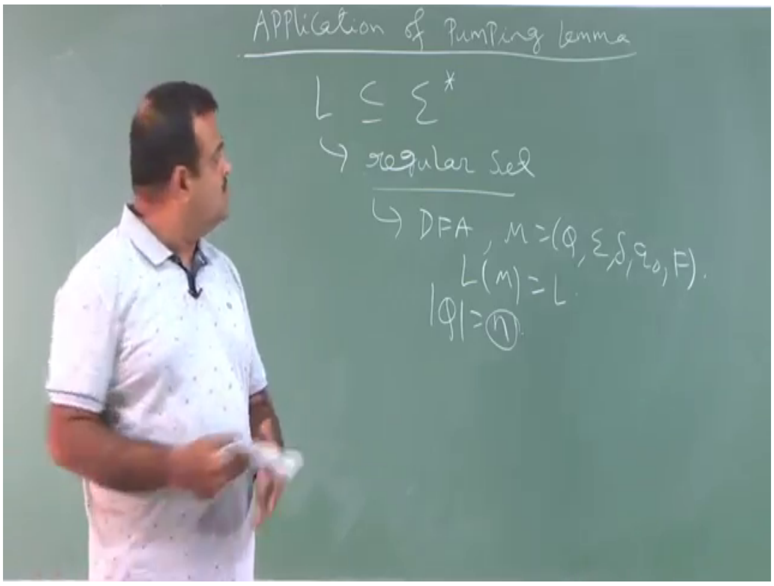
Introduction to Automata, Languages and Computation
Prof. Sourav Mukhopadhyay
Department of Mathematics
Indian Institute of Technology, Kharagpur

Lecture – 28
Application of the Pumping Lemma

So we are talking about Pumping Lemma. So, in last class we have this we have defined the pumping lemma like what is the pumping lemma? This is just to recap this is necessary condition to be a language to be a regular language.

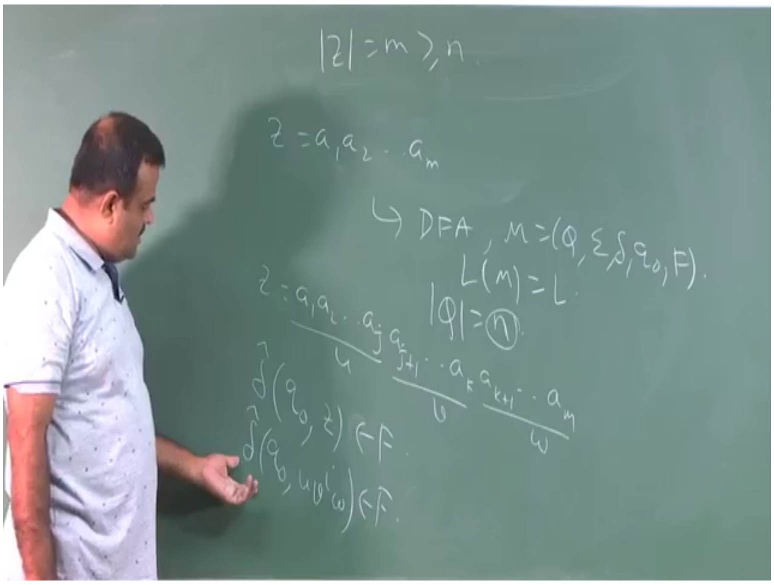
(Refer Slide Time: 00:33)



So, given a language L which is a subset of star if this is regular suppose this is regular set or regular language, then we have seen that regular means there has to be a DFA which is accepting this language, any member of this language must be accepted by this DFA L and the size of this is n.

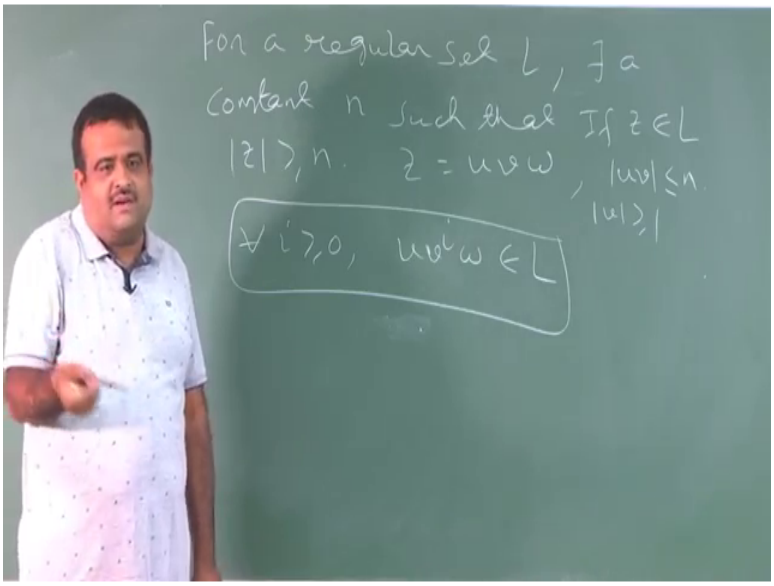
So, this given this L, the language if it is a regular then what we have? We there exist a n, this n is nothing, but this cardinality of this set Q there in n number of states. Then if we choose a string which is more than n from L.

(Refer Slide Time: 01:39)



If we choose a string whose length is more than n like this, say we are choosing a z of length m which is greater than equal to this n. Then we have seen this z can be written as a 1 a 2 a j then a j plus 1 to a k, then a k plus 1 to a m this is our u this is v this is w. Such that the delta hat of q 0 z this has to be in one of the final state. So, this is say belongs to F then delta hat of q 0, u v to the power I w these also belongs to F. I mean these also has to be a this string also has to be accepted, so, that is the pumping lemma. So, let me just write it, so, let me just write it again.

(Refer Slide Time: 03:01)

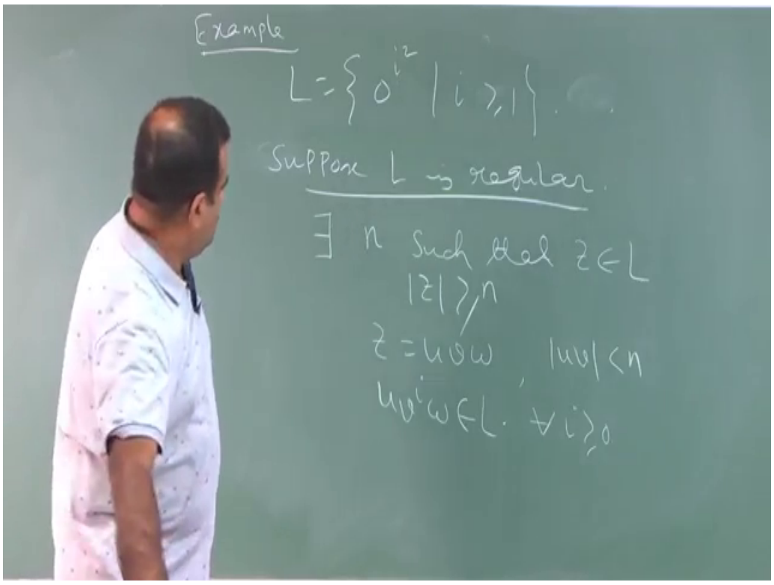


So, for the regular set L, then exist a constant n such that if we choose a such that if we choose a word whose length is more than n, then we must able to write z to be u v w where, u and v length is less than n and length of u has to be greater than 1. Then for every I greater than equal to 0 this also has to be accepted by the DFA this will also belongs to L for every i; if i is 0 this is u v u w if i is 1 this is L if i is 2 then u v square w like this.

So, this is what is called pumping lemma this is the necessary condition. If we have a L then there has to be a there is a constant n that is nothing, but the number of states in the DFA, then we can given any string of more than n that is z, we can write z as this where you will have a substring v which can be pump like this and that has to be again in L, that has to be that string new string has to be accepted.

So, this condition is necessary condition. If a set is regular then we must have this condition satisfied, but this is not a sufficient condition; that means, if it is true then for a given set, then we cannot say that set is regular. So, we will use this condition to check the non regularity of some set. Given a set if this condition is not satisfy then we can straight away say this language is not regular. So, we will apply this pumping lemma to check whether a set is non regular or not ok.

(Refer Slide Time: 05:41)

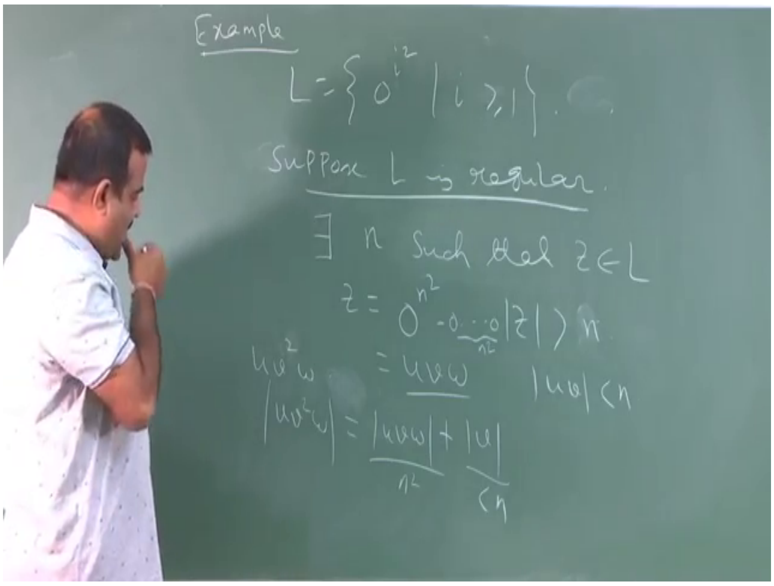


So, first example is let us take a set which is 0 to the power i square i is an integer; i is an integer greater than 1, is this regular I mean is this language is regular that is the question that is we have to find out we have to check it using the pumping lemma ok. Now this will show; so, this is meaning the this is say 0 0’s 1 so, 0 1, so, 2 means 0 0 0 0; 2 means it is four like this. So, these are the string.

So, this is the string of 0 with length perfect square string of length 0 this is any string of length 0, if this is k then k has to be a some n square perfect square that is the set, now this set is you have to show whether this is regular or not. So, suppose this is regular, if possible or suppose L is regular will prove using contradiction then if a L is regular.

Then it must satisfy the pumping lemma, that is the necessary condition of a regular set then we must have a then there exist n which is nothing, but the number of state in that DFA. There must exist n such that if we choose a z from this L whose length is more than L or more than equal to n then z has to be written as u v w where, u v are less than n then this u e to the power i w has to belongs to L for all I ok. So, now let us check that whether this is possible or not. So, we take a we have to take a z which is more than n.

(Refer Slide Time: 08:17)

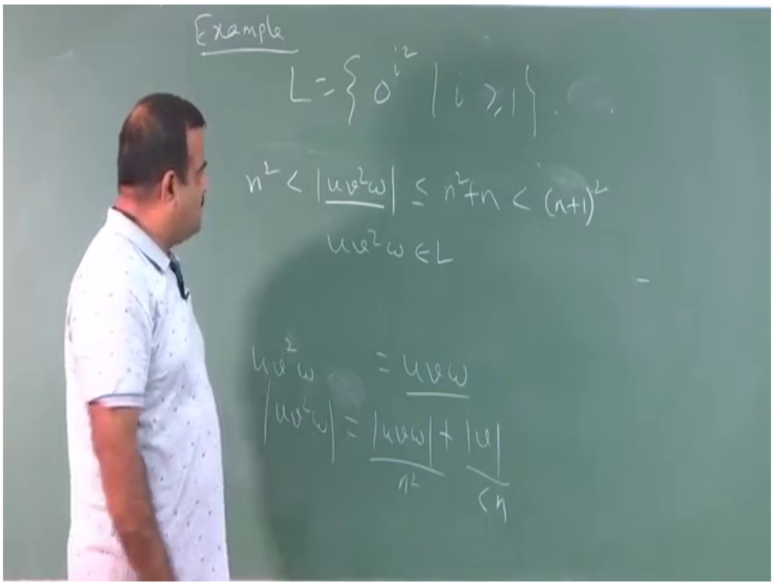


So, let us take length n square say we take z equal to this n square so; obviously, mode of z is less than n mode of z is less than n ok. Now if we write this as u v w where. So, these are all 0’s this is all 0’s where length is n square.

Now, if you write this as u v w have u and v are less than n then what will happen? So, this is the n square now this is u v w we are writing; now if we take then all the u v to the power i w has to belongs to L now if you take i is equal to 2. So, in particular this has to be in L now what is this? So, if you take u v squared w, then this set length of this.

So, these are all 0's that is a no issue, but only thing length of this has to be a perfect square what is the length of this? Length of this is nothing, but length of u v w last length of v because we have added only 1 v this curve this is,. so, length of v. Now, length of this is n square, a length of this is less than n so; that means, let me just do it here.

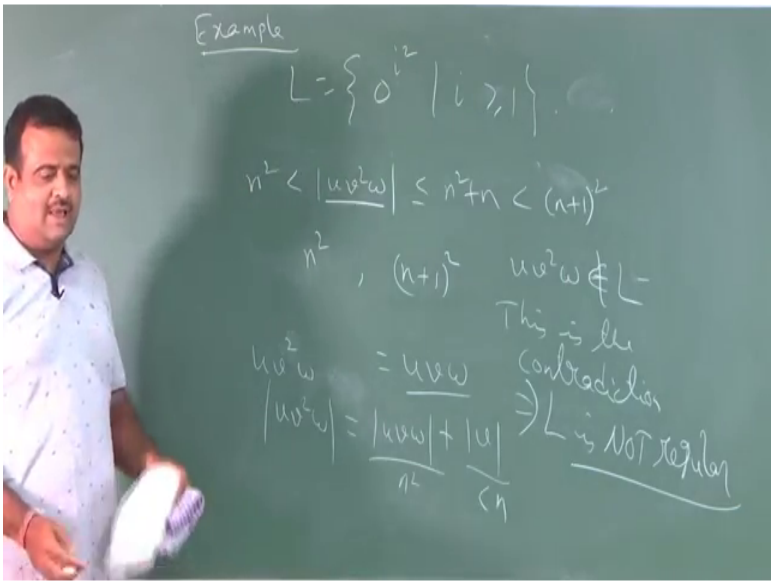
(Refer Slide Time: 10:07)



So; that means, length of u v squared w is less than equal to n square plus n, which is less than n plus 1 whole square. And n square is less than this because we have added u, n square was the length of u v w.

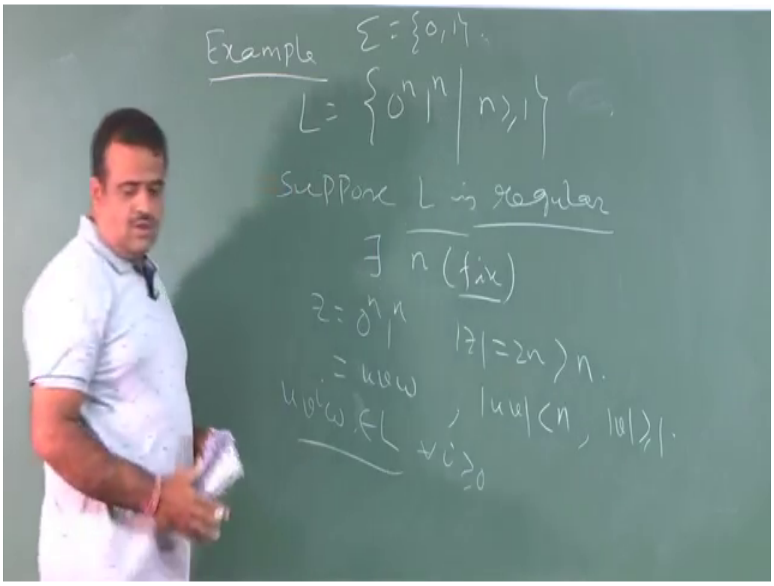
So, now this has to belongs to L, because that is the necessary condition from pumping lemma. So, for that this has to be a length of this has to be a perfect square.

(Refer Slide Time: 10:52)



But that is not possible why? Because it is lies between length of this is lies between n square and n plus 1 square. There is no number over here which can give us the length of this to be a perfect square; so, that is the contradiction; so, that is the contradiction that u v square w does not belongs to L. So, this is the contradiction contradictions. So, by pumping lemma we can say this imply L is not regular straighter you can say that. This is the use of pumping lemma we can state away say L is not regular we can take another example.

(Refer Slide Time: 11:45)

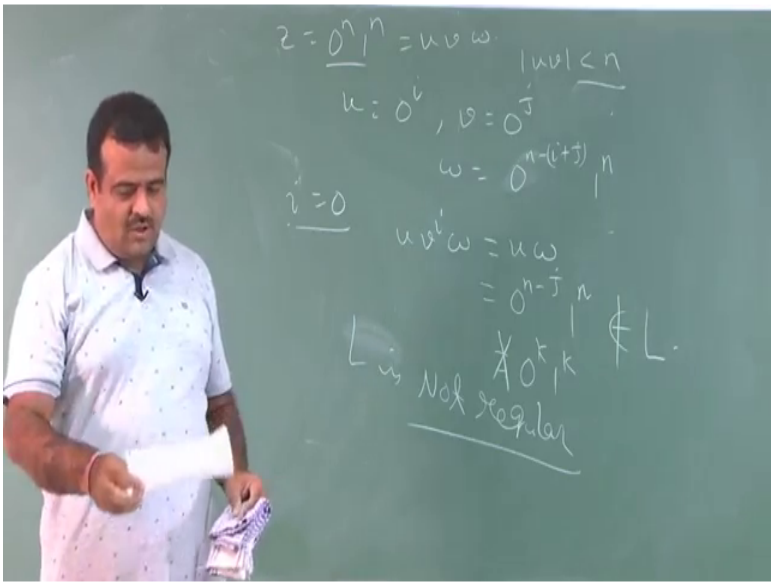


We will check 0 to the power n 1 to the power n, this we have seen earlier also where n is greater than equal to 1 is regular or not this is sigma is 0 1 ok. So, we have to check whether this is regular or not. So, again by similar way, suppose this is regular suppose L is regular. Now if L is regular then there has to be a n, we fix that n such that if we take any string from this L whose length is more than L that z, then there has to be an n there exist a n we fix that n, which is nothing, but the number of state in that finite state machine we fix that.

Now, if we choose a z whose length is more than n, suppose we choose z to be this whose length is 2 n, so, it is more than n. So, length of this is 2 n which is more than n. Now z has to be written as u v w, such that mod of u v is less than n and mod of v is greater than 1, length of this mod means length and this is true for all such z where length is greater than n that fix n.

So, now this pumping lemma is telling us u v to the power i w must also belongs to L. So, these we have to check and this is true for all i. So, this you have to check whether this is happening for some i or not ok. So, how to check that?

(Refer Slide Time: 14:05)

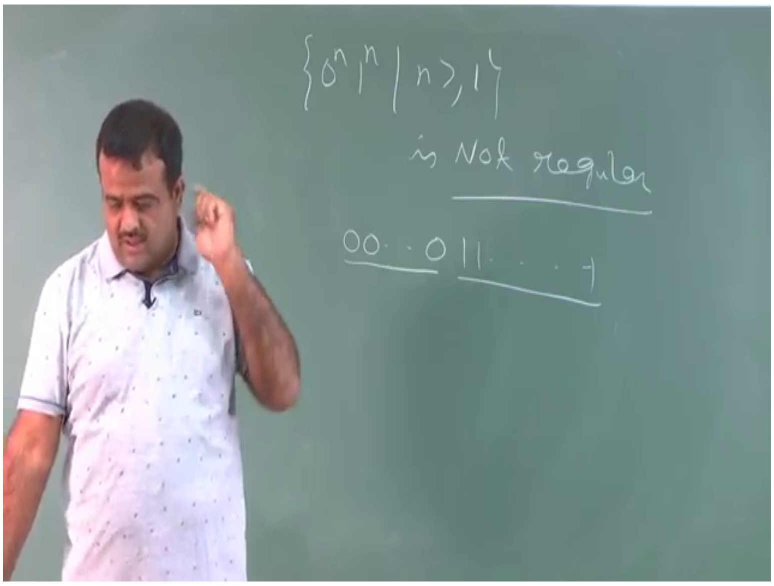


So, let us, so, let me write it here. So, z is 0 to the power n 1 to the power n in case u v w. So, u v is less than n so; that means, u v is of the form. So, u will be 0 to the power i something like that and v will be 0 to the power j like this and w is having w is. So, 0 to the power n minus 0 to the power n no one part is there. So, 0 to the power n minus i minus j or I plus 1 and 1 to the power n this is our w, but anyway we are not much worried about w.

Now, if you take some i say i is equal to say 0 then what is this expression? U v to the power i w. So, i is 0 means this is missing this is u and w. So, this is nothing, but; this is nothing, but 0 to the power n minus j 1 to the power n and this is supposed to be is n, but this is not of the form 0 to the power k 1 to the power k so; that means, this does not belongs to L, because L is of the form 0 to the power n 1 to the power n. So, that is the contradiction.

So, this has to be true for all such strings. So, this is the contradiction by pumping lemma we can say L is not regular; L is not regular ok. So, we can have some more example also.

(Refer Slide Time: 16:32)

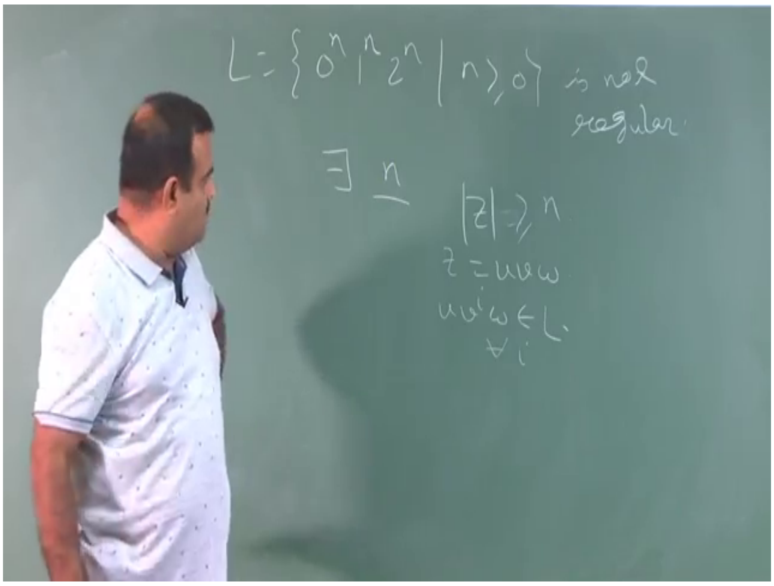


So, we can have. So, this 0 to the power n, 1 to the power n is not regular. So, there is the we do not have any DFA which will accept this why it is true? This is not regular 0 to the power n and 1 to the power n its not regular.

That means there is no finite automata which will accept this. This we have seen by pumping lemma, but there is a another way we can convince this because this is what? This is accepting all the string which is number of 0's is same as number of 1's number of 0's is followed by number of the number of. So, first we have number of 0's 0 0 0 something and 1 1 1 something. So, now this number this number is same, but in the automata finite automata. So, far we have seen has no way we can just store this number how many times we are encounter is 0, how many times we have encountered 1.

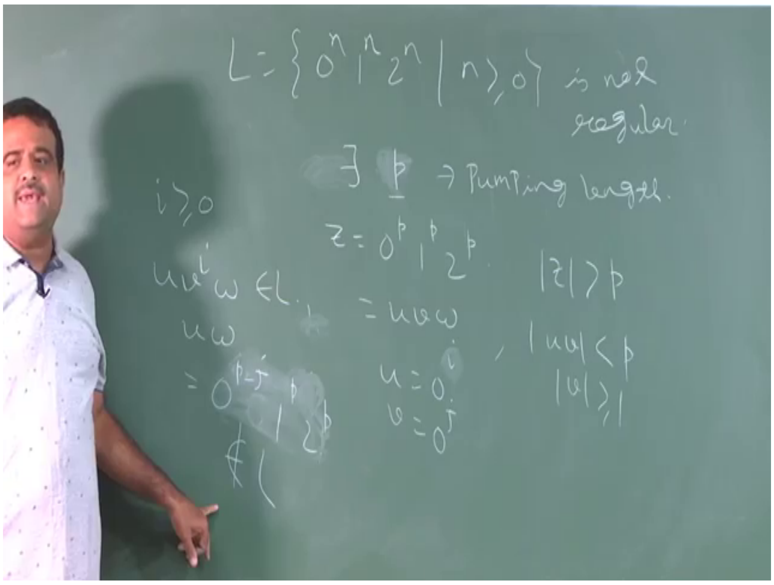
So, that is not so, far we have seen. So, we do not have memory there anyway we will talk about this on more details when we talk about push down automata ok.

(Refer Slide Time: 17:41)



Similarly, we can prove this also 0 to the power n, 1 to the power n, 2 to the power n here n is greater than 0 is not regular is not irregular. So, how to prove this? So, similarly we can prove this. So, suppose this is regular then there exists a n such that is the that n is the number of state in that finite automata, such that if we choose a z which is length is more than n then z we can write is as this then u v to the power i w must belongs to n for all i. So, this is the necessary condition of a language to be regular.

(Refer Slide Time: 18:53)



Now, here we can choose this, suppose there exist a n and we choose a z suppose there exist a k n is n may be k, but we can take n also does not matter or we can take p this is called pumping length then we p is nothing, but the number of state of that automata which is accepting this.

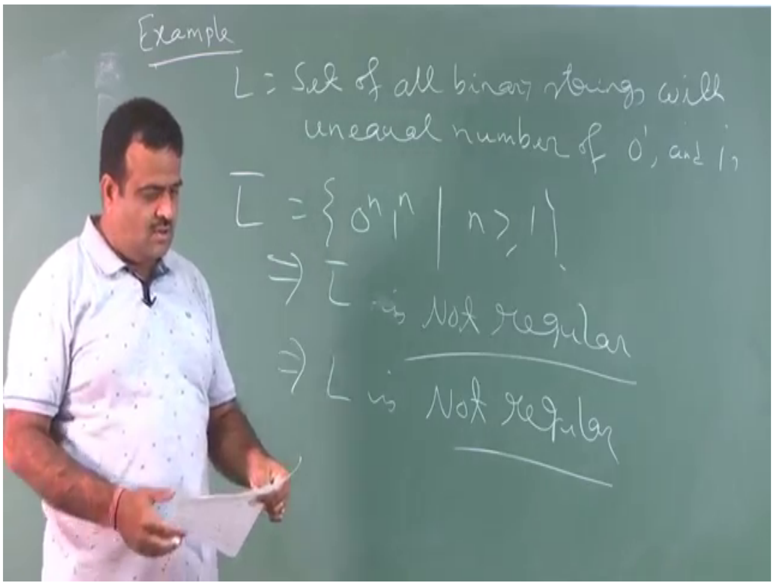
So, we are assume this is regular. So, now, we have to choose a z which is more than p. So, if you take 0 to the power p, 1 to the power p, 2 to the power p. So, this length is more than p 3 p which is more than p. So, now, we should have this we should have what? We should have u. So, this should be written as u v w, where mod of u v must be less than p and mod of v must be greater than 1 ok.

Now, in this case; so, u will be of the form 0 to the power i or 0 to the power say yeah i v will be of the from 0 to the power k j and w is like this. Now we know u v to the power i w for all i greater than equal to 0 must belongs to L, now in particular if you take I is equal to 0 this is u w must belongs to l. So, what is u w? U w will be 0 to the power i and w is nothing, but w has p q also. So, some 0 part will be there 0 to the power p minus i plus j 1 to the power.

So, this if we add it out it will be 0 to the power p minus j and then 1 to the power p 2 to the power p. This does not belongs to L because L is has to be of this from this. So, there is a contradiction, if it is a regular language this has to be belong still for all such things all such i. So, this does not belongs to l, so, this imply L cannot be a regular language.

So, that is also coming from pumping lemma ok. So, we take some more example and we will use some properties of the regular language to show that the language is not regular.

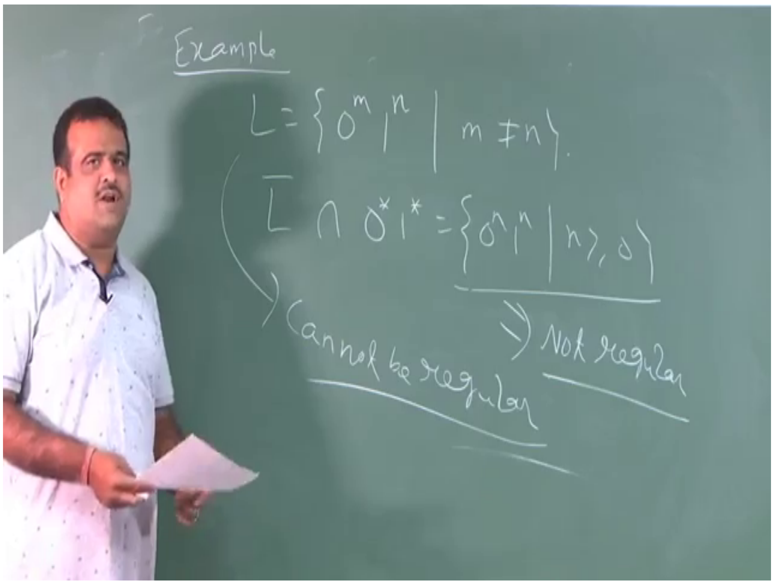
(Refer Slide Time: 21:38)



So, for example, suppose we have a language L set of all binary string with unequal number of 0's and 1's with unequal number of 0's and 1's. So, this will so, this is also not regular set this is not a regular set. So, how to show this? Either we can directly apply the pumping lemma or we can so, using some properties of the regular set. So, what is the complement of this? Set of all binary strings whose length of 0's and length of 1's are not equal.

So, what is the complement of this? So, complement of this is nothing, but length are equal ok. And we know a this is not regular just now we have seen we know this is not regular. Then this imply L is also not regular, because if L is regular then L prime is complement is also regular that is the property we have seen. So, that is the contradiction. So, if; so, this is sometimes we do it by this way in instead of the directly going to the pumping lemma, we know some result we know this is not regular and this is the complement of this. So, then this is will be also not regular ok. So, we take another example ok.

(Refer Slide Time: 24:04)



Say suppose we have some language L, which is nothing, but 0 to the power m 1 to the power n, where m not equal to n; m not equal to n. So, now we want to check whether this is regular or not. So, now, we observe that complement of this intersection with this set, this is this now this. So, now, we are looking for whether this is regular or not.

Now, we know this is not regular and then if this is regular if this is regular then this has to be regular ok. So, but from the pumping lemma we already know this is not regular; this is not regular. So, from here this cannot be regular cannot be regular; because if this is regular then this will has to be regular because this is regular if this is regular, then this will be regular and this is the regular this is regular.

So, then we know the intersection is also regular. So, this has to be regular then this will be regular, but we know this is not regular, so, this is not regular. So, this is the otherwise we could directly try to prove using pumping lemma. So, these are the example in the lecture note, there is more example on this using the pumping lemma to show the a language is not regular.

Thank you very much.