Introduction to Automata, Languages and Computation
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Lecture – 35
Equivalence of Moore and Mealy Machine

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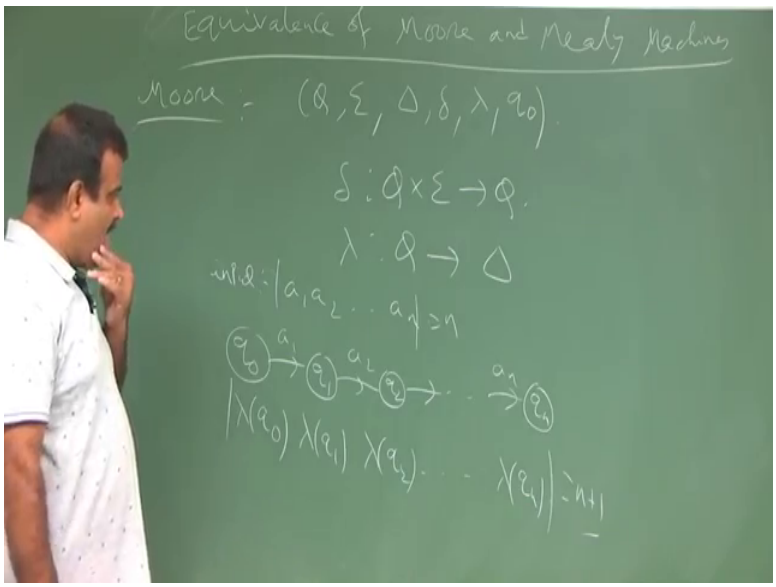

So, we are talking about the Automata, finite automata with output. So, we have seen two machine, when which is Moore Machine and Mealy Machine just to recap and then we will just go for the equivalency between two. So, just to recap the Moore machine is it is 6 tuple. So, every we have output here. So, this Q is the finite number of states, this is the input alphabet, this is the output alphabet. And you have a delta, delta is the state transition function which is taking the input alphabet and which is changing the states and we have lambda this is a output function, for output and q 0, so, 6 tuples.

So, this is delta is Q cross sigma 2 Q, like DFA and lambda is q 2. So, every state is outputting a output symbol. So, this is the set of output symbol, everything is finite over here. So, this is the set of output symbol everything is finite over here this is the set of output symbol which is outputting it output symbol.

So, now, if we have to read a string, suppose we are going to read a string say x equal to a 1, a 2, an and suppose so we start with q 0 say, we start with q 0 we go to q 1 with a 1 q 2 like this way dot dot dot, a n we are going to q n. So, this is the input; this is the input

and these are the state it is going by the rule of delta. Now, what will be the output? Output will be then we have a delta of q 0 I sorry lambda of q 0 because this is the first state, then lambda of q 1, lambda of q 2, lambda of q n. So, this is of length this is of length n but the output sequence of more machine is not yet n plus 1 ok.

And we have seen an example where we have seen an example of the Moore machine. We can take the example quick (Refer Time: 03:04) again, just this is just to recap of the previous lecture.

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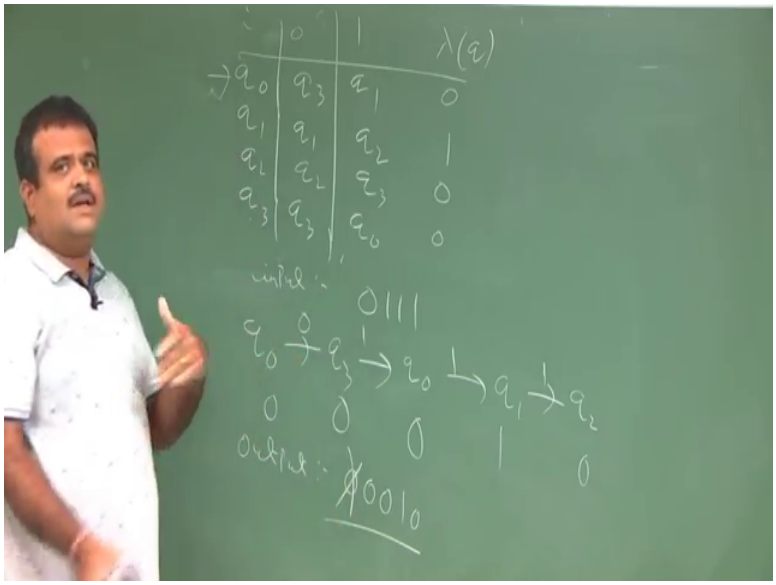

Say for example, we have a message we have 0 1 is the input alphabet, 0 1. And we have the delta rule is like this, q 0 is the starting state with 0 it will go to q 3, with 1 it will go to q 1. And lambda is like this, so lambda of q; q is q 0 for with it is 0 and q 1 say it is going to q 1 q 2, so lambda of q. Say for 1 it is going to this is q and then q 2 q 3. So, q 2 it is for lambda it is q 2 q 3 and it is 0 and q 3, q 0 and it is 0 for q 3 ok.

So, now, suppose we have reading a string like this say our input string is 0111. So, we are starting from q 0 with 0 we are going to q 3, now with 1 from q 3 we are going to again q 0 and with 1 so this is 0 1 from q 0 with 1 we are going to q 1 and from q 1 with 1 we are going to q 2. So, what is the output? Then output is, so with 0 we are going to this is the outputting 0 this q 3 q 3 is 0 and q 1 again 0 outputting q 1 is q 0 0 output is 1 it is 0. So, output is, this is input and the output is this one 00010, 5 length and this is of length 4.

Now, when you talk about the Mealy machine, then that will be of the same length. So, just quickly recap then what how do you mean by equivalency? So, that means, we have to remove this then we will see whether they can output the same stream ok. So, let us just quickly have the Mealy machine, this is the Moore machine, we have seen. The Mealy machine is the difference is, so output will not depend on only on the state output will depend on the transition also.

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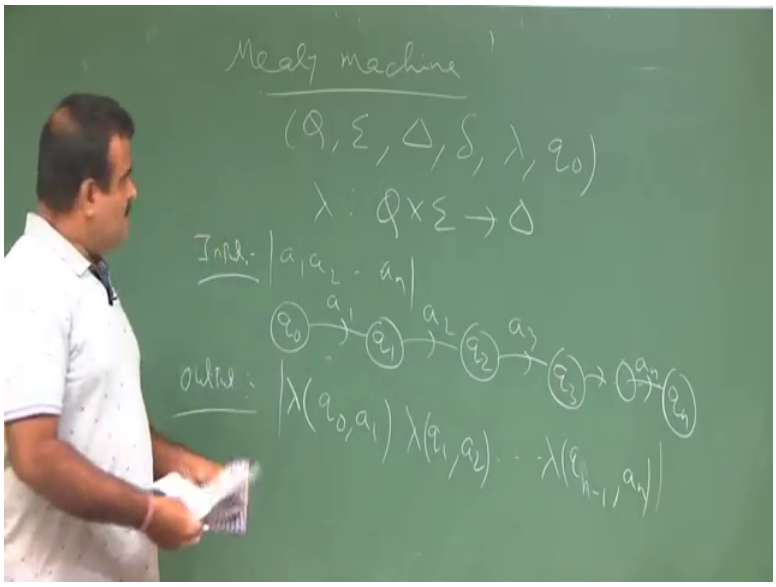

In the Mealy machine what we have, it is also the same 6 tuple, this is the output alphabet, this is the input transitions, this two are different, this is different. So, here it is a there lambda is a function of this will depend on the chip on a transition rule ok, not only on the state it will depend on the input also whatever the input we have. So, that is the difference between Moore machine and Mealy machine.

So, suppose we are at, so suppose we are reading q a 1 a 2. So, we are at q 0. So, suppose by a 1 we are going to q 1. So, what is the output? Output will be, so this is the transition. So, delta of q 0 a 1 is q 1 and say again by a 2 we are going to q 2, let us see we are going to q 3 like this ok. So, if it is nth length we are going to q n after this nth length.

So, now, this is the input and corresponding state we are visiting. Now, what will be the output? So, input is this a n and the output will be like this. So, it will be delta of, I mean not delta this is the lambda of q 0 a 1 because it is a function of state and the corresponding input alphabet. Then we are at q 1, q 1 a 2. So, lambda of q n, so not q n it

is q n minus 1 a n, because this is an and we write q n minus 1 this is the input ok. So, this is of length n this is the sorry, this is output, this is of length n, this is also up length n.

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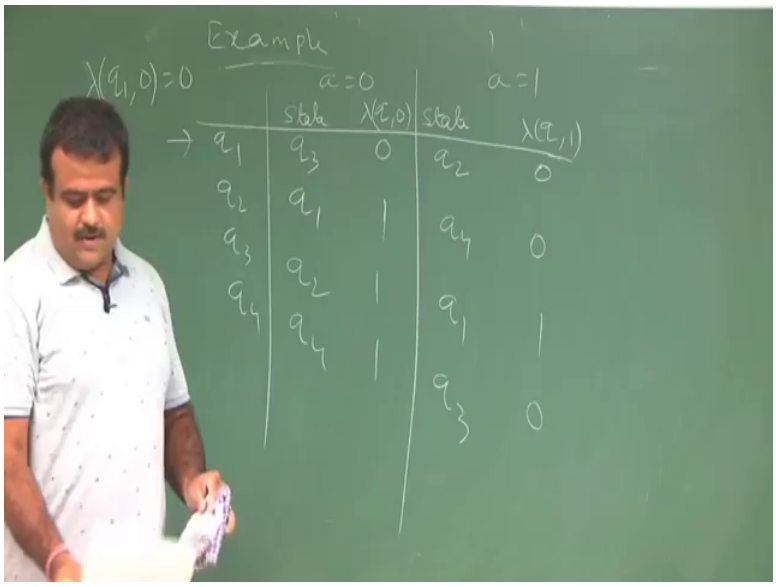

And we have seen one example of this Mealy machine like this. So, say we have 4 state q 1, q 2, q 3, q 4 and q 1 is the starting state. So, let us write the transition rules. So, this is for input 0, this is for input 1; so, there are two input. And this is say which state it is going and this is our lambda, q comma 0 and which state it is going and this is our lambda q comma 1. So, suppose it is going to q 3 the state and the corresponding output will be say, delta of the lambda of q 1 0 say 0.

So, this is q 1 then it is going 1, q 2 it is going 1, q 4 it is going 1. So, similarly q 2 it is going 0 and q 4 it is going 0 and q 1, 1 and q 3 it is 0 like this ok. So, this is one example where we have these outputs also function of this. Now, the question is how we can make the equivalence between these two because.

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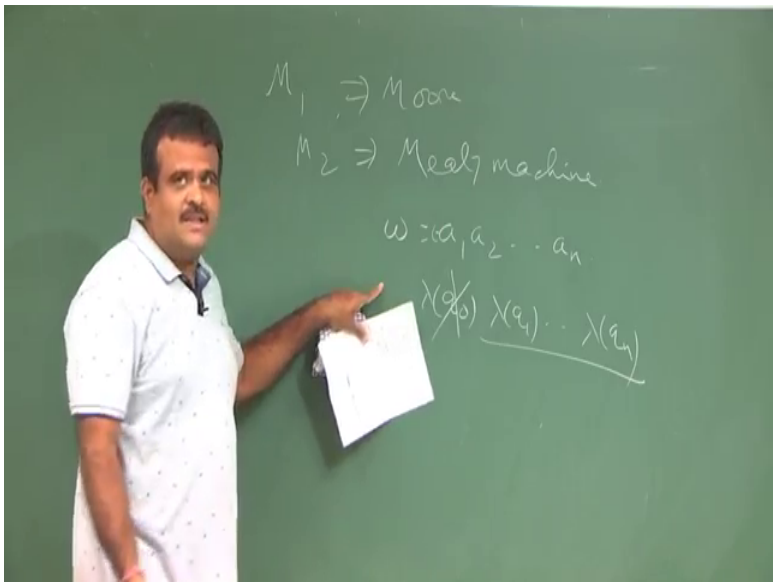

So, how we can make the equivalence between these two? Because this is one length is suppose we have given a Moore machine and this is a Moore machine and how to construct the corresponding Mealy machine from Moore machine that we will first do. And then we will see they are outputting the same string, because we are their equivalents means they are outputting the same string. To see this we have to ignore the first string because if we have a w which is say 1, a n.

Now, for Moore machine what we have if it is going to say delta of this is delta of q 0, this is for epsilon, then delta of q 1, these are the state it is visiting delta of q n like this. So, here we just omit this and then we this will be of length n and that for Mealy machine it is of length n, so, that way we can verify. So, let us do that. Let us taken transformation from the Moore machine to Mealy machine, given a Moore machine how we can transfer that how we can get a Mealy machine then we can say their equivalence.

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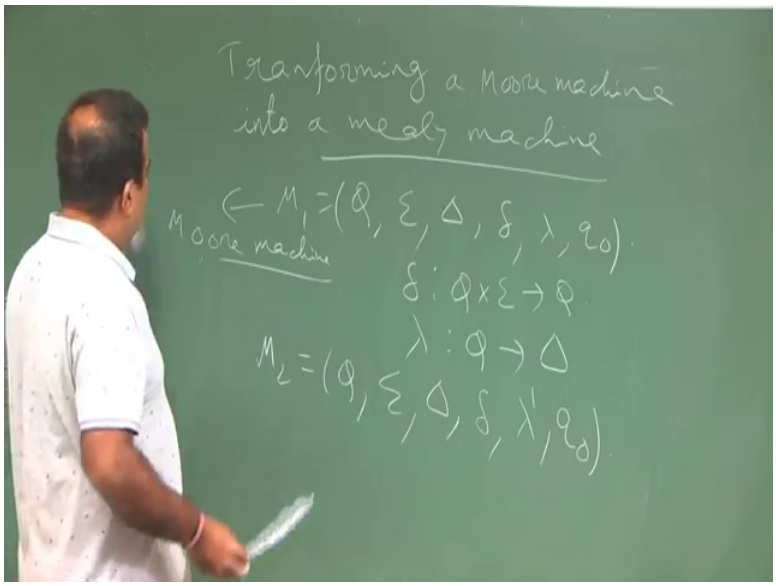

So, transforming a Moore machine in to a Mealy machine ok. So, in the Moore machine in the Mealy machine what we have in the Mealy machine, our output function is a function of state and the input symbol, but the in the more machine it is a just a function of state. So, that is suppose we have given a Moore machine, so this is Q delta lambda q 0, this is a Moore machine ok.

So, now here we know the delta is a function of Q cross sigma to Q, lambda is a function of Q to output alphabet. Now, we have to construct a Mealy machine which is equivalent. So, how to construct that? So, to construct that we just need to change this lambda that is all. So, this will be Q will be same, sigma output alphabet is same, my transition rules is same, now this will be lambda hat q 0. Now, how to define this? Let me define (Refer Time: 14:14).

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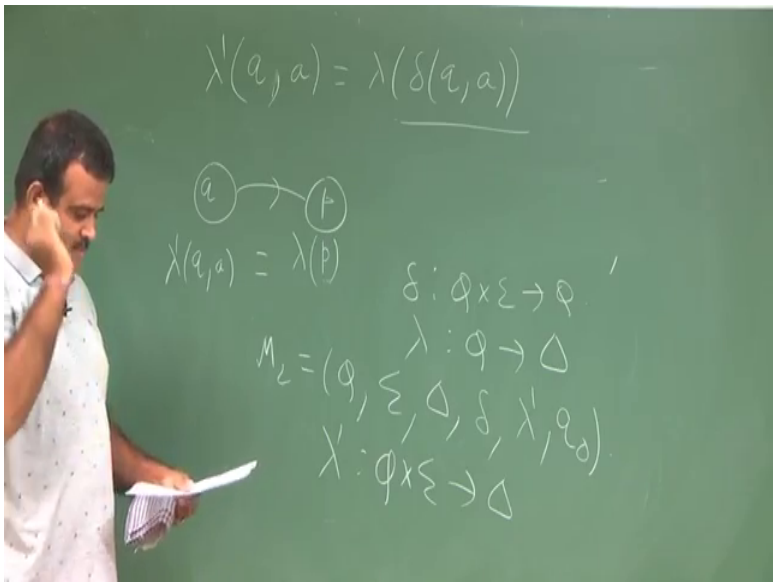

So, a lambda prime of q 0 a. So, this will be basically lambda of delta of q a that is all, because this for the Moore Mealy machine this lambda is a function of lambda prime is a function of Q cross sigma to the output alphabet. So, for that we need to take a Q from here and we need to take a and we need to give the output. So, output will be same as. So, it will reach to some state.

So, like suppose here at say we are at q 1 or q say this is q we are at q, with delta suppose we are going to p, now with this p where we are going under this. So, that is nothing but our lambda prime q a this is the way we define ok. So, let us take an example. So, actually the Mealy machine, a Mealy Moore machine is a particular case of Mealy machine ok.

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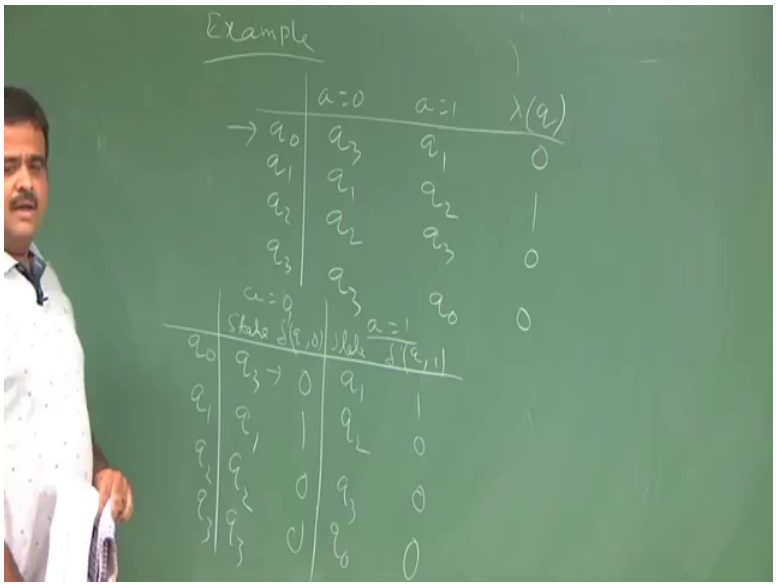

Let us take an example. Suppose we take a Moore machine like this we have two input alphabet a 0, a 1 and we have lambda of q. And we have this state q 1, q 0, q 1, q 2, q 3 there are 4 states over here and this is the starting state. So, this is a q 3 the earlier one 0, earlier example, q 1, 1 and q 2 q 3, 0 and q 3, q 0, 0ok.

Now, we want to convert into Mealy machine. So, what will be the equivalence Mealy machine? For equivalence Mealy machine we then need to have the delta transition. I mean the lambda; lambda will be for state and the input alphabet; so, that is very easy. So, we can have say a is equal to 1 sorry, a is equal to 0 is equal to 1 and here we can have state and here we can have delta of q delta prime of q comma 0 and here we have a state and delta prime of sorry q comma 0 and q comma 1.

So, we have q 0, q 1, q 2, q 3 and this is going to q 3 and the state will be, for q 3 why we are going? 0 but it is combining these two. So, this is q 3 0 and again q 1 and it is going to for q 1 it is going to 1, q 2 for q 2 it is going to 0 and for q 3 q 3 to q 3 and it is going to 0 again, like this. So, similarly you can fill up this q 1, q 2, q 3, q 0 and these are all 0 0, I mean straight away the Moore machine is a particular case of Mealy machine.

But the reverse is not directly true, reverse is reverse we have to construct the Moore machine from Mealy machine; so, that we will do now. So, given a Moore machine how we can construct a, I given a milling machine how we can construct a Moore machine so that they will output the same stream for a given input ok.

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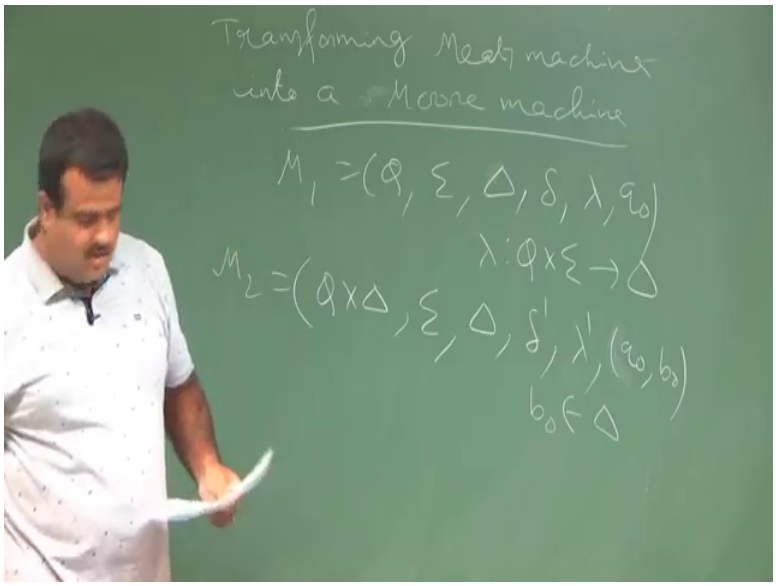

So, this is the transforming Mealy machine into a Moore machine, ok. So, suppose we have given a Mealy machine Q, this is the output alphabet, lambda, q 0 ok. So, now, here lambda is a; lambda is a function of Q cross input alphabet to the output alphabet ok. So, now to Moore machine you have to construct like this.

So, it will be the Cartesian the state will be Cartesian product of Q and the output alphabet set, because it is going to the output alphabet set. This is same then this will also change lambda prime and the starting state is q 0 and q 0 b 0, where b 0 is the arbitrary element selected from this. So, b 0 is coming from this. Now, what is the rule? How we get the rules? Rules are you can define here.

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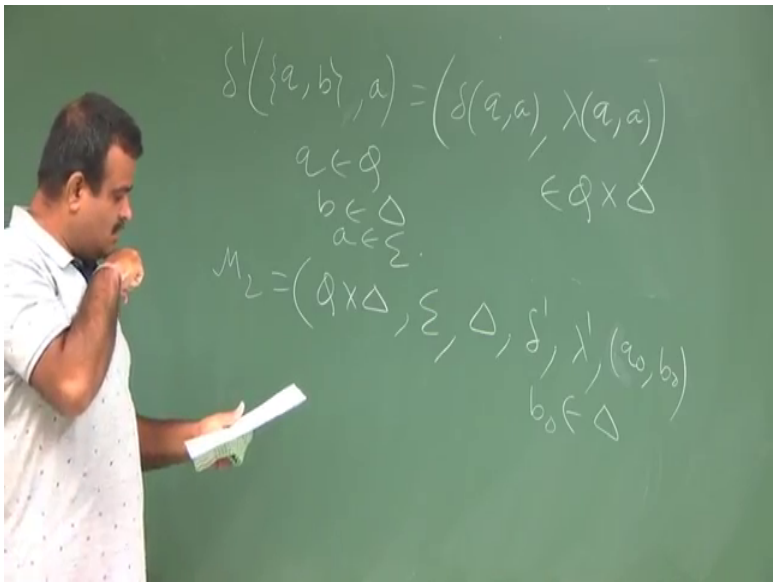

So, basically, we have to define the delta prime. So, delta prime is what? Delta prime of now q 0 will go up this form or any state of this form q comma b because this is a Cartesian product of this with a.

So, where it is going? It is going to the state is changing state is going to delta of q comma a. This is the state change and the output lambda of q comma a ok. And this is again a subset of q comma q cross output alphabet set. And then this is b, where q is coming from state and b is coming from b is a alphabet set output alphabet set and q is a state and a is a input alphabet. So, this is the way we defined this ok; so, let us take a quick example.

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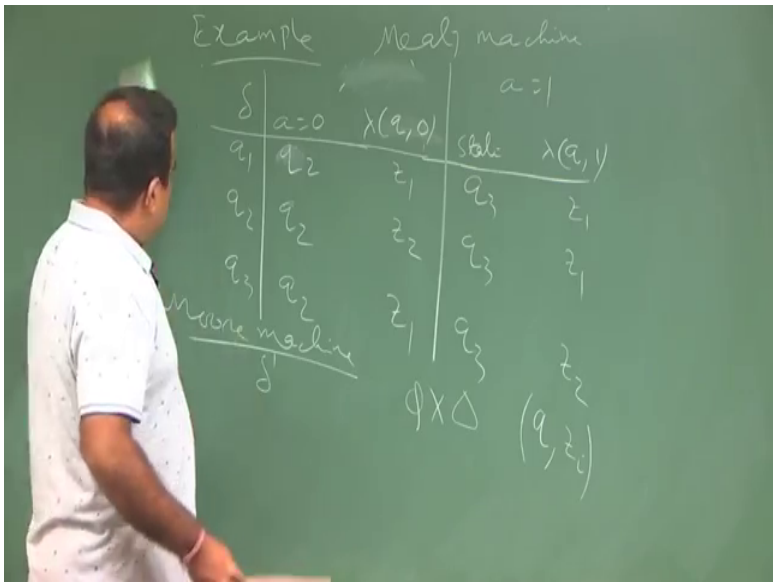

So, we have given suppose you have given the Mealy machine which is say this is our delta and these are our state q 1 q 2. They said there are 3 states only and this is our delta transition with a is equal to 0 a is equal to 1 and this is the lambda of q comma 0 and so this is say this is going to say 2 and this is z 1 say sorry, this is lambda of q comma 0 and so this is say this is going to say 2 and this is z 1 say sorry this is lambda of q comma 0. This is going to z 1 this is say q 2 and this is also q 2, this is z 2, this is q 2, this is z 1, these are the output alphabet.

And with a is equal to 1 these are the states and these are the lambda. So, states are going to say q 3, q 3, q 3 and lambda is a z 1, z 1, z 2 ok. Now, if you draw the corresponding Mealy machine it will be like this I sorry this is the Mealy machine, now if you draw the corresponding Moore machine it will be like this, so like this. So, we have delta prime, so delta prime. So, we have our states are basically what? The states are basically for the new states is this Q comma output alphabet. So, all states are of the pair of say Q comma some z i.

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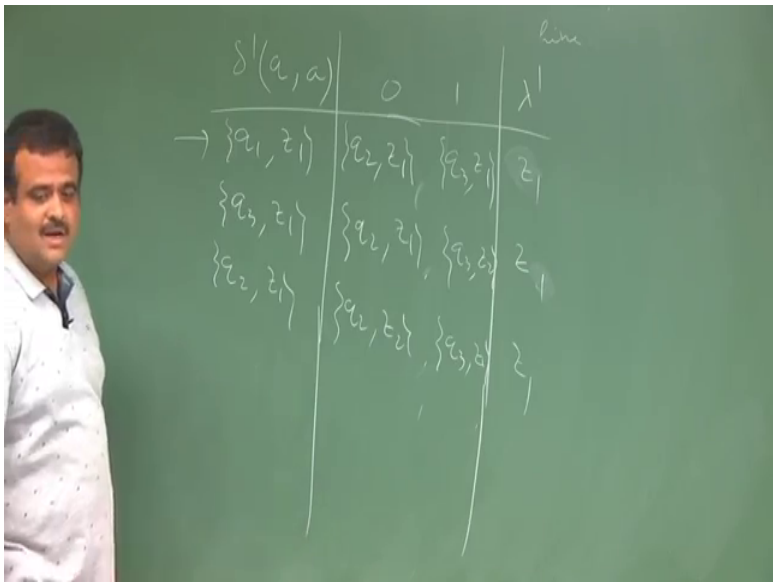

So, if we do this then the corresponding Mealy machine Moore machine will be like this q comma a. So, this is our transition role 0 1 and this is our ok. So, q 1 z 1 this is the starting symbol. It is going to be, you can verify this q 1 z 1 q 2 z 2 and this is q 3 z 1 and this is going to z 1 ok. And then we have q 3 z 1, you can easily verify this and q 3 z 3, I sorry q 2 z 2 and this is z 2, no this is z 1. So, like this we continue.

So, q 2 z 1 and then we have q 2 z 1, q 2 z 2 and this is q 3 z 1, z 1. Like this we can continue this to fill up this table. And this will we can show that, this will accepting the this is outputting the same stream omitting the first alphabet, first output. But here we do not need to omit the way we constructed it will output the same string as the two machine. And then we can have this some more example on this equivalency which will be given in the lecture note.

Thank you very much.