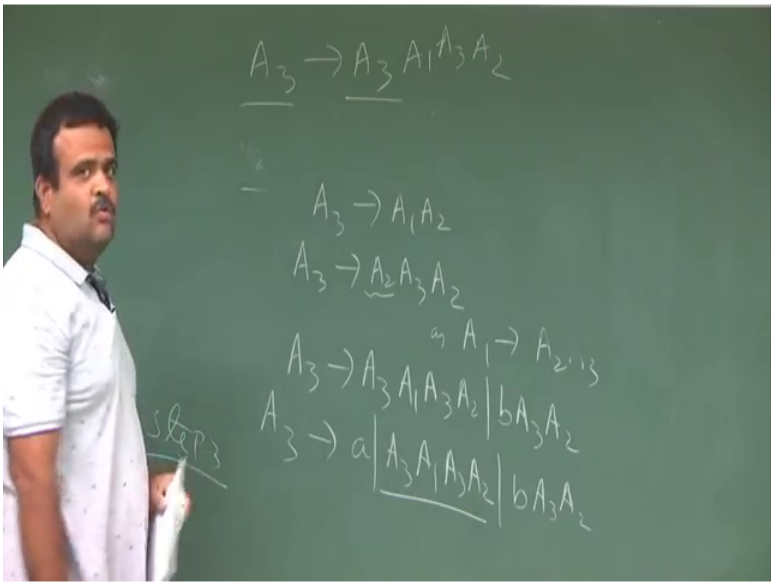

So, we have to do something for the this A 3 is going to A 1 A 2. So, this we have to change. So, we replace this by using lemma 1. So, you replace this by following A 3 is going to. So, we check all the A once where it is going lemma 1. So, A 3 is going to A 2 A 3 A 2, as A 1 is going to A 2 A 3, this is production in A 1 ok. Now, this is also having problem because this is 2 this is 3.

So, now, we are going to replace using the A 2 production. So, if you do that it will be like A 3 is going to we are repeatedly applying the lemma 1, A 3 is going to A 3 A 1 A 3 A 2 then b A 3 A 2 ok. So, this is one now ok. So, now, resulting production is like this A 3 is going to we have the A 3 production there. So, A 3 is going to a or A 3 A 1 A 3 A 2 coming from here, or b A 3 A 2 ok. Now, this is the A 3 production. Now, in this is the step 2.

Now, in step 3 what we do we check all the production like this, like here this A 3 is going to this one, A 3 A 1 A 3 A 2. So, this you have to change, we have to introduce a new variable Z that will change this. So, to do that we have to bring this alpha beta gamma and so, then we can check that this will be like this.

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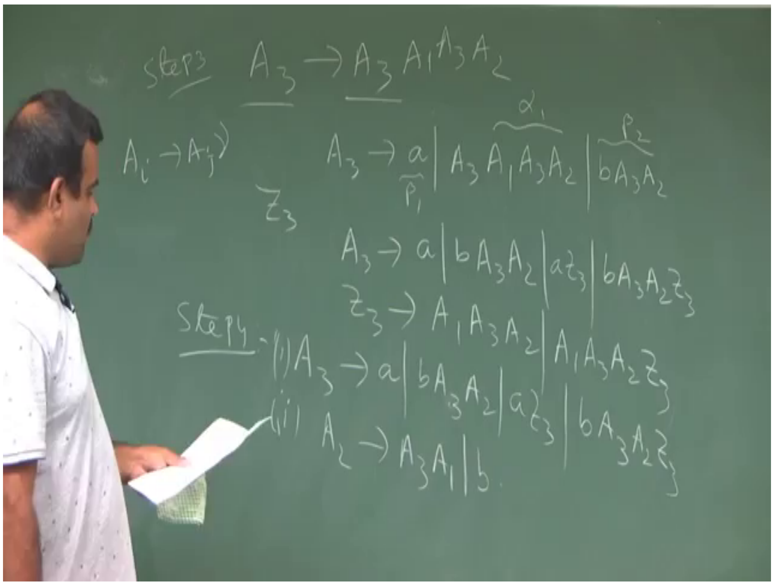

So, let me just write A 3 is going to A this is our beta 1 A 3 A 1 A 3 A 2 this is our alpha 1 and we have b A 3 A 2. So, we are just applying the this is our beta 2 ok. So, if we apply the lemma 2 for this so, we are introduce a variable Z 3 say. So, A 3 is going to a b A 3 A 2 just we are applying lemma 3, a Z 3 and b A 3 A 2 j 3. And then Z 3 is going to A 1 is alpha 1 A 1 A 2 A 1 A 3 A 2 and alpha 1 Z 3 sorry A 1 A 3 A 2 Z 3 ok.

So, this one alpha and Z so, this way we continue this is the step 3 and step 4 again we repeat so, this is step 3. So, in step 4 what we are doing. So, now, in after step 3, we have the all productions in the format of this A i is going to A j gamma where this is not same as this. So, now, so, we have some A n is going to this so, that will be doing in here.

In step 4, if we apply A 3 productions A 3 will be going to a or b A 2 A 3 sorry A 3 A 2 and A Z 3 this one and b A 3 A 2 Z 3. These are all in the required format and for the this is and for A 2 productions what we have A 2 is going to A 3 A 1 and b ok. Now, we writ10 this if A 2 is going to b this is of the required form, but these we have to change, we have to do something on A 2 is going to A 3 1. So, then we will apply lemma 1 again.

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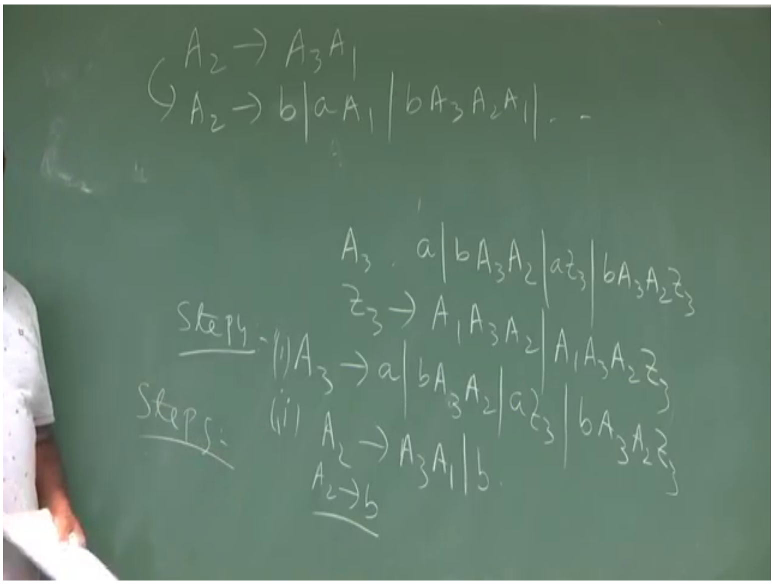

So, A 2 is going to A 3 1 how we can change this we know this A 3 is we know the A 3 rules we apply lemma 1 to change this. So, that will be like A 2 is going to so, this will replace 2 we applied all the A 3 rules. So, a sorry so, b is there so, b is there so, a A 1 then A 3 A 1.

So, this one a A 1, then b A 3 A 2 A 1 like this all the rules. So, this is the lemma 1. So, we will keep on apply lemma 1 then this is step 4, we will do it for all then the step 5 if we apply for this on the Z variable we will check and finally, we will get the required form so, that the details will be given in the lecture note.

Thank you very much.