Introduction to Automata, Languages and Computation
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Lecture – 33
Two Way FA

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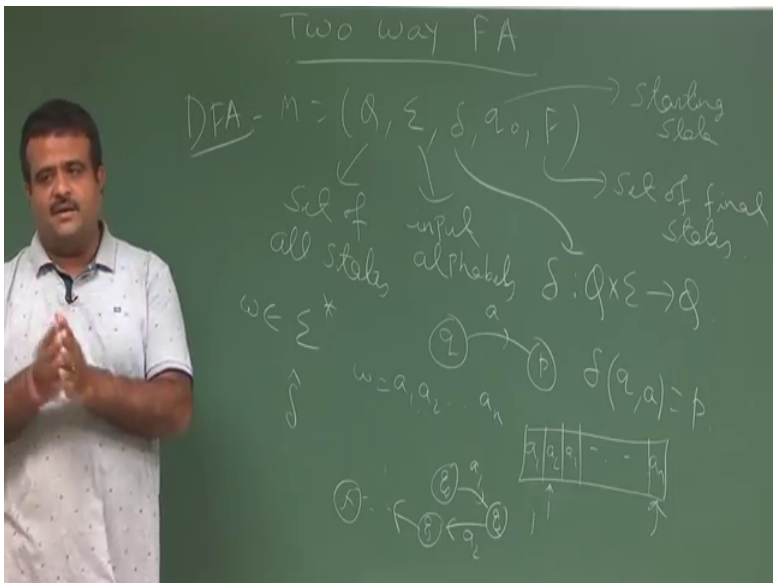

So we are will start the concept of Two Way DFA, two way finite automata. Two way means so far we have seen given a finite automata say DFA, NFA suppose you have given a DFA which is Q sigma delta q 0, F and the Q is set of all; set of all states and this is the input alphabet, all are finite and this is the transition rule. For DFA it is, if it is DFA transition rule is like this delta is a function from Q cross sigma to Q.

So, given a state suppose we are at state q and suppose you are reading a input alphabet a will go to state p. So, delta of say q comma a is equal to p this is for deterministic move this for DFA, but in case of NFA the move is non deterministic, we cannot define we cannot have a definite p, I mean there is a option; I mean this move is not deterministic it is a non deterministic that is why.

And this we know is the starting state or the initial state and this is the set of all final state final states and there may be one or more or many final states, but starting state is 1. Our automata our finite automata is starting from a one starting state there is that is

represented by q 0. So, far what we are, how to read a string? Suppose we have a string which is a consists of the input alphabets.

So, this is we what we do we, suppose w is a 1, a 2; a 1 a 2, a n implementing. So, how to read this? So, we use the extended delta to read this. So what we do? We put the string in a tip a 1 a 2 a 3 like this and we start our automata at q 0 state and our tape is heading like this and every time we are moving the tape right wise. So, that is only one way it is the movement so, tape is moving tape head tape head is moving only one direction right side.

So, first it will read the delta of so suppose with a 1 it is go to some q 2 or q 1 with again it will move to here like this. So, again with a 2 from q 2 we are going to q 3 like this. So, finally, with a n if we are reaching to some of the state say f and if f is happened to be a final state, then we say this is the accepted string. So, this will know about the finite automata.

So, this the tape is moving only one direction so right side, it has no two way moving so tape head header this is the header of the tape. So, header of the tape we are reading that string left to right like this. So tape is at each point of time tape is moving from left to right ok. But in case of two way automata, tape can move left also. So it can suppose we are here and our present state is say our present state is say q and we are here.

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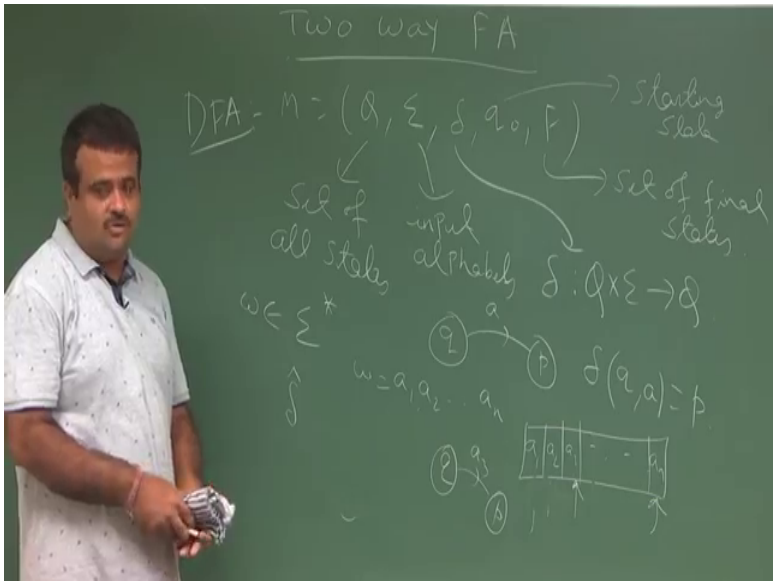

Now, with this input a 3 we can go to either the tape can go point to here or point to there and it will move to some state p that is ok, but the tape header can go to right side or left side so there is a option there, so that that is the sense it is called two a. So, we will formally define that two a finite automata, this is which is something different so far we have seen.

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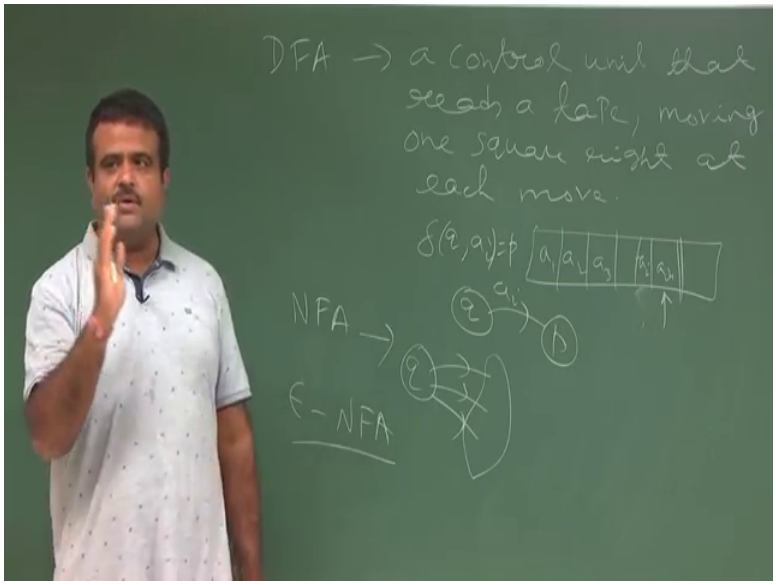

So, let us just recap. So we have seen the DFA which is a control unit DFA has a control unit that read the tape from left to right, tape that read the tape from they read the tape and it is moving one square; and it is moving one square right at each move each move. So; that means, given a tape we have given this a 1 a 2 this is the 6 string say a i and suppose at some point of time, tape header is pointing there.

So, suppose at that point we are at state q. Now it will go to so suppose delta of q comma a i is say p. So, it will go to state p and the header will move to the next a i plus 1; next input of the of the string. So it is moving one square right at each time. So, there is no move the in the left side which is there in that two a automata.

And then we have seen the NFA. NFA is basically the it is a it is also finite automata with one side move and only difference is the non deterministic; the move in move is non deterministic. So, from q with the same input we have many options so this move is non deterministic and then we have seen the epsilon NFA. Epsilon NFA is the tape head

moves without reading the input symbol. So, without reading the input symbol we can move the we can change the state so that is the epsilon NFA.

Tape is pointing header is pointing somewhere, but we are not reading that symbol without reading the symbol the state can move our automata can I mean. Now 2 way DFA is 2 way finite automata is basically we have movement both the side 2 way finite automata.

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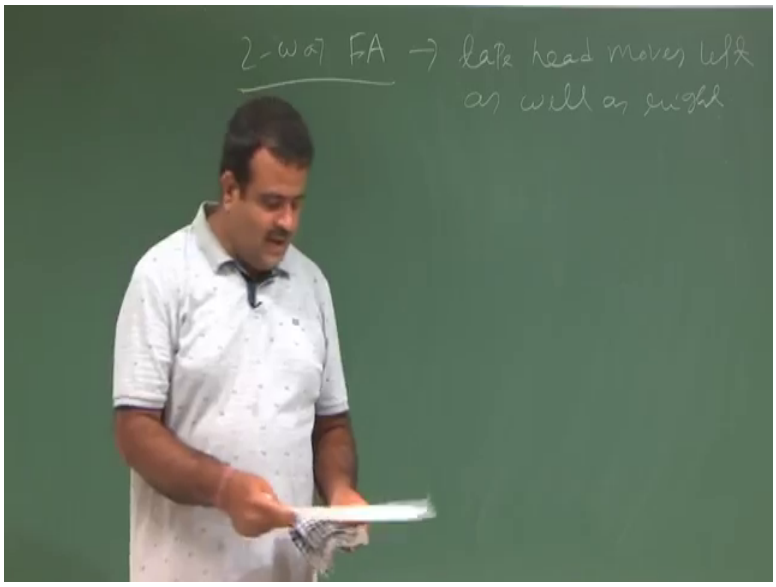

So, in this case the tape head; the tape head moves left as well as right. So, that is the that is called two way finite automata so we have a option to go to left or right. So, its increased little more power than the conventional finite automata which is, but eventually it will accept the I mean it will give the same regular set. There is no I mean in terms of language there is no benefit of having this so eventually, it will accept the regular sets only. So, let us define the two way deterministic finite automata [noise].

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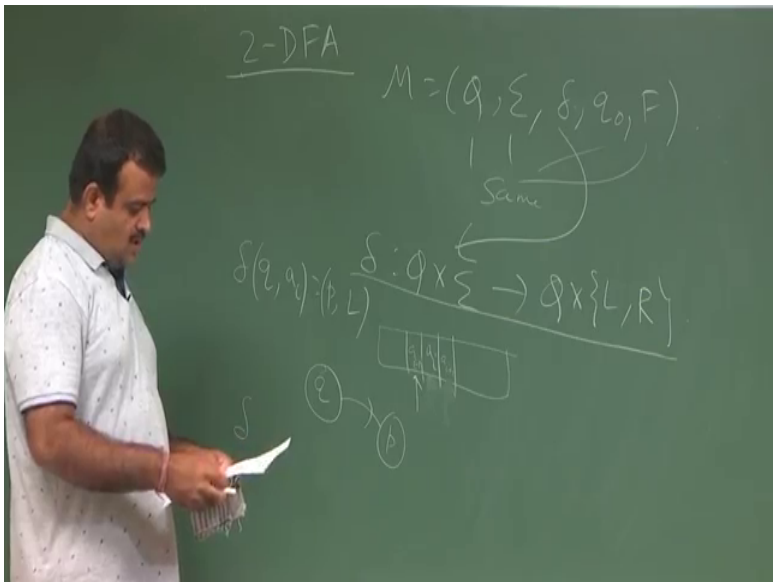

So, this is called 2 DFA; Two way Deterministic Finite Automata. So, it is basically M tuple a five tuple again Q delta q 0, f, we know all this symbol Q is the set of set of finite number of states possible all possible state which is finite and the sigma is the input alphabet and delta is the transition rule. But here we have a option to move left or right the tape head next step and q 0 is the starting state and f is the set of final states.

So, those are all the things, so these are same; these are same as earlier all the differences here this is different. So, this is now a function to it will go to another state, but the movement so, tape head movement will be either left or right so, that we have to point out L or R ok.

So, suppose we are we are just here a i and this is say a i plus 1 a i minus 1 and a i plus 1 suppose it is pointing. So suppose at that point of time we are at state q. Now here suppose delta of q a i so now, this is a output will be a state along with a the sign I mean movement of the taper tape head; it will either go to left side or right side.

So, if it is p comma R p is a state so, it will go to p and the tape head will point to the next it is going to the right. But if it is but if it is p comma L; p comma L so, it will go to p, but tape head will point to this that is the difference. So, tape head for the next move will point to this, so again you have to scan this. So, it is like movement of the head.

So, we may eventually sometimes we may we may put into the infinite loop; so that is why when you talk about tuning machine whether tuning machine will hauled or not. So, depending on the movement over here we may eventually end up with a infinite loop, we go then a comeback, go come back like this ok. So, this is the only difference; so this is the only difference in the function transition function or transition rule ok.

So, in that case how we define the language of this two way DFA so you have given a two way DFA. So, for one way DFA we know how to for one way DFA this is our DFA q 0 f.

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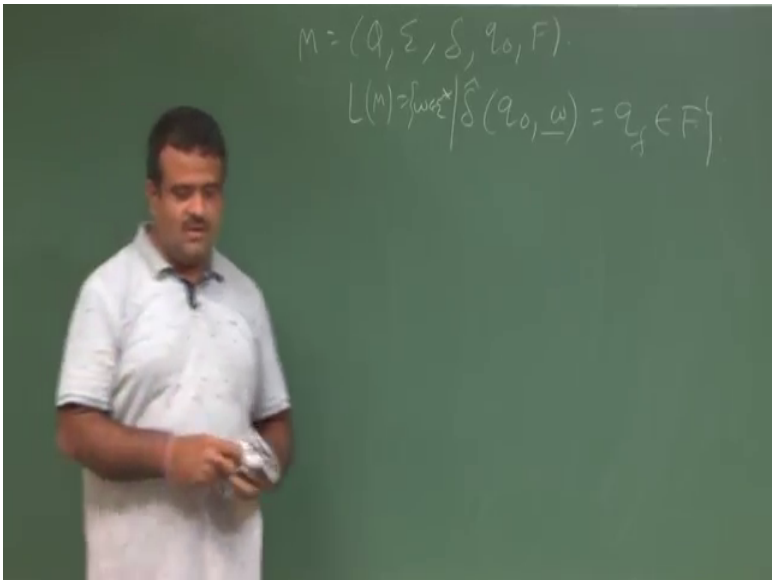

Now, for one way DFA what we did? We extend this delta to delta hat; that means, which is accepting the string and then we start with q 0 and if w we reed and if it is end up with some f or q f which is belongs to f, then we say this is the set of I mean this is the language of this M say for wall w such that this happened ok.

Now, now we are in two way DFA. So for two way DFA extending this delta hat is insufficient I mean it is not I mean we because you are moving left to right again. So in that case what we will do we will take help of what is called Instantaneous Description or ID. So, it is basically a snap sort of the process, then in the next move the snap sort of the process again like this. So, we will define the ID or Instantaneous Description of the finite automata.

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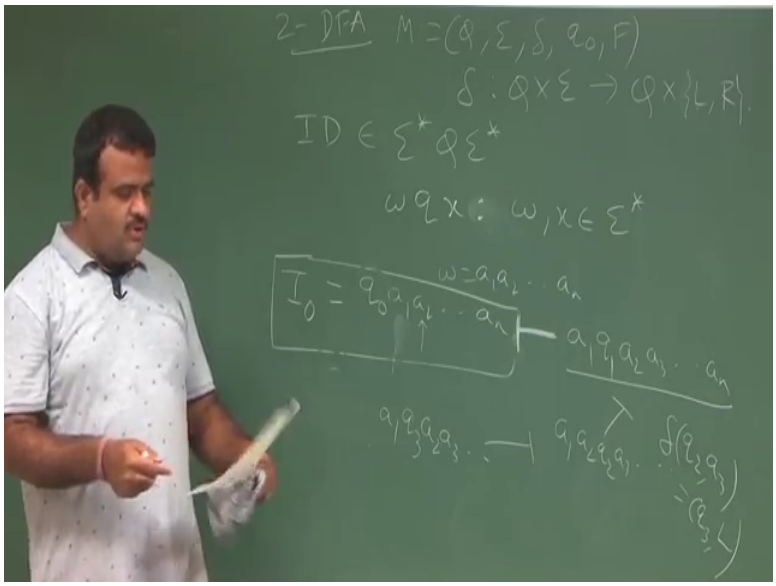

So, we have given a two way DFA M which is Q sigma delta q 0, f and we know here delta is Q cross L comma R so our tape head can move to the left to right left or right ok. So, M is the two way DFA. So, ID or instantaneous description is basically of the form is coming from this q sigma star ok. So, basically it is of the form w q x so w is w, x are coming from; w, x are coming from this is w x are coming from this sigma these are all string.

And q is the state that is that point of time state the starting I mean it is not starting state starting state is q 0. So, at some point of time this is the snapshot of the system of the automata of the finite machine. So, like for initially what you have? We have given a tape, we have given a string say w; w is a 1, a 2 say an and initially ours automata is at state q 0.

So, this is the initial ID q 0 a 1 a 2 an this is the initial ID of the this is the I 0 we can say. This is the initial ID of the M because initially we are at q 0 this is nothing, but a snapshot we are taking a photo of the present situation of this system of this machine finite machine. So, at the starting we are at q 0 our tape this is the position of containing the tape position and our header is here ok. So, header will be pointing always the right side of this q 0 that is the convention will follow.

Now if it is say for example, after this suppose we start with q 0 and suppose our delta is like this so, a 1 and say it is going to q 1 and it is right say. So, what will happen? So, it

is going to basically we can say this is going to. So, next so what is the next position? So, now, the next position is we are at q 1 and our tape is moving right so tape is here. So, we will write this a 1 then q 1, now we are at q 1 because our state change to q 1 and next you have to read a 2 a 3 an this is the way.

So, this we can say this is the current snapshot or current ID of the machine and this will say by this is going here like this symbol. So, this is the initial instant initial instead ID and this is the next ID if the our delta is like this ok. Now suppose in the next time if it is going to q 2 and if the right, then it will go to again it will go to say it will go to a 1 a 2 say q 2 a 3 like this. Now suppose so, again you have to read the symbol which is write to q 2.

Now suppose so now, we are going to read q 3, now suppose if delta of q 2 q 3 is q 3 but if it is left; if the header is pointing to the lefts then where it will go, then the header will be pointing left; that means, you have to read here. So; that means, this state will be pointing here and the state is q 3. So, this will now go to like this a 1 then q 3 a 2 a 3 like this. So, because that is the next state is next symbol is q 2 which we are going to this is the this is the wherever we are putting the state, the next symbol is the pointing the tape head so that is the conventional will follow.

So, this is going there because next we are going to read again q 2, since it is a left moved over here. So, this is the way we take the snapshot of the finite machine and we go to that from one state to one state to another state like this ok. So, now, when we say yeah so when we say we are accepting a string.

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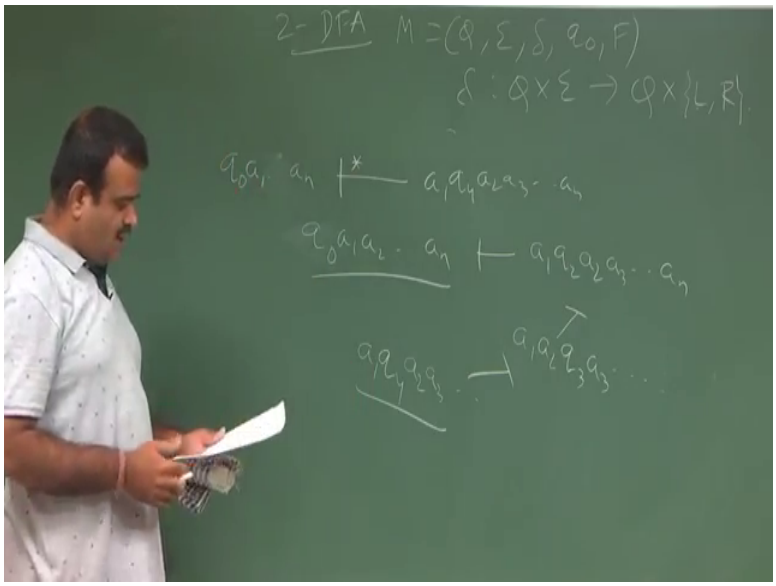

So if we so, this star means if we apply this recursively then it will give us a star so; that means, if 1 push or suppose a 1. So, initially it is q 0 a 1 a 2 a n say. Now suppose next step will going to say a 1 q 2 a 2 a 3 a n. Suppose in next step we are going to a 1 a 2 q 3 a 3 like this, suppose in next step our tape is pointing to the left side so; that means, insert so after reading this we have to read this. So, that means, and suppose it is going to q 4, so q 1 q 4 our current state, but we have to this q 2 will be the next symbol to be read I mean header is pointing to there because it is a left move a 2 a 3 like this.

So, now this means this is this id is going to this id by more than one operation of the transition so that is the star so; that means, you can say q 0 a 1 a 2 a n is going to a 1 q 4 a 2 a 3 a n so this is the star. So, by this using the star we can just define the language accepted by this. So, when you say a language accepted by this if eventually it will reach to a final state and all string has been explore; that means, if eventually we can reach to like this.

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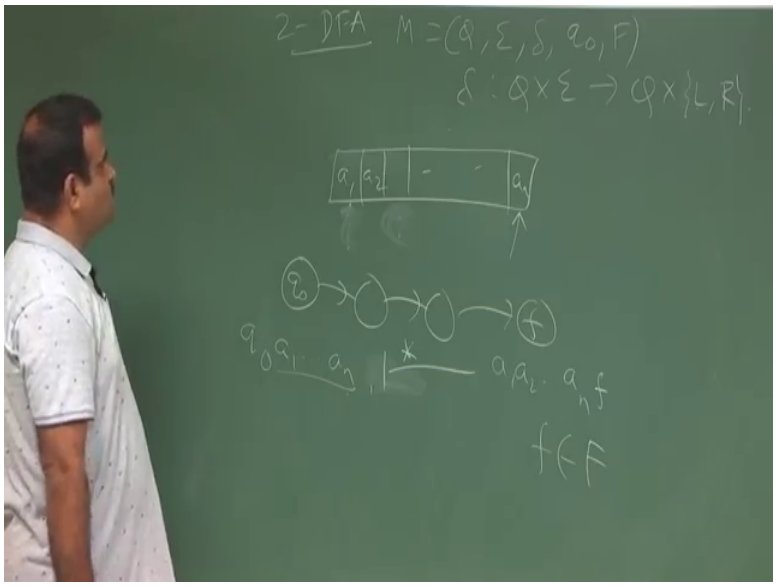

So, suppose we are starting with a one a two suppose we put this in a tape a 1 a 2 a n and we keep on reading you start with q 0, we read the tape will go to some state some state and tape will move here and there like this. Now suppose at the end we reach to the some of the state f and if it is happened to be a final state.

And if we exhaust of reading all these symbols then and if we exhaust to reading all the symbols means then what is the position there at the end at the end it will be a 1 a 2 a n and f will be pointing here that is one. So; that means, if q 0 a 1 a n if it is going to this where f is a final state, then we show a this string is accepted by this two way DFA. So, this set we are this set is our language so, we can formally define this.

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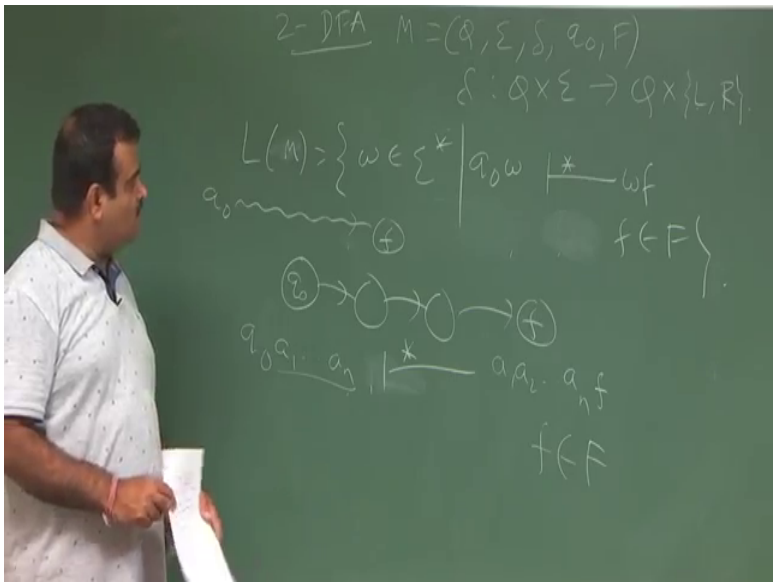

So, language of the same is nothing, but set of all w such that q 0 w we will end up with a f, for some state f belongs to or some state I mean that should be at least I mean for some state f belongs to this. So, that means, we start with q 0 and eventually will reach to f and we exhaust thereading that tape ok. So, tape header is just reach to the end of the string. So, then we stop and we say this string is accepted by this DFA. Now we will take an example.

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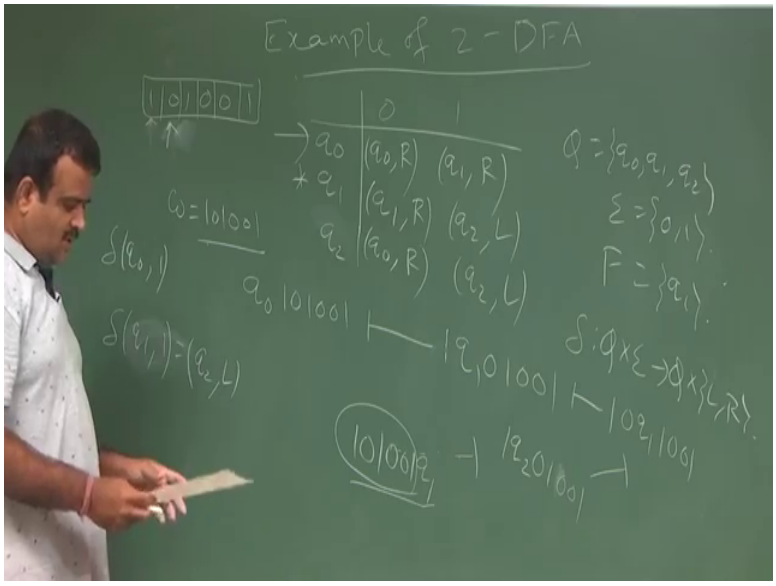

So, let us take an quick example, suppose we have given this two way DFA we have two input alphabet 0 and 1 and these are the state q 1 q 2 q 0 q 1 q 2 and this is the starting state and this is a final state ok.

Now, delta we have to define so Q is basically q 0 q 1 q 2 and sigma is 0 1 and f is q 1 and delta we have to define. So, delta is basically a function form q cross sigma 2 q cross L R ok. So, if we add q 0, then with the input 0 will go to q 0 and it will be R and this is q 1 and it will be R again and with q 1 with 0 input is go to q 1 it will be R, q 2 it will be L and same q 0, R I mean with q 2 R and with q 2 here at q L ok.

So, this is the transition rule and here we are indicating the next position of the tape after reaching to that state. Now if we want to see if we want to read a string say w which is 101001, then how to read so we start with q 0 and we put the w over here this is the initial idea of the initial position we are at q 0 and our tape is tape is this 1 0 1 0 0 1 this is the tape and we are header is pointing here. So, if the header is pointing here, so we have to read that delta of q 0 1.

So, delta of q 0 1 is basically q 1 L so L means header will go here so, this is basically one we have exhaust. So this will be now q 1 and header is pointing this so this is 1. So, now, next so now, we are at; now we are at q 1; we are at q 1 and we are reading the 0 so we are at q 1 you are reading 0 so we will go to again q 1 and header will be moved to right so header will move here and we will again go to q 1.

So, that will be 1 0 q 1 again 1 0 0 1 so like this. So, now, we are at q 1 now it is 1. So, now, header will move to the left so, this is the we are at q 1 so delta of q 1 now we are reading 1. So, this is again q 2 L so L means header will come back here. So, this will be basically this is going to q 2, but we have to read this so 1q 2 1 0 I sorry 0 0 0 1 like this.

So, if you continue like this we will end up with this 1 0 1 0 0 1 q 1. So, you have to do little more step and eventually we will reach to this that is all. So, that; that means, we are reaching to this position means this string is accepted so this string belongs to L of G. So, this is one example where yeah where this is this is belongs to the language of this two way automata.

Thank you very much.