Introduction to Automata, languages and Computation
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Lecture – 42
Ambiguity in CFG

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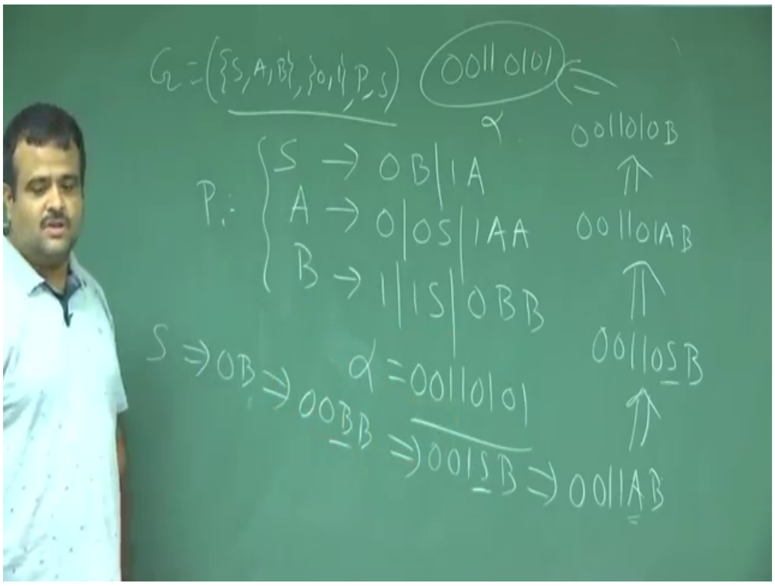

So we are talking about the leftmost derivation and rightmost derivation of a string given a grammar. So, we have given a grammar this is one example we have discussed in the last class we have given this grammar. So, they are variables are S, A, B and we have terminals 0 1 and this is our P and S; so this is the grammar we have given. And we want to derive the string alpha now this is the way we derived this called leftmost derivation. That is; that means, whenever we have the we have the variables we are taking the leftmost variable and we are applying the one of the production of the leftmost variable and we are reaching to the alpha.

So, every time we are doing this ok. So, now, we want to see the, we want to see the rightmost derivation of this alpha as well as we want to draw the derivation tree of this. So, let us draw the first the derivation tree of this. So, this is the leftmost derivation which you have discussed in the last class so this is our alpha.

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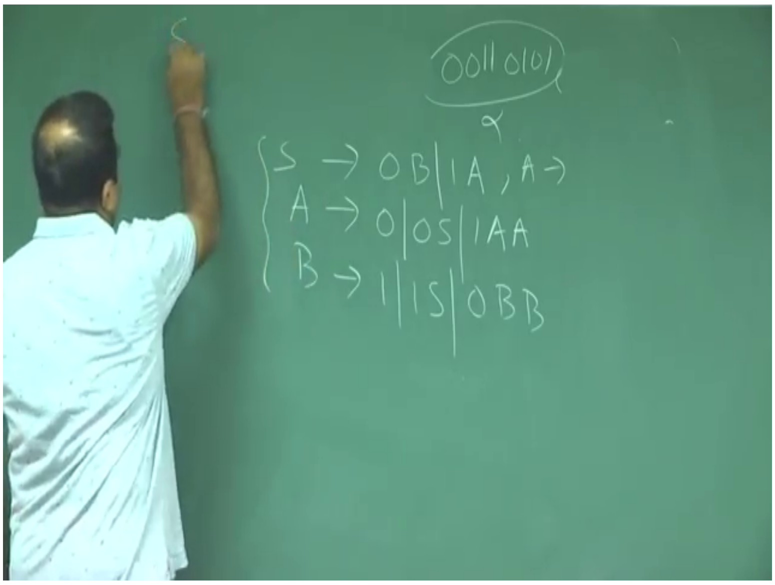

So, let us first we take this here this side or we can put it here.

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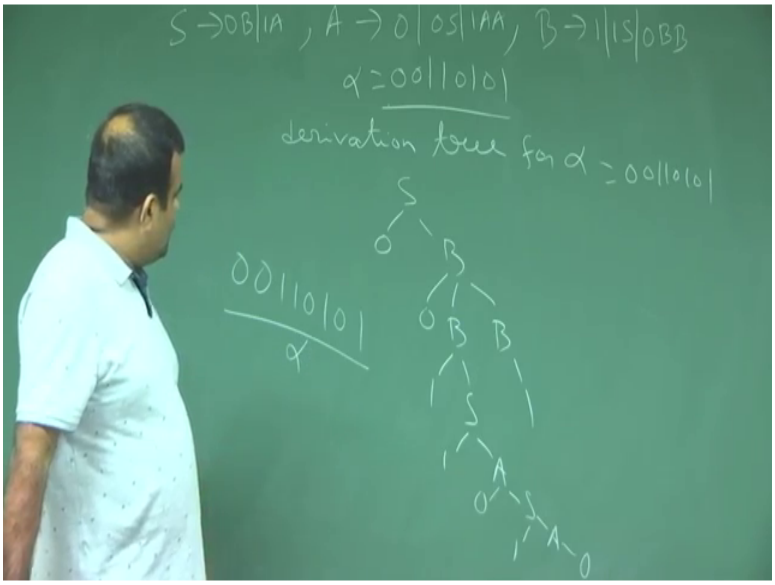
S → 0B|1A , A → 0|0S|1AA, B → 1|1S|0BB
α = 00110101
derivation tree for α = 00110101
S
0
B
0
B
B
1
S
1
1
A
0
S
1
A
0
00110101
α

So, S is going to 0 B 1 A 0 B 1 A so this is the two production for S. Alpha we can write here 0 0 1 1 0 1 0 1 ok. And then S A is going to 0 0 S 1 A A so this is the production for A. And B productions for B is B is going to 1 1 S 0 B B now we can erase this we can rub this.

And alpha we are getting alpha we want to this is our alpha we want to. So, last class we have seen how we can get A leftmost derivation of alpha from S. Now we want to see the we want to see the corresponding graph. So, there is a sorry derivation tree the derivation tree for alpha 0 0 1 1 0 1 0 1.

So we can start with S then the way we got S is going to say 0 B then this is a leftmost. So, this is all the variables we can apply one of this. So, the way we got the derivation you have to apply the same way. So, 0 BB then this is the leftmost one so we apply 1 S. Then this S is going to 1 A then A is going to 0 S.

And then this S is going to A 1 and then this A is going to 0 and this B is going to 1. And if we what this tree is healing? If we just read this leaf from the left right so 0 0 1 1 0 1 0 1 so this is nothing, but alpha ok.

So, this is the corresponding parse tree. Now we consider the rightmost derivation of this alpha then we draw the parse tree for that ok. So, correspond so this is our alpha.

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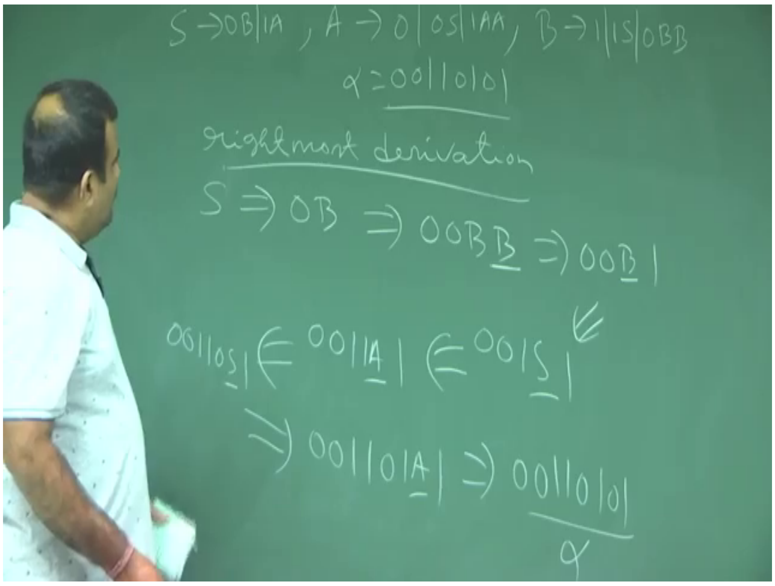

$S \rightarrow 0B \mid 1A$, $A \rightarrow 0 \mid 0S \mid 1AA$, $B \rightarrow 1 \mid 1S \mid 0BB$

$\alpha = \underline{00110101}$

<u>rightmost derivation</u>

$S \Rightarrow 0B \Rightarrow 00B\underline{B} \Rightarrow 00\underline{B}1$

$0011 0\underline{S}1 \Leftarrow 001 1\underline{A}1 \Leftarrow 001\underline{S}1 \Leftarrow$

$\Rightarrow 001101\underline{A}1 \Rightarrow \underbrace{00110101}_{\alpha}$

So, you want to get a rightmost derivation; first we want to get a right most derivation then we get a right most that corresponding parse tree. And then we will so then we will try to get another leftmost derivation for this alpha. So, then we will say that this is A ambiguous we will talk about that ok.

So, let us draw A rightmost derivation let us just try to get rightmost derivation. So, S is going to 0 B so B is the only variable we have to apply one of the rule. So, suppose you apply 0 0 BB. Now this is the rightmost variable so we need to apply the production for that variable.

So, suppose we are taking the B is going to 1 so this is going to 0 0 B 1. Now this is only one variable this is rightmost also. So, we apply the rule so suppose we are going 1 S. So, this is 0 0 1 S 1 now this is rightmost we apply the rules in S. So, suppose we are applying 1 A, so this is 0 0 1 1 A 1.

So, this is now the rightmost this is the only variable which is rightmost also suppose A is going to say 0 S so 0 0 1 1 0 S 1 ok. So, now this is only again only one variable S so we go we apply this S is going to again 1 A.

So, this is 0 0 1 1 0 1 A 1. Now this is only 1 variable it is going to you can go to 0 to get alpha. So, 0 0 1 1 0 1 0 1 so this is the rightmost derivation of this string alpha. Now what is the corresponding tree; corresponding tree is the way we derived this, it will give us the tree.

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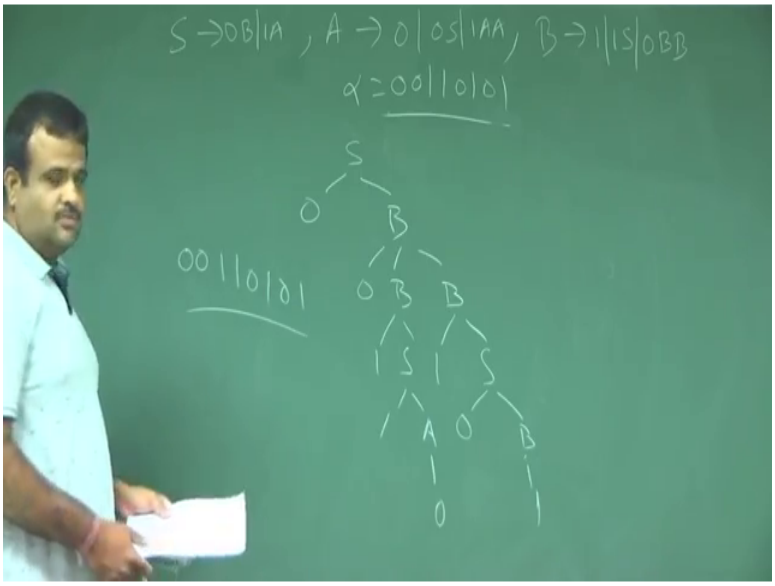

So, S we use this B now this B we use like this B is going to 0 BB now use the rightmost derivation. So, this is going to 1 S and this is again this is going to 0 B. And this is going to 0 B and then 1. And this is again going to this P is going to 1 S and this S is going to 1 A and this A is to 0.

So, if we just read the left to right leafs this will give us 0 0 1 1 0 1 1 1 then S is going to 0. So, then B sorry this B is going to 1 B is going to 1 0 0 1. So, this is our alpha this is the corresponding tree. Now we will see there is this than a corresponding rightmost derivation then we will see there is a another leftmost derivation for this and that will give us the ambiguity of the string.

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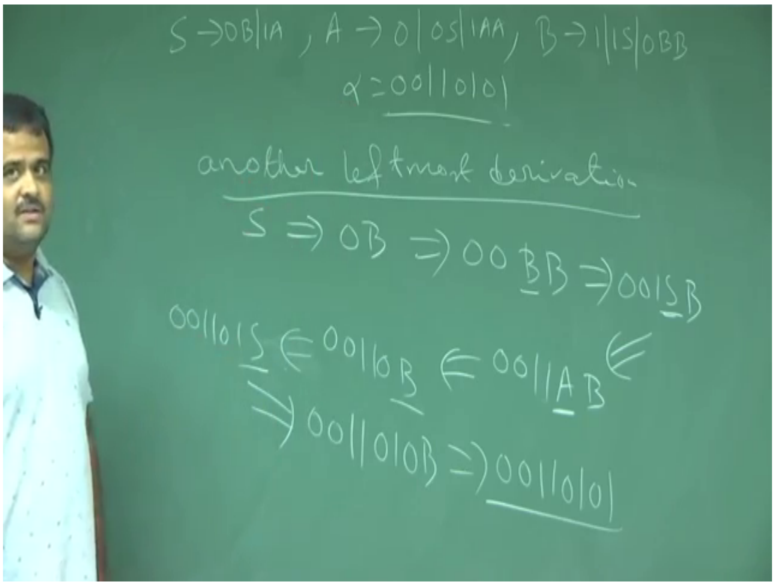

$S \rightarrow 0B \mid 1A$, $A \rightarrow 0 \mid 0S \mid 1AA$, $B \rightarrow 1 \mid 1S \mid 0BB$

$\alpha = 00110101$

another leftmost derivation

$S \Rightarrow 0B \Rightarrow 00\underline{B}B \Rightarrow 001\underline{S}B$

$00 11 01\underline{S} \Leftarrow 00110\underline{B} \Leftarrow 0011\underline{A}B \Leftarrow$

$\Rightarrow 0011010B \Rightarrow \underline{00110101}$

Another leftmost derivation.

So, S is going to 0 B then leftmost derivation this is only B. So, B we can make it to 0 0 B B. And now this is leftmost so you have to use this. So, you can take 1 B is going to 1 S 0 0 1 S B. So, now, this is leftmost we can just make it this S we have to use one production is S. So, this we can make it 1 A so 0 0 1 1 A B.

So, 1 A B now this is again rightmost like sorry leftmost. So, we can use these two we can use a production in. So, this is going to 0 0 1 1 A is going to 0 B. Now this is the leftmost one is the only variable we have. Now suppose this is going to be B is going to 1 S so 0 0 1 1 0 1 S. Now this is the only one variable which is the leftmost one also.

So, this is now going to say S is going to 0 B say says 0 0 1 1 0 1 0 B. Now we can have this B is going to 1 0 0 1 1 0 1 0 1 B is going to 1. So, this is the, this is another leftmost derivation for this so this is the ambiguity. So, we have another leftmost derivation for this. So, we have A we have many many derivation for this string alpha.

So, we say the string alpha is ambiguous so we will formally define that. So, let us first have the understand what you mean by ambiguity in a context free grammar.

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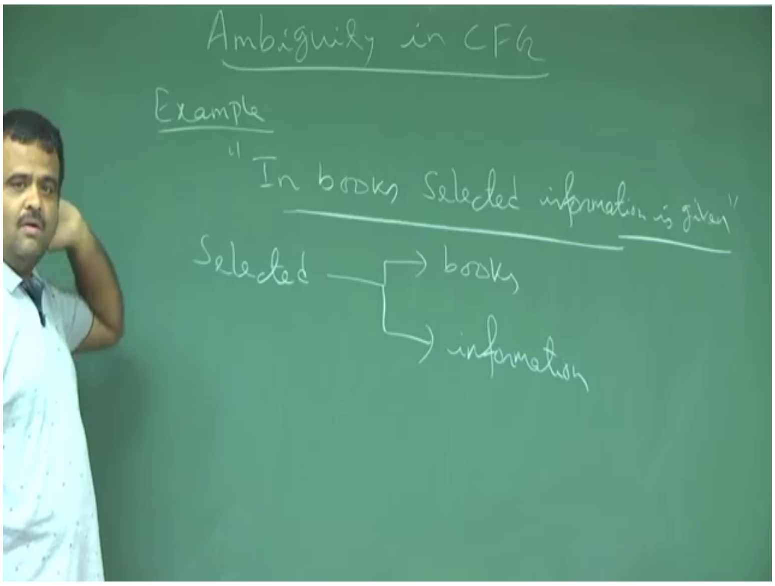

So, let us take English sentence we are talking about ambiguity in context free grammar ok. So, let us take a English text; suppose we have in books we have this sentence in books selected information is given ok. This is sentence English sentence.

Now this sentence is ambiguous why? Because this selected one, so it can be read like this; in books selected information given, or in book selected the information are given. So, the ambiguity is like this and selected may refer two ways one is the books or information ok. So, in book selected information are given or in books selected information are giving. So, this is the ambiguity emigrate sentence.

The sentence may pass in two different ways so that is called that is referred as ambiguous sentence or this, this, this then this grammar will be called ambiguous grammar. So, we will formally define the ambiguous string ambiguous ambiguity in our context free grammar.

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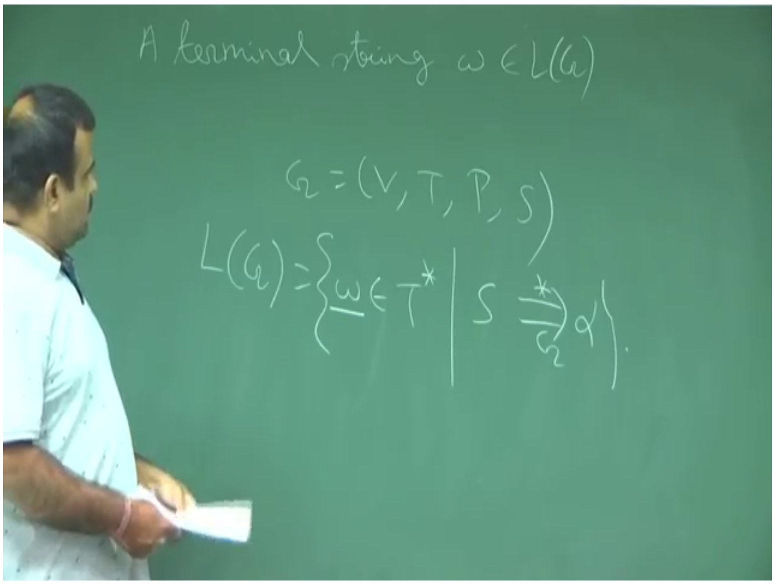

A terminal string $\omega \in L(G)$

$G = (V, T, P, S)$

$L(G) = \{ \underline{\omega} \in T^* \mid S \overset{*}{\underset{G}{\Rightarrow}} \alpha \}.$

So, this is the definition a terminal string W or alpha we use alpha anyway which is coming from the context free language.

So, what is this? We have a grammar G which is V T P S. Now how we different L of G? L of G is nothing, but setup all terminal strings W there is no variable here this is only contain the terminal string or the input alphabet string. Such that S is the alpha is derived from S or alpha there is a parse tree which that yields alpha.

So, this is the context this is the language generated by this grammar. And this is also called context free language we may have many grammar for a given language. But anyway so this is a string of terminal this W so W is a string of terminals.

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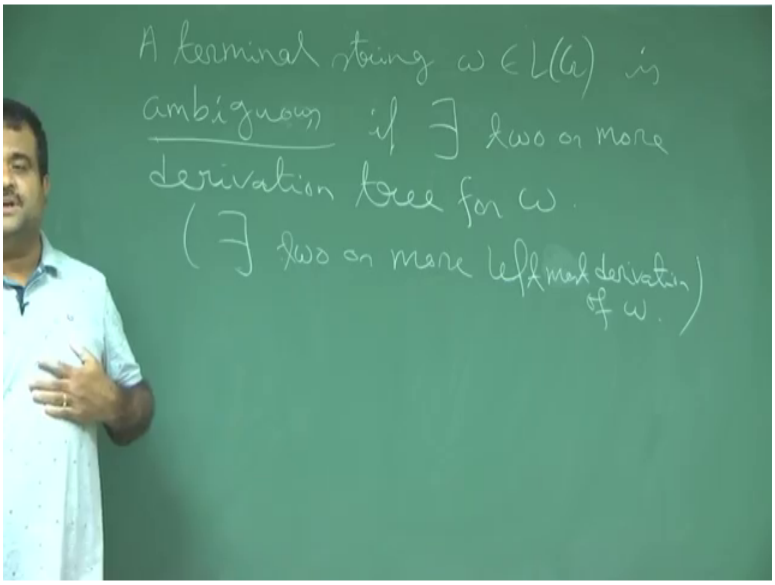

A terminal string $\omega \in L(G)$ is ambiguous if $\exists$ two or more derivation tree for ω.

($\exists$ two or more leftmost derivation of ω.)

Then the W is called is ambiguous is ambiguous g u o u s ambiguous. If there exists two or more derivation tree for W two or more parse tree or the derivation tree for W which will W actually.

So; that means, if there exists two or more left derivation of W if there exists two or more at last two left derivation leftmost derivation of W leftmost derivation of W. Then this W is called the string W is called ambiguous string or ambiguity ambiguous what this has ambiguity so then this is a string ambiguity.

Now when we called a context free grammar is ambiguous if we have such string in this grammar I sorry context free language. When we call context free language is ambiguous if we if there is such strings are exist in the language then recall the language is ambiguous. So, we say yeah context free grammar first.

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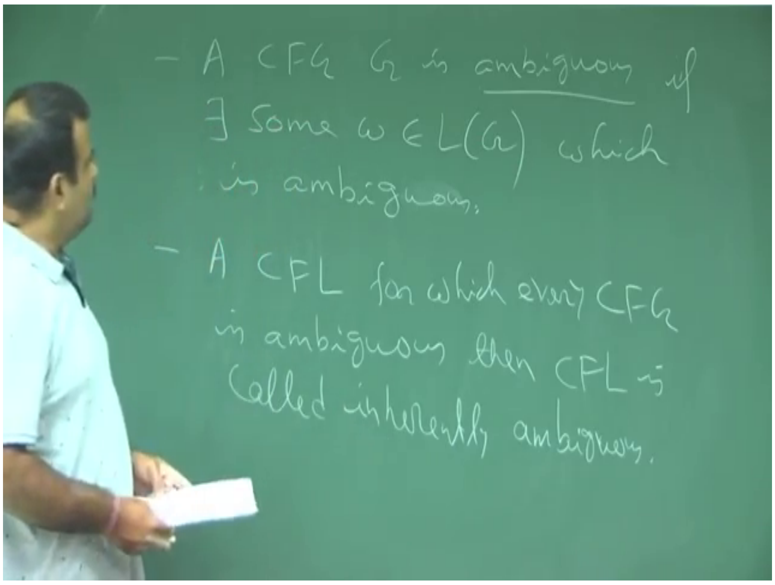

- A CFG G is ambiguous if $\exists$ some $w \in L(G)$ which is ambiguous.
- A CFL for which every CFG is ambiguous then CFL is called inherently ambiguous.

We say a context free grammar is ambiguous G is ambiguous if there exists such W. If there exists if there exists some W belongs to L of G which is ambiguous which has more than two leftmost derivation which is ambiguous g o u s ambiguous.

Then we say that context free corresponding context free grammar is ambiguous. And when it is a context free language is ambiguous a context free language is ambiguous. If for corresponding to every because the context become the language can have more than 1 context free grammar. If every context free grammar is ambiguous a CFL for which every CFG context free grammar is ambiguous.

Then it is called inheritedly ambiguous then CFL is called inheritedly ambiguous ok. This is the definition now it takes a quick example for a ambiguous language, ambiguous grammar, ambiguous string, we have already seen one; let us take some more example.

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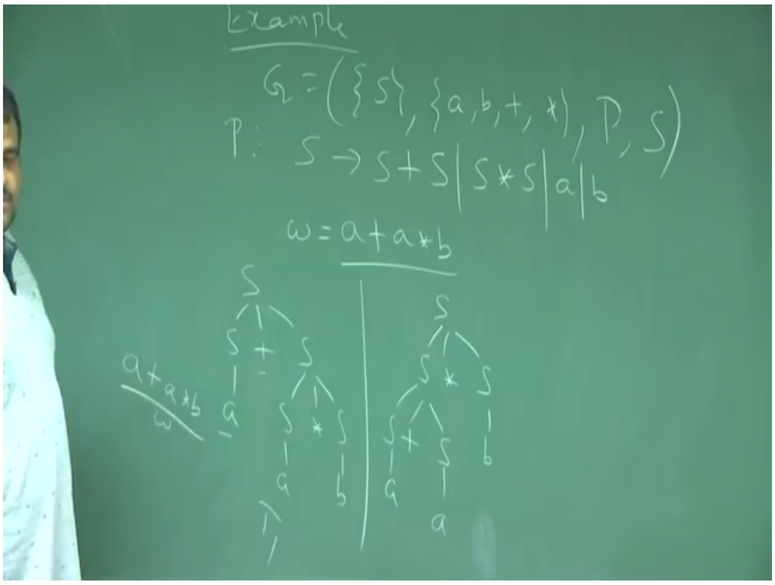

So, let us take a grammar; you have only one variable and we have few terminals A B plus star P S. So, what is the P? P consists of these rules, S is going to S plus S or S star S or A B A or B this is the rule ok.

This is our P this is the production ok. Now we want to get a alpha like this W. This is a string we want to we will see this W is a ambiguous string ambiguous string of terminals. So, for that what we need to have? We need to have two different parse tree for this; so let us try to draw two different parse tree for this.

So, S is going to say or we can try to get to left most derivation, but anyway we can try it parse tree. Now this is directly going to A and this plus is there this is say S star S and this is A this is B. So, what is the corresponding alpha it is illegal. So, A. So, we just read from left to right the leaf nodes, so this is leaf A star B. So, this is same as this our W.

So, this is our T 1 one parse tree. Now can you have another parse tree which is yielding the same W yes. So, let us try that S S star S, then this S is going to S plus S then this is going to A this is going to A and this is going to B so this is our T 2 I mean the parse tree.

Now what is the string of terminal it is yielding we can just read A plus A star B. So, that is W so this is our T 2. So, we have two different parse tree corresponding to this string so this string is a ambiguous even we can have the corresponding derivation of this.

So, this string is ambiguous string ok. So, we can have 1 more example even I can draw the leftmost derivation of this so how to draw this? How to draw this? We should have two leftmost derivation.

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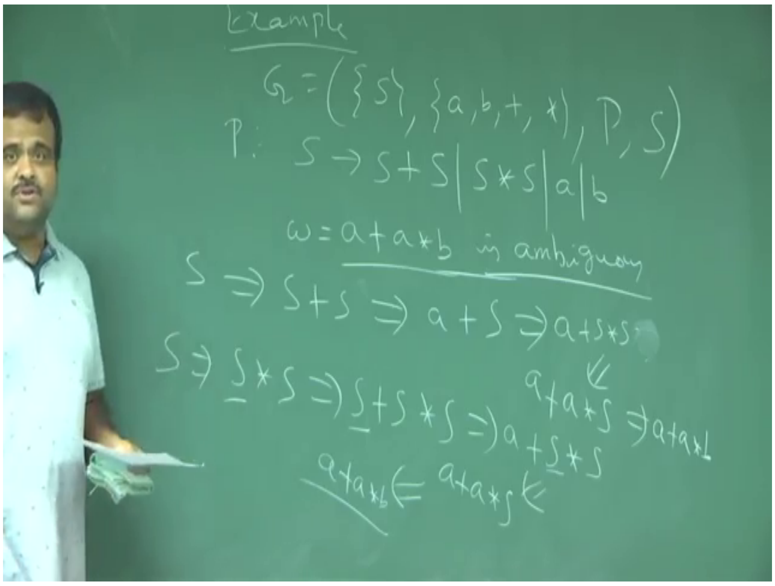

So, this S is going to S plus S the way we have derived the tree the same way we can have this. So, now this so the leftmost derivation so this is the leftmost so this S we can reach to A plus S. Now this S we can go to this rule. So, A plus S star S now this S we can have so this is the leftmost one is this one.

So, you can have A plus A star S now we can have this is to be A plus A star B. This is one leftmost derivation; we can have another leftmost derivation like this. So, we can have A star S instead of plus then we can take this is the left most case so we can take S plus S star S now this is the leftmost variable. So, we must replace this fast for the leftmost derivation. So, this is going to A plus S star S.

Now this we can this is the leftmost variable. So, this we can replace by A A star S and this we can replace to the now this S can be going to B so A plus A start B. So, you can have two leftmost derivation corresponding to this so then this is ambiguous. So, this is a ambiguous string this is ambiguous. So, once this is ambiguous we have to ambiguous tree so then these grammar is ambiguous. So, we take quick one another example.

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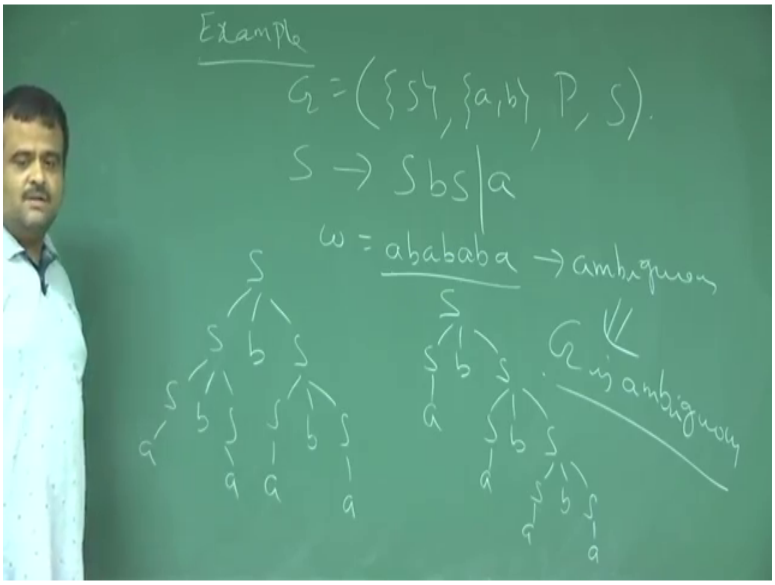

So, we take quick example one more so suppose we consider the grammar G which is having say only one variable s and we have terminals a b and have a P this s ok. And suppose rules are like this s is going to s bs a ok. So, now how to show this is the ambiguous grammar.

To show this is a ambiguous grammar you have to have a string which is ambiguous. So, let us have a string. So, so let us take w is ab ab ab a ok. And we will see there are two parse tree corresponding to this then we can say that w is this w is ambiguous. So, let us draw two parse tree for this.

So, s is going to s b this, again this is we can take this s b s and this is going to a this is going to a. And this is again going to s b s and this is going to a this is going to a. So, what is the string yielding? So, A B A B A B A this is the string it is yielding string of terminal. Now we can have another derivation tree for this s is going to s b s and this s is going to a.

. So, a b we got this a b now we have to generate this remaining from this s. So, for that we can have s b s now this s we can go into a this will give this a b now we need to have another a b. So, s b s now this is going to a and this is going to a so you have to parse tree corresponding to this. So; that means, this is s a ambiguous string this implies this G is ambiguous ok.

So, grammar is ambiguous if we have a string of terminal which is having more than one derivation left most derivation. Or we can have more than one rightmost derivation. That means, we can have more than one parse tree I mean 2 or more parse tree which derived which yields that then it is a ambiguous string. And if grammar is just such ambiguous string then the grammar is called ambiguous grammar.

Thank you very much.