Introduction to Automata, Languages and Computation

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Lecture – 45
Elimination of Null and Unit Productions

So, we are, will be talking about how we can eliminate the null production and unit production. So, first we will check how we can eliminate the null production or epsilon production from a grammar.

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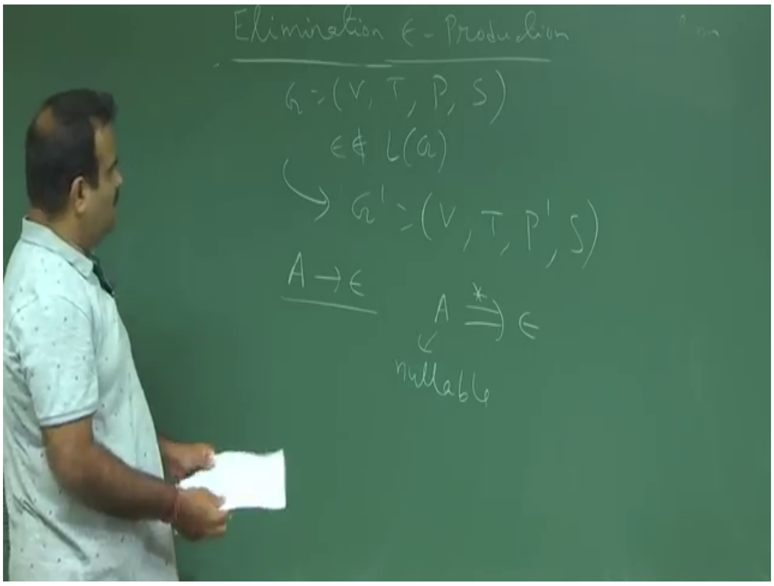

So, elimination of epsilon production, ok; ao basically what do you want? We have given a grammar G which is V, T, P, S. Well, V is the set of variables final set of variable, T is the final set of terminals and P is the production that is all final and S is the starting variable. Now suppose L of G, L of G suppose does not contain epsilon.

Then we can construct a construct a grammar G prime which is V, T, P prime, S such that there will be no null variable, I mean there will be null no null production in this grammar, ok. There will be no null variable, null variables means if suppose A is suppose there is rule A is going to epsilon, this A is called null variable or it is the production is called an A, A is called null variable; that means, we use this type of production to erase A. Suppose you want to erase A so, you use this type of production. Now we do not need this and also nullable means if some variable A is going to Epsilon then this is called A is called nullable, nullable which can be erased like in not only directly some more steps, it is there.

So, suppose say we will take an example; so we want to get rid of from these type of variables and this type of production from this production set.

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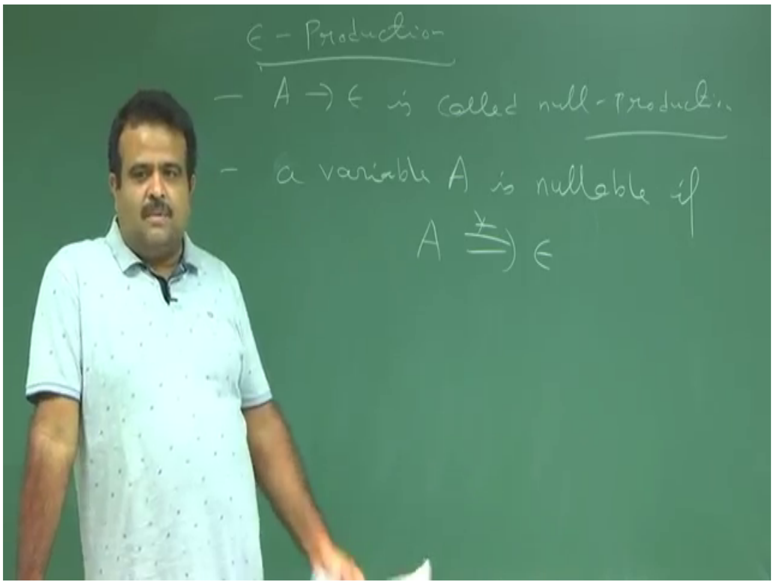

So, let us define what is the null production or epsilon production? A production of this form is called null production, ok. And a variable is, is nullable variable A is nullable if epsilon can be derived from A by few productions by applying few or more productions. So, if epsilon can be derived from A then we call this A nullable. So, then we want to omit the all the null productions, because if our grammar is not accepting epsilon then we can reduce the grammar by omitting the all the null productions.

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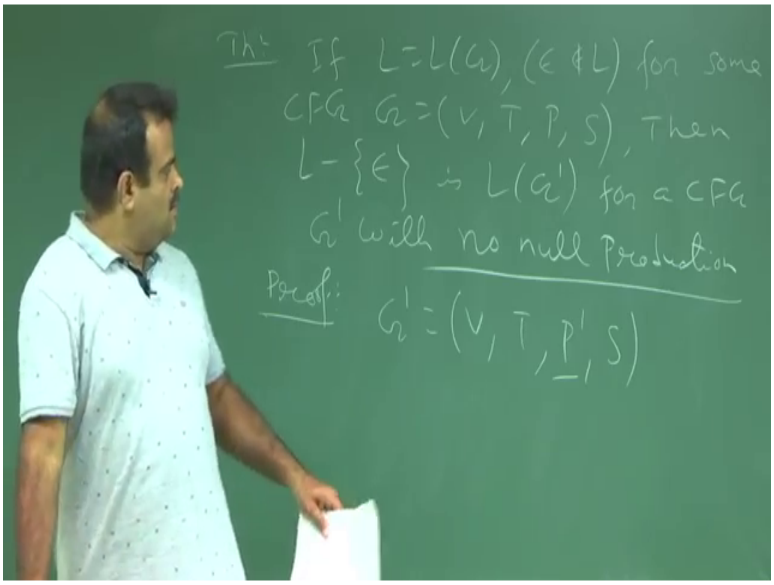

So, then the theorem is telling, then the theorem is telling if L is a L of G and G is the grammar and Epsilon does not belongs to L for some grammar context free grammar G which is V comma T comma P comma S then L minus Epsilon, because Epsilon is anyway not belongs to L. So, we can we are not going to derive Epsilon from starts state for star T symbols or start variable is L of G prime for a CFG G prime with no null production. So, that is the way we want to minimize our grammar.

Ultimately, it will accept the same language. What do you want to omit?,we want to get rid of from the all the null productions, because they are not going to contribute anything because epsilon is not at all in the language. So, how we can do that?

So, basically you are going to construct a grammar G prime yeah, G prime like this. So, VT will be same only the P will be different the rules, I mean the transition will be different. So, for that we need to find the nullable variable recursively. We first find the nullable variable recursively and then we construct the P prime. So, now, the question is how to get the nullable valuables recursively?

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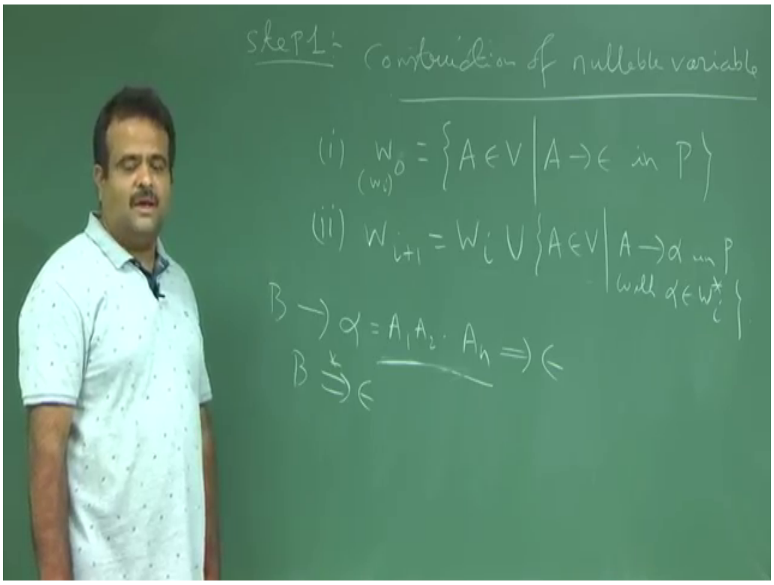

So, this is the first step construction of nullable variable, I mean the variable which are going to Epsilon, I mean Epsilon can be derived from those variables those are called that is A is nullable if Epsilon is derived from A. So, that is then is a useful value you want to find out all the nullable variables, ok.

So, we will do it recursively like we form that Wi, W i is equal to 0 0 say W 0 0, it is all the variables which are directly going to Epsilon. This is the first step, this is we can say W 0 0. And then W 1 that it means basically Wi this is say W i say now we are going to find the next one that is say W 1 for i is equal to 0, i is equal to 1. So, W 1 is i plus 1 is nothing, but W I, because they are already there and suppose we put a variable in the variable is nullable. Now if a variable is going to that variable or a string of nullable variable suppose B is going to some alpha and alpha is consists of say some variable like A 1 A 2 An.

And suppose these are all nullable variables they are already included in the previous W. Then we can say these are all going to this; that means, B is Epsilon is derived from B; so that is the idea. So, this union with all the variables V such that there exists a production A is going to alpha in P where with alpha belongs to W i star with alpha belongs to W i star, because if these are all W i start that if we know these are all nullable variable the previously then B us also nullable, because this is B is having production like this and all are going to alpha I mean if you are more step; that means, this is B is also going to alpha, I sorry, Epsilon. So, epsilon can be derived from B. So, this is the way, we construct the nullable variable.

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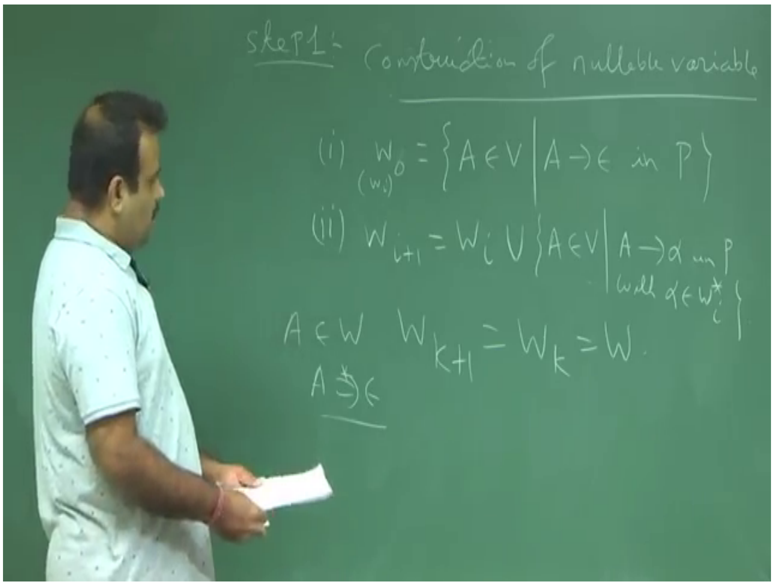

And eventually it will converge so, after some time we will see we have captured all the nullable variables. Eventually we have this WK plus 1 is equal to WK for some K, you refer this sorry WK plus 1 is WK, we refer these are W. So, W consists of all the nullable variables; that means, all the variables if it is belongs to W; that means, this property is there, ok. So, now once we construct this nullable variable then how to construct the P prime, the production new productions.

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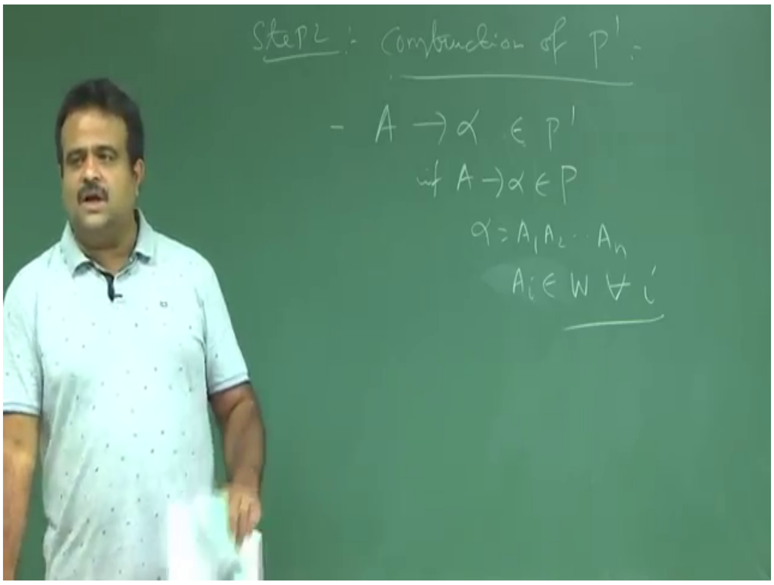

So, that is the step 2 so, construction of P prime, ok. So, first of all so, this is a subset of P this is a subset, this is a not a subset of P, this is coming from the production of P. Now, we take all the first of all we take all the productions whose right hand side does not contain any nullable variable. Then that means, we take we take this in P prime this is a belongs to, this is belongs to we take this since P prime where I mean this is in P, because this is the production coming from this.

And alpha does not belongs to W star I mean alpha is not having any nullable L variable alpha is. So, alpha we can write say A 1 A 2 A n and A i's are any A 's, any A 's and some all the A is, A is are not belongs to W for all i; that means, we just include all the productions whose right hand side, whose right hand side does not contain any nullable variable that is the first step.

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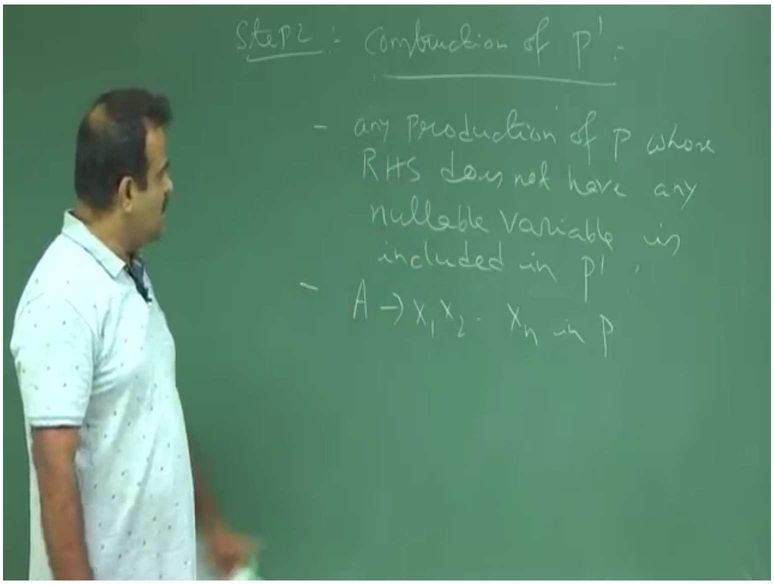

So, this is any production in P will be added in P prime, any production of P whose right hand side does not have any nullable variable have any nullable variable; they added in P, is include included in P prime that is the first step.

And the second one, now we, we take we consider all the such productions which are having no nullable variable in the Right Hand Side. And now suppose, we have we have a production like this, A is going to X 1 X 2 Xn in P. Suppose this is a production and we know a few are the nullable variable, if it is not if none of these are nullable visible that will come in the first potion. Now if few of this is nullable variable then we have to have the rule like this.

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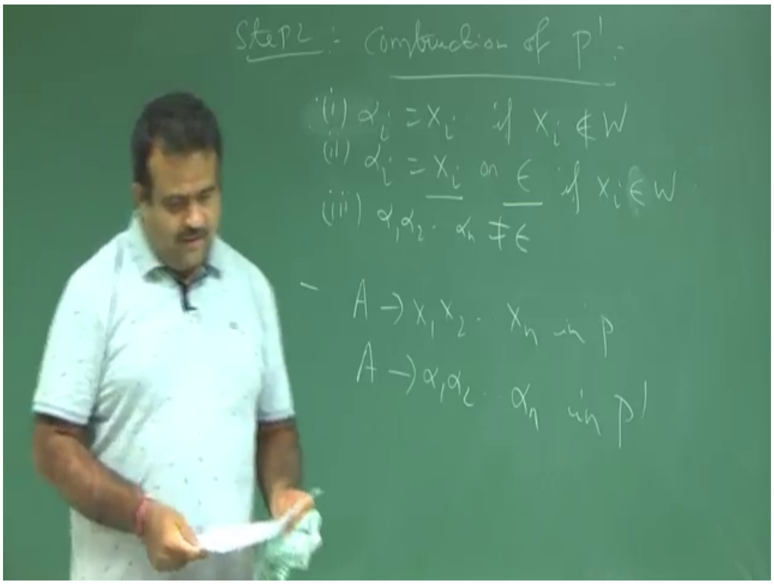

So, what we are going to do? So, yeah, so, this is we are going to do, we are going to add the production like this. A is going to alpha 1 alpha 2 alpha n in P where in P prime where alpha i is equal to Xi. If Xi is not a nullable variable and then if X i is nullable variable we will take this X i. And also you have another rule which is to erase Epsilon. If X i does not belongs to if X i belongs to W, we will take an example. So, suppose if X i does not belongs to W, if X i is not a nullable variable then we will tell you keep the X i as it is. Now if X i is nullable variable then you have two production, one is we keep X i and (Refer Time: 14:19) Xi.

So, that is the thing and not all alpha alphas are epsilon, not all alphas are epsilon. So, this is the rule, we will follow to construct the P prime we will take an example, then it will be more clear.

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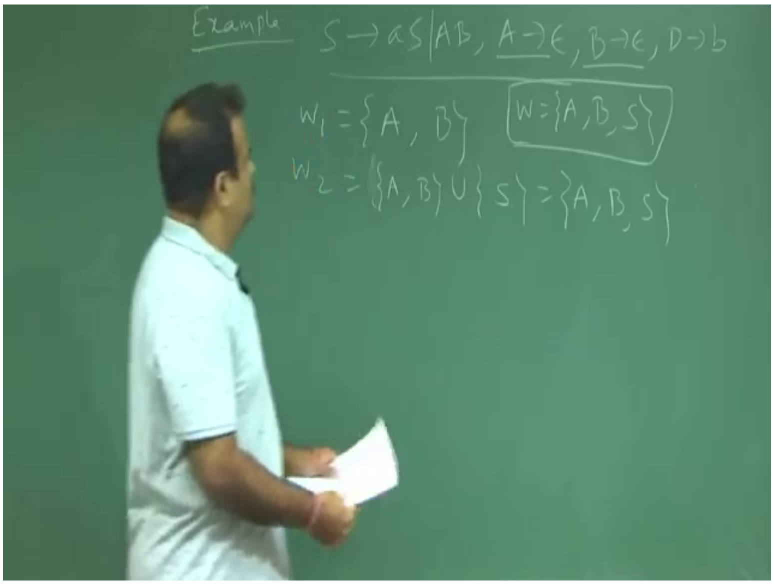

So, we will take an example, how to construct this how to reduce this how to reduce a grammar. Suppose we have given a grammar like this, we have few variable S A B and terminals are small a small b.

And we have the rule like this S is going to a S capital S capital A B then A is going to epsilon, B is going to epsilon and D is going to b. So, S A B D are the variables, ok. Now, suppose this is given, this is our G now we want to construct a G prime. So, first of all you have to find the nullable variable. So, these are the variable which is directly going to epsilon.

So, this will come into W 1 or W 0, whatever you say if you start with W 1 it is. So, W 1 is the all the variables which are daily going to epsilon, these are the W 1.Now to construct W 2, W 2 will be W 1 and those variable which are going to some nullable variables are some string of nullable variable. So, like here S, S is going to A B and A B is nullable variables. So, S will be added here A,B,S. And next we stop here because this will S W 3 will be same.

So, this is our W or W is nothing, but A,B, S this is our W that is the set of nullable variables.

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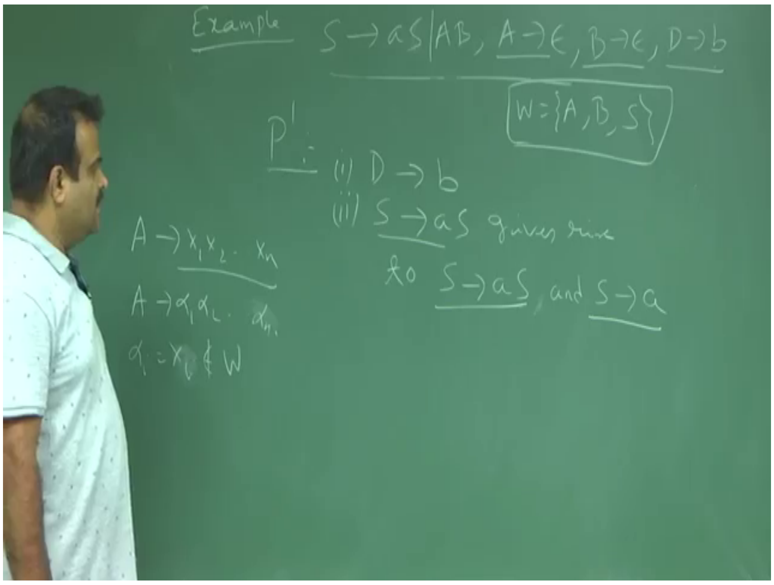

Now using this, we want to construct a P prime the rules, new rules. So, first of all we, we take the variables whose Right Hand Side is having which takes the production whose Right Hand Side is having known nullable variable. So, this is the one so; that means, V is going to this, this is the one we are going to add.

So, this is the construction of P prime, ok. Now, there is no, no such rule which is Right Hand Side does not have any, any variable or which is nullable. So, now, we take the rule like so, this we directly omit this Epsilon. So, now, we take S is going to a S. So, this gives rise as give rise to. So, like as we said if A is going to X 1 X 2 Xn, then we are going to replace this by A is going to alpha 1 alpha 2 alpha n, sorry.

So, there are few step X i if X i is not nullable variable X i is not W then we keep sorry alpha i it is not I mean it sorry X i this is alpha I, it be equal to X i. If X i is not a nullable variable otherwise, if it is nullable variable, you have two options you can erase Xi or we can keep X i. So, if you do that, it will give rise as S is going to a S this is our X i and S is going to a, ok. So, this is the two rules, it will these two rules will be added in this. So, it can erase X i we may keep X i that is the nullable variable.

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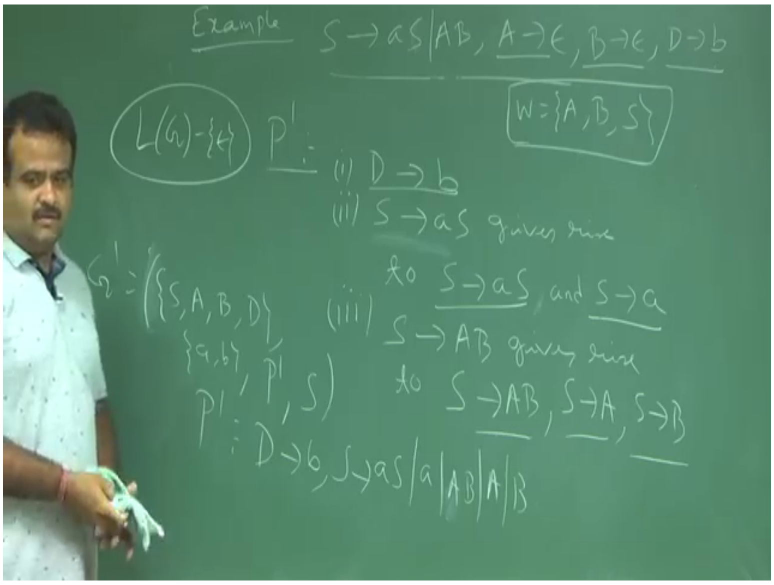

So, this will give rise as this and the third one is S is going to A B, this is also give us what this gives us give rise to rules again S is going to AB 1 and we can erase A B and then we can erase A. So, these are the production will be in will be in P prime, ok. So, P prime consists of this, this, this. So, this, this, this and this, this, this all this production will be in P prime.

So, if we write this so, our G is G prime is G prime is we have all the variables S A B D and the terminals is A B and P prime and S were P prime consists of D is going to b and S is going to a S or a or A B or capital A or capital B. So, these are the rules it is going.

So, then we can show that this is eventually the, this is eventually giving us the language L of G (Refer Time: 20:39) So, that proof will be there in the lecture note. So, now, we will go for the unit production elimination of a unit production.

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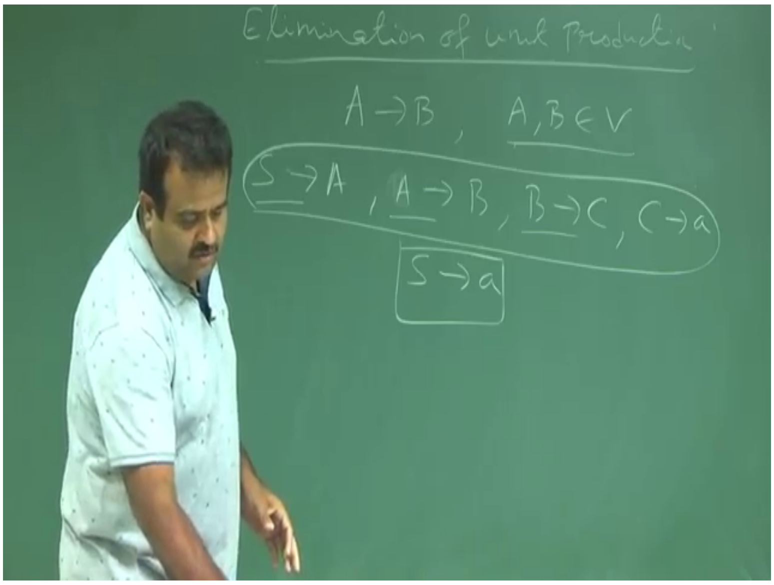

So, now we will go for the elimination of unit production, ok. So, unit production or a chain, chain rule in the g is a production of the form A is going to B where A B are two variables only, not the string of variables; so these are called units production. So, for example, if you have a grammar like this A is going to A, B is going to I sorry S is going to A, A is going to B then B is going to capital C and then say C is going to small a, ok.

Now, these are called these are all the unit productions. So, this is you can easily see these are all can we reduce further, because ultimately C is reachable from C can be derived from S, A, B and C is going to this. So, this is state away can be all this thing will be, because this is generating the same grammar like a. So, S is going to a this is the only production can be represent this. So, this is the, this is called the.

So, we are eliminating all the unit productions and it will eventually give us the same grammar, but formally we will define that how we can do that. So, let us try to do that. So, we are going to remove all the unit productions, because we want to reduce the grammar, ok.

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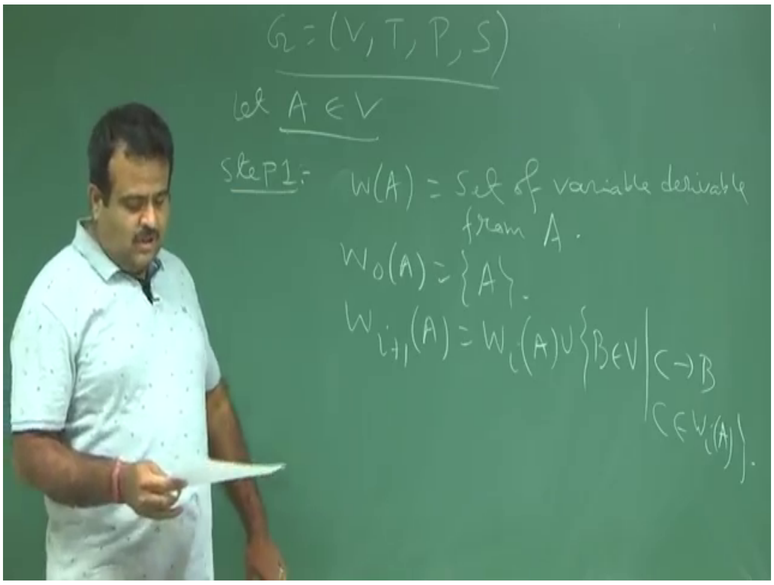

So, so, we have given a grammar G then we have to find another grammar. So, first we will remove the null productions and then we will go for the removing the unit production by this. So, for that let for any variable we are going to construct a set of variable which is deriving from A; so that is the step 1.

So, that we denote by W A. W A is the set of variables which are derivable form A. And that we do recursively like this so; obviously, A is derived from A. So, you put a first and then the recursively like this. So, Wi A union of all the variable B such that C is going to B where C can be derived for A. If C can be derived from A then, then B also can be derived for A.

If C is belongs to Wi A this is the recursive way, we can construct this and eventually we will see this will converge, this will converge and give us this. So, we will find all the variable W, W of A all the variables and then we will construct a grammar like this. So, let us first take an example, then it will be more clear.

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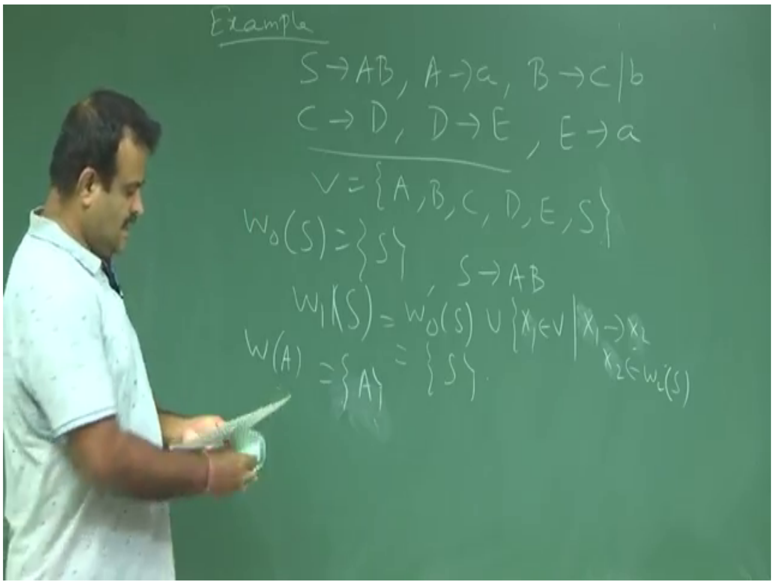

So, let us take an example suppose we have a grammar like this C comma B and C is going to D, D is going to E and E is going to a. So, first we want to a get a grammar which is after eliminating, we want to eliminate the all the unit production and get an equivalent grammar. So, for that what we do, what are the variable set? Variable set is A, B, C D, E, S. So, this is the variable set, ok.

Now first of all we are defined W of each variable, so W of each phase, so first of all you have to find this W of W of S. So, this is S then S is A B. So, so, W 1 of S these are the variables which are reachable from S, W 1 of S is W 0 S and union of all the A belongs to this such that A is going to B where B belongs to sorry, this AR that is mixing.

So, this is X 1 this is yeah all X 1 X 2 where X 2 belongs to W i of S. So, that is null. So, basically this is S, ok. Similarly, we can construct W of A. So, W of A for this we can just write W 0 of and this will be again only A, we find out this. Now let us take one example how it will be more W of B; so these are the variable.

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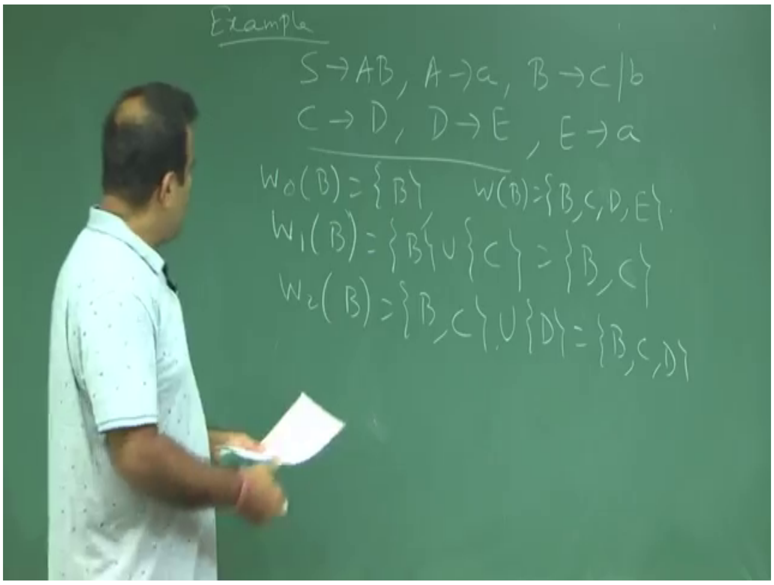

So, W 0 of B is B and so, W 1 of B it will be B union of. So, all the variables which are going to a, variable which is derived from B so, which is derived from B. So, C is derived from B directly, because B is going to C. So, in a first step it will be B C capital C. Now then W 2 or B; so B C and then union of that those variables which are derived from which is derived from B; that means, which will go into any one of these. So, that; that means, it will be D, because D is going to C D can be derived from C and C can derive from B. So, it will be D; so this will be the B C D like this.

And if you continue this ultimately you will be added, because once D is added e is derived from D and D is derived from B. So, E is derived from B so; that means, in the next step U will be added so; that means, W of B is nothing, but BCD and E; so this way we will construct the derivable set.

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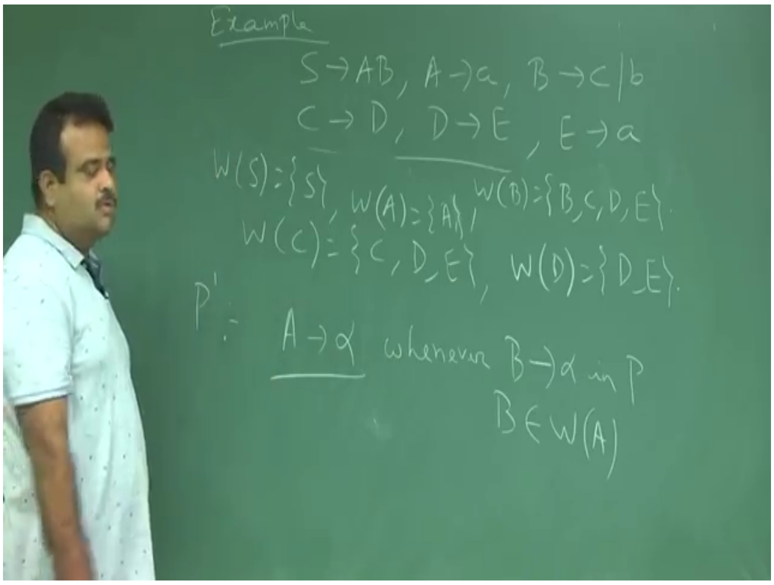

So, let me let us write all the deliverable set. So, W of S is only S, W of A is A and W of B is this set and W of C, you can similarly find out C D E and W of D is DE like this. So, you can get this W S, ok. Now, once we get these W s - how we can get the P prime from here? So, the rule is if we have yeah, so, if we have A A is going to alpha this will be added in the P prime whenever, whenever B is going to alpha in P, I mean in G such that B belongs to W of A. So, if B is reachable from A and if we have a rule that B is going to alpha that; that means, eventually A is going to alpha. So, we can get rid of B, we do not need B. So, like this we will find the rules over here.

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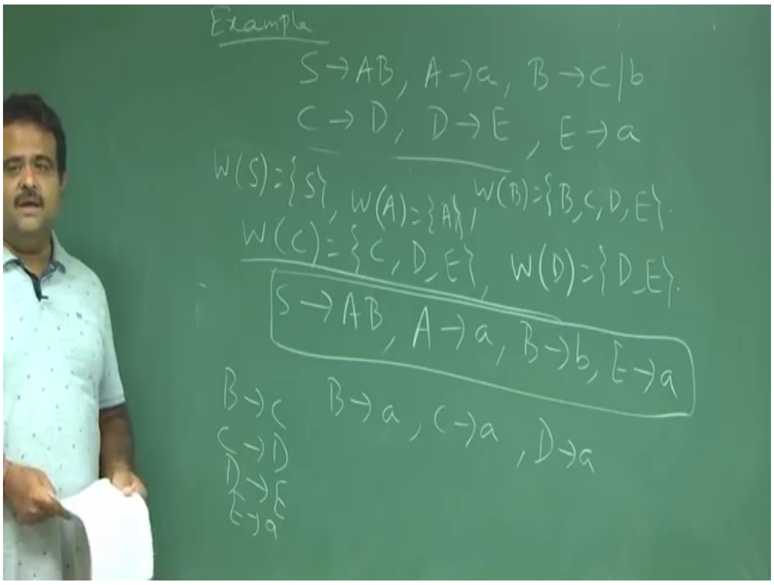

So, if we do that if you do that we will get the rules like this, yeah. So, S is this is the production here writing S is going to A B and A is going to a, B is going to b E is going to a, this is the productions. Now B, B is going to a and C is going to a D is going to a, this is because B is going to c and c is going to D and D is going to E and then finally, E is going to a.

So, we can we can use this by we can get rid of all this thing and we can just write this E is going to a in G and this. So, this is the way we reduce the all the unit production in the form the grammar. From the reduce grammar, it will accept the; it will corresponding to the same language.

Thank you.