Introduction to Automata, Languages and Computation
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Lecture – 34
Finite Automata with Output

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So, we will talk about the Finite Automata with Output. So, so far we have seen the output is either we are going to be rejected state or a accepted state. So, output is just so accepted state accept or reject, so that is the output. We have given a string, we have to say whether the string is accepted or rejected, so that means at the end of the I mean at the end of the string after reading this, whether we are reaching to a accepted state or rejected state t that is the output ok.

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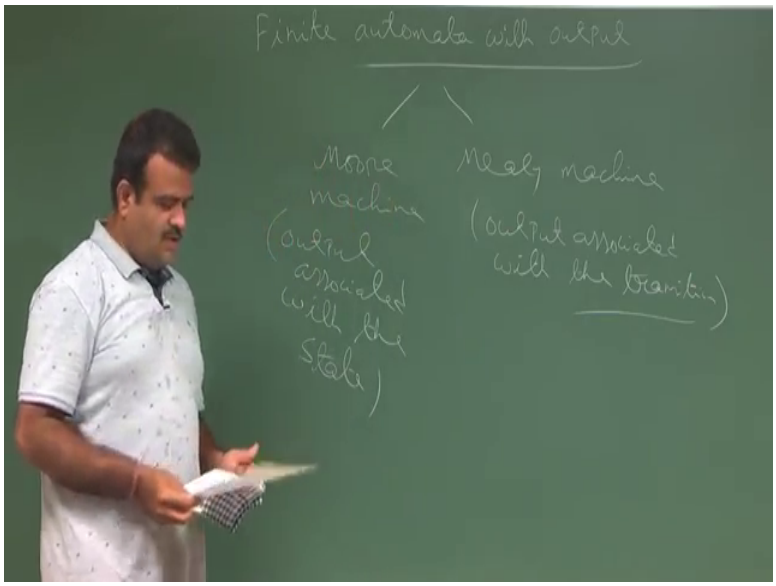

So, but there are some other output of this finite automata, so it is discussed basically we will discuss the two machine Moore machine and Mealy machine. So, we will define that, so we will discuss the two machines like Moore machine, and Mealy machine ok. Moore machine is the output associated with the state output associated with the state. So, associated with the space and state.

Suppose, we are at this state q. So, the if we are at state q, we have the output like this. So, output associated with so each state is I mean the output is a function of state output associated with the state.

But, here in the Mealy machine output associated with the transition, so not only state, it is the input alphabet also. So, the symbol you are going to read, and the current state it will depend on the where I mean what will be the output. So, output associated with the transition function with the transition ok. Anyway, we will formally define this two machine, and there equivalences.

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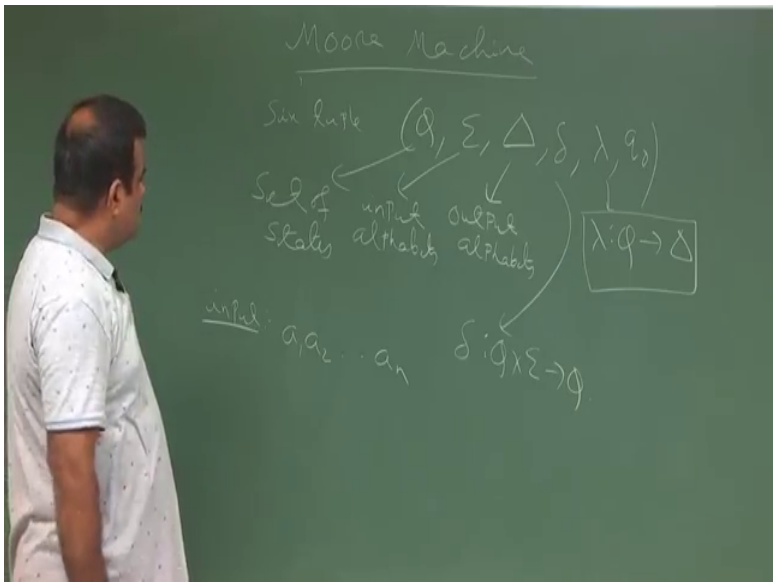

So, first let us define the Moore machine. So, it is again a six tuple, so Q, Q is the final finite set of state. Sigma, sigma is the input alphabet, and this is the output alphabet, this is the new one delta, lambda is the output function, and q 0 ok.

So, we know everything. This is the set of set of states which is finite this is the input alphabet set of input alphabet which is also finite weights, and this is the new one, this is the output alphabet which is also finite alphabets. And this delta is the transition function, so delta is same. So, delta is function from if it is I mean depending on the which suppose we are in d f, so delta is the function form Q yeah. So, this is a deterministic finite automata.

And this lambda, lambda is the output function like lambda is a function from Q to output alphabet. So, this is the name, lambda is the output function. So, suppose we are at state Q, each state is associated with their output symbol output alphabet, so that is that that will be determined by this function lambda ok. And this is the output alphabet set so like this.

So, now suppose we are going to suppose it so this is the situation, suppose you have an input alphabet say input alphabet say a 1, a 2, a n a string ok. And suppose this the delta under this transition, this is going like this.

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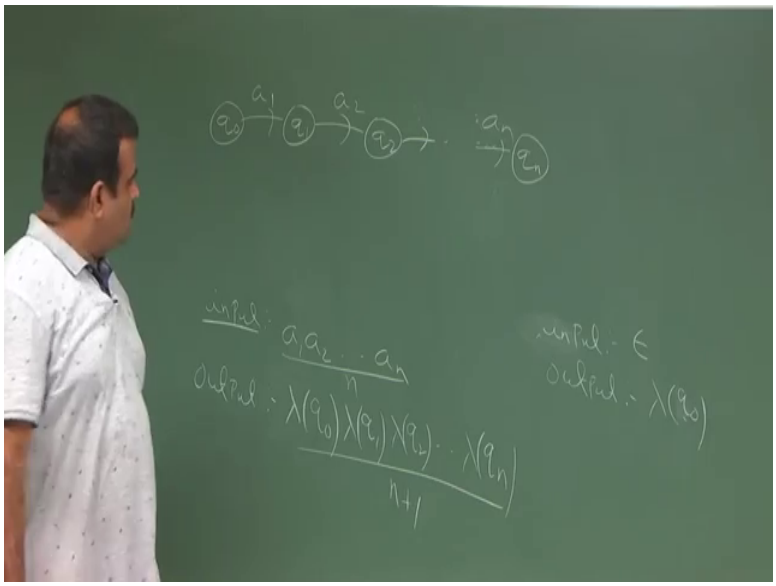

So, say let me rub this yeah. So, suppose we are having you are starting with q 0, you are reading this. Suppose, it will go to q 1 with a 1 with a 2 q 2 dot dot dot, we are reaching to starting from 0 q n. So, these are the state it is visiting. And when you go to one state, would you have the output.

So, what is the output string, output string is so we are visiting this so lambda of q 0 lambda of q 1, so all the states lambda of q 2 dot dot dot lambda of q n. So, length the length of this output string is n plus 1, so it is n plus 1th length, because we are starting with q 0 without reaching without reading any symbol and this is length is n.

So, output string is having one more length than the input string ok. So, if input is epsilon, then the output will be lambda epsilon lambda q 0 ok, this is the way. Now, can you can you just think of that DFA is a special case of this Moore machine yeah, DFA is a special case of Moore machine. Because, DFA what is the output, output is either it is accepting or it is rejecting.

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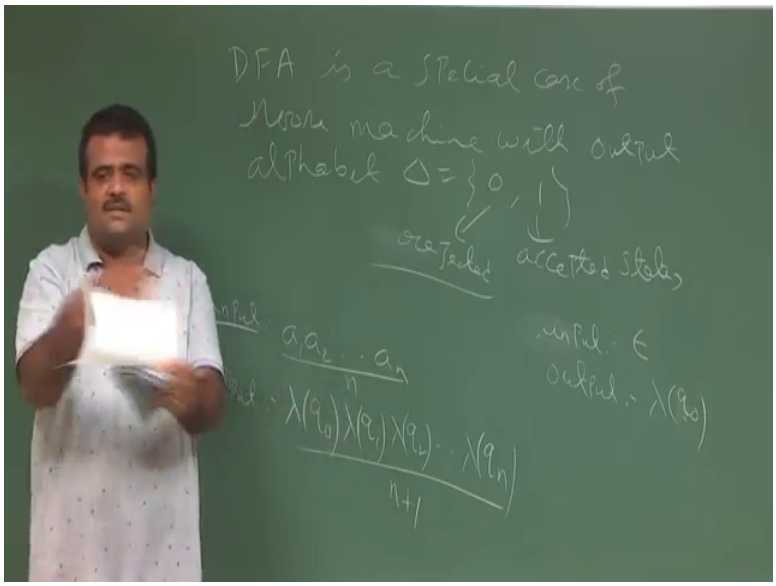

So, DFA is a Moore machine, where lambda is DFA is a special case of Moore machine with lambda 0 or 1 I mean 0, 1 means, it is accepted; 0 means, it is rejected state. So, we are labeling the states. So, in the DFA few states are accepted, few states are non-accepted.

So, those states which are accepted, we can level them by one, so that is the lambda. And those states which are non non-accepted we can level them by 0, so that is the way with output alphabet 0, 1. So, maybe this is level two this is level two accepted state, and this is for rejected state. This is the way we can view the this Moore machine I mean DFA as a Moore machine.

But, in the Moore machine this could be anything, I mean it is not just 0, 1, it could be any finite number of output alphabet, so that is the general case. So, in that case it would not be a DFA, so it is a general case. We have we have out we are outputting the many many symbols, so it is not just 0, 1 ok.

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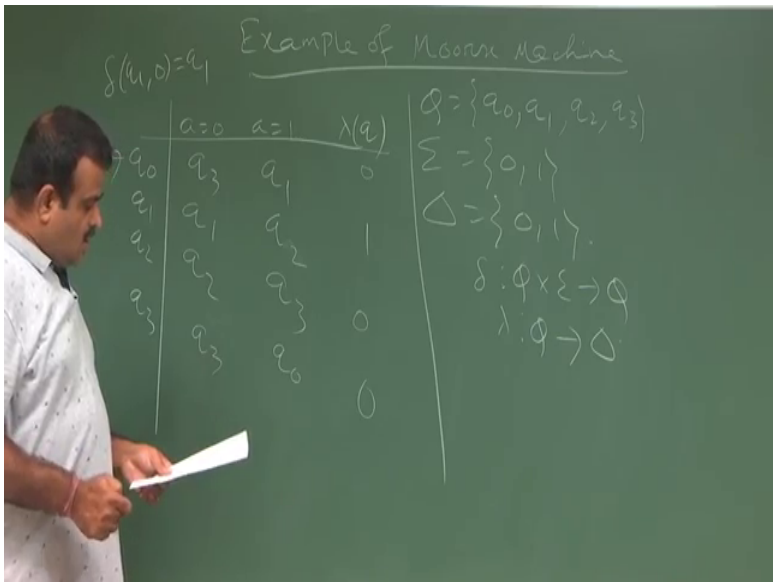

Now, let us take an example of Moore machine, then we will go to the Mealy machine ok. Let us take an example of Moore machine ok. Now, suppose we have Q is we have few states the q 0, q 1, q 2, q 3. And this is 0, 1 and output also says 0, 1 output alphabet. And now we have to defined this two terms; one is lambda, one is delta which is Q comma Q, and another one is lambda.

So, let us have this. So, these are the state q 0, q 1, q 2, q 3, so this is the starting state ok. Now, this is this is a is input alphabet a is equal to 0, a is equal to 1. And suppose here, so with q 0, so we can have a lambda over here also, this is lambda of q ok.

So, now with q 0 if we are reading that 0 will go to q 3, and lambda of q 0 is say yeah. So, q 3, and lambda of q 3 say in the lambda of so this is basically lambda, so lambda of q so this is 0. This is I will come back to that, this is q 1. And say q 1 of delta of q 1, 0, this is say q 1 again. So, this is q 1, this is q 2, this is q 2, q 3, and this is q 3, q 0 ok.

Suppose, this is a situation, and this side it is 0, 1, 0, 0. So, this is given the transition rules. And these are the corresponding value of the state. And for q 0 say this is this is a 0. So, lambda of q 0 yeah lambda of q 0, where is the q 0 q 0 is there lambda of q 0 say 0 ok.

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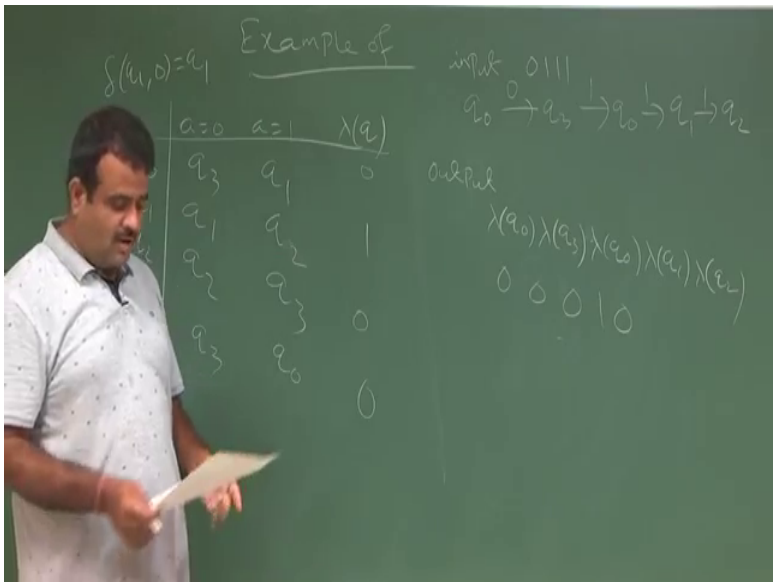

So, now suppose we have a input alphabet given this suppose you have a input alphabet say 0 1 1 1 four length 4. So, we are at q 0 if we read 0, where it will go? If we read 0, it will go to q 3. And then we read 1 from q 3 with 1 will go to q 0 again, and again so with 1, so this is 0. So, we read again 1 so with 1, we go to q 1. And from again we read 1 so with q 1 with 1, we go to q 2 q 2. So, this is the transitions of this for this input alphabet ok.

So, now what will be the output? So, output will be lambda of q 0, lambda of q 3, lambda of q 0, again lambda of q 1, lambda of q 2. So, lambda of q 0 say 0, and this is also 0 0 1 0. So, this is of length 5, and this is of length 4, so because that is the so q 0 is the starting state, so that will contribute in the output.

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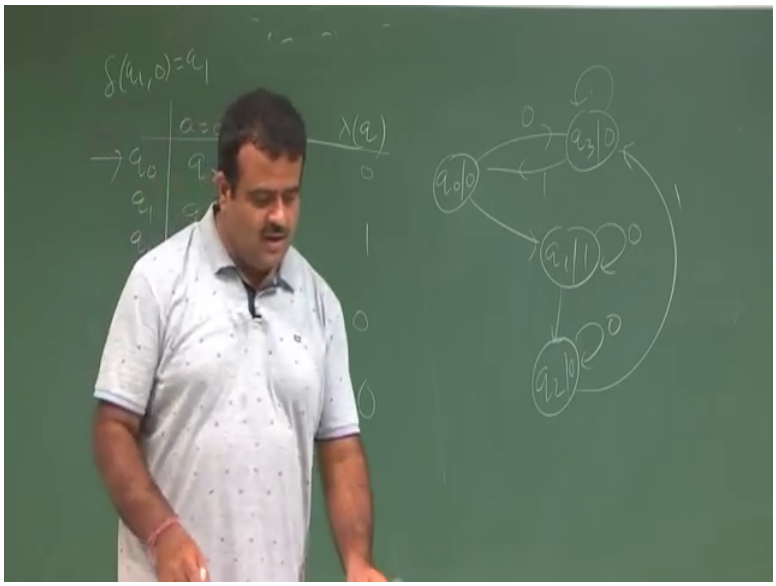

So, if you draw this, it will be like this if we draw this picture in a graphical form, so we are at q 0 so q 0 say output is I mean if we add q 0 will the output 0. So, from q 0 with 0, we are going to have q 3. And if you have q 3, what is the output 0, this is with 0. And with 1, we will go to q 1.

And for q 1, what is the output 1. And from q 1, if we read 0, we will go to q 1 again. And if we read 1, we go to q 2. And q 2 output is q 2 output is 0. And then from q 2, this is ok. And from q 3 from q 3 if we yeah again from q 2, q 2 if you see a 0, we will we will have a sell flow over here. And if we see a 1, then we will go to if we see a 1, we will go to q 3 with this. And from q 3, where we are going from q 3, if we see a 0 here, and if we see 1, so this is the corresponding graph of this Moore machine is the transition ok. So, this is the example of a Moore machine.

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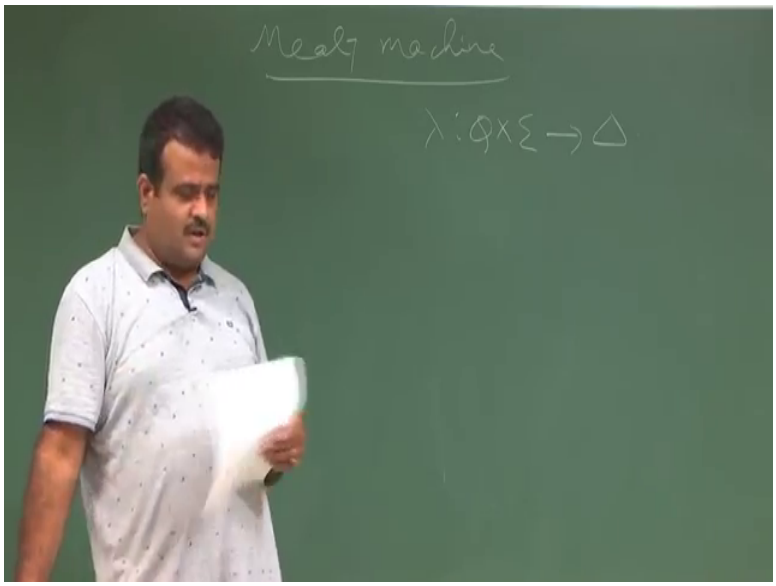

Now, we will define the Mealy machine. So, let us define the Mealy machine. So, Mealy machine that the output is not only depend on the states, it will depend on the input tape which we are reading. So, it is depend on the transition function.

So, earlier the for the Moore machine, it is depending on the wholly on the fully on the states, but here it is depending on the state as well as the current state as well as the input alphabet we are reading. So, it is a function of delta I mean it is a function of Q cross this to I mean the which step we are which input symbol we are reading ok.

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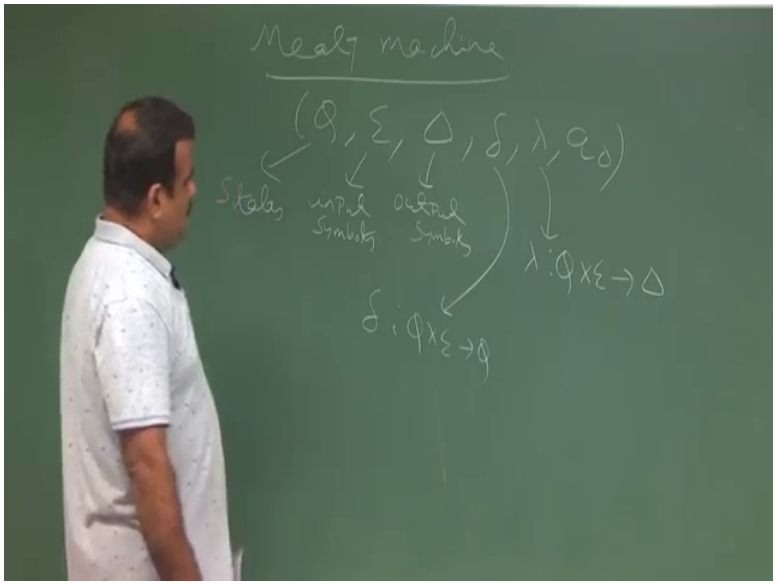

So, let me formally define this, so this is again a six tuple Q, sigma, delta, lambda, q 0. So, all are same over here. This is the finite number of set of states. So, for this is the state set which is finite, this is the input alphabet set or input symbols, which is also finite. And this is the output symbols, which is also finite I put symbols.

And now this is the del transition rule transition rule again a function from Q cross, this is the deterministic noise this one. And here the difference is here this delta now sorry this was lambda, this was this was for Moore machine, if as a function of the state directly. But, here it is a function of state and the input alphabet; it will go to the output alphabet. And this is the starting state ok.

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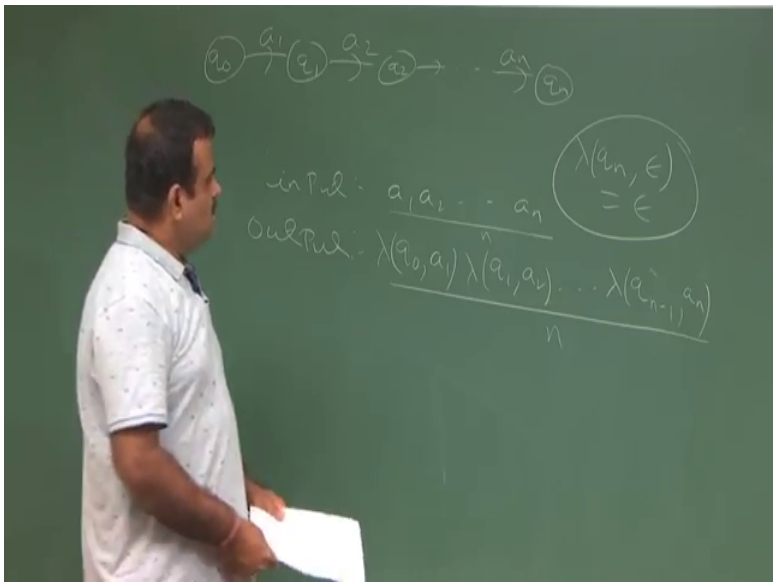

So, now how to read a input. So, suppose we have given a input say a 1, a 2. So, suppose you have given a input like this a 1, a 2, a n ok. Now, suppose with this input our state transition is like this. So, we are at q 0 with a 1, we are going to q 1, with a 2 we are going to q 2, with a n we are going to q n ok. So, this is the state transition for this input string. So, you are putting the string in the tape, and we are reading the tape like this ok.

Then what will be the output, so earlier for Moore machine, it was the lambda of this states only, but here it is a state as well as the input symbols. So, this is basically lambda of q 0, a 1, because a 1 you are reading. So, depending on this q 0, and this is a 1, it will output some value. Lambda of q 1 a 2 dot dot dot lambda of q n minus 1 a n.

And at the a n what we have at the a n we have q n, so lambda of q n, but you do not have anything to read. So, epsilon so this is epsilon, this is our convention. If we if the input is epsilon, output will be epsilon, so that is why the size this is n input alphabet is input string is n, this is also size n, this is the difference. So, size is n in the both the cases. Earlier it was n plus 1, if the input is n n symbols, then the output was n plus 1 symbol, because every state has a output. But, here once we reach to the this once there is no symbol in the input alphabet, then we are no had to go, so that is so that is their ok.

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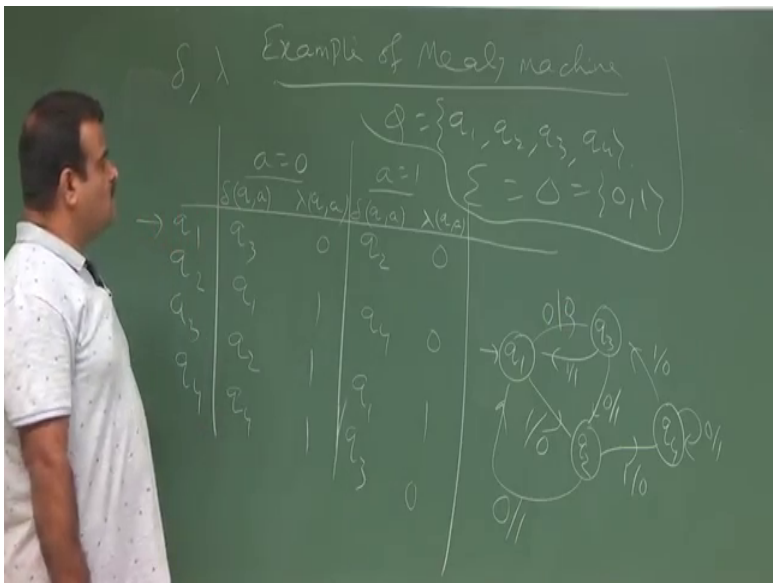

So, will take an example of a Mealy machine, then it will be more clear. So, let us take quick the example, example of Mealy machine ok. So, so let us have the earlier one. So, we have Q is equal to there are few states q 1, q 2, q 3, q 4. And suppose q 1 is the starting state.

So, now say sigma and lambda is just 0, 1 ok. Now, let me define the transitions. So, transition is like this, so here at q 1 is the starting state, q 2, q 3, q 4. And here we have say input symbol is 0, and here you have input symbol is 1.

So, for a given input symbol, we have for a given input symbol, and the state we have to define two functions. One is lambda, and another one is delta. So, this is the two function, you have to defined. So, let us defined the states, where it is going this is lambda of q, a, and this is delta of q, a. So, this is output state, where it is going and this is the output alphabet. So, suppose this is going to q 3, and this one is 0. And it is going

to q 1, this one is 1. It is going to q 2, this one is 1 again. It is going to q 4, this is one example going to one ok.

And similarly here also you can have this lambda of q comma a, delta of q comma a, suppose this is like this. q 2, it is going, it is 0. Then q 4, it is 0. Then q 1, it is 1. And q 3, it is 0 ok, suppose this is the situation.

And how to draw a graph for this? So, to draw a graph, suppose you are at qs q 1, this starting state. So, from q 1, where we are going, we are going to with one we are going to q 3, we are going to q 3, so this is sorry with 0 with 0 input we are going to q 3, and we are outputting 0. So, this we can write as this is the reading, and this is the output.

And with 1, so with 1, we are going to q 2 with 1 we are going to q 2, and you are outputting 0. So, this is 1, 0. So, if we do this, complete this, then it is coming here 1 1, because from q 3 from q 3 with 1. We are going to q 1, and outputting 1, this is the output, what we are outputting this other part.

Like this so we have a q 4 over here, and this is going 1 0, 0 1, this is 1 0. And we have a sell flow over here 0 1 ok. And then from q 2 you can go to q 1, from q 2 with 0, from q 2 with 0, we can go to q 1 and output 1. So, its 0 output 1 like this, anyway we can complete this graph. So, this is the input alphabet, this is the output alphabet. So, now this output alphabet is a function of this.

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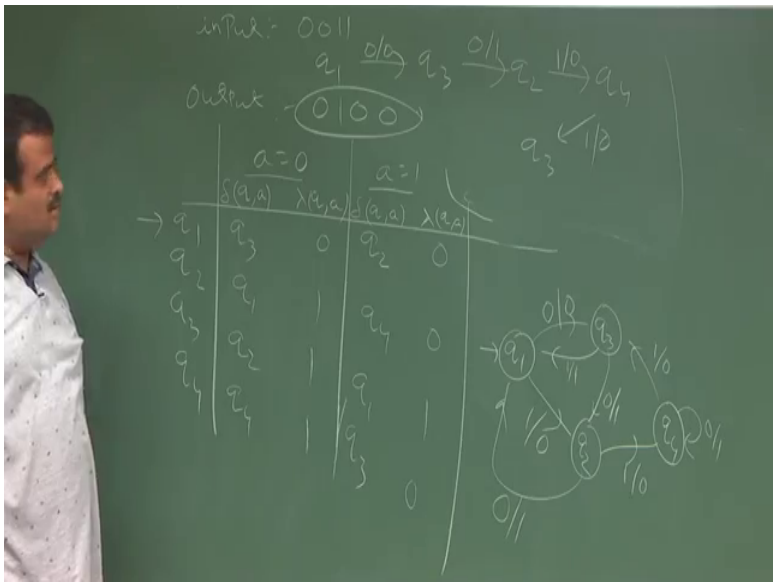

Now, how to read a input string? So, we can just quickly do that, suppose our input is say 0 0 1 1, then the supposed 0 0 1 1 where we are going. So, q's we are starting from q 1, so with 0, we are going to have, with 0 we are going to q 3 and the output is 0.

And from q 3, again we are seeing a 0. From q 3, we are seeing a 0. We are going to q 2, but output is 1 0 output is 1. So, from again we are seeing a 1 from q 2 from q 2, we are seeing a 1, you are going to q 4, but output is q 4 output is 0. And from q 4, you are seeing a 0 I sorry 1. So, we are going to q 3, and we are outputting 0. So, what is the output, output is this one, the other part of this 0 1 0 0, this is output string. So, this, this, this in this example, it is of same length ok.

Thank you.