Introduction to Automata, Languages and Computation
Prof. Sourav Mukhopadhyay
Department of Mathematics
Indian Institute of Technology, Kharagpur

Lecture – 23
Construction of Regular Expression from a DFA (Example)

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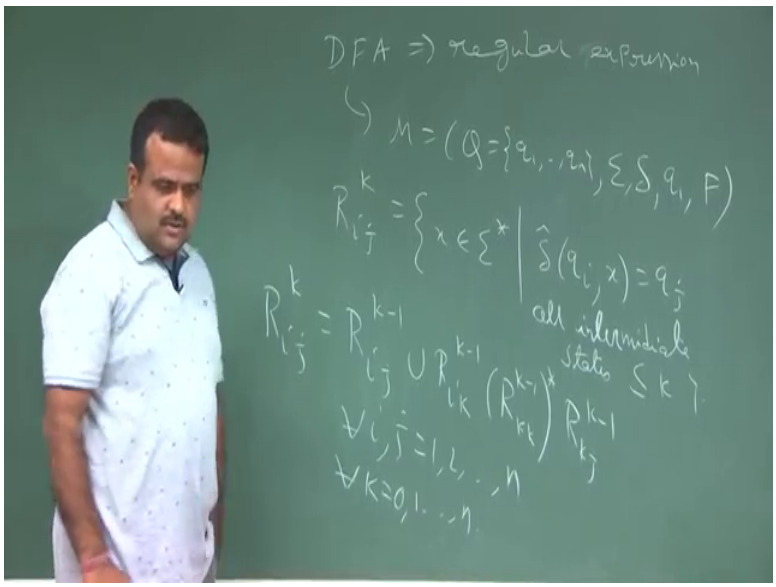

So we are talking about DFA at a regular expression. So, given a deterministic finite automata we have seen how we can construct a regular expression. So, this is just to recap q 1, q 2, q n and we have a sigma delta, here q 1 is the starting state.

So, now, just to recap we have defined R ij k which is nothing, but set of all x string such that we start with q i with x will go to q j. And no and the intermediate state we visit, all intermediate states all less than equal to k this we have seen.

Now, we have also proved this recurrence k minus 1 union of R ik k minus 1 R kk k minus 1 star in R kj k minus 1. So, this recurrence we have seen and this is true for all i and j 1 to n i also for all k n ok.

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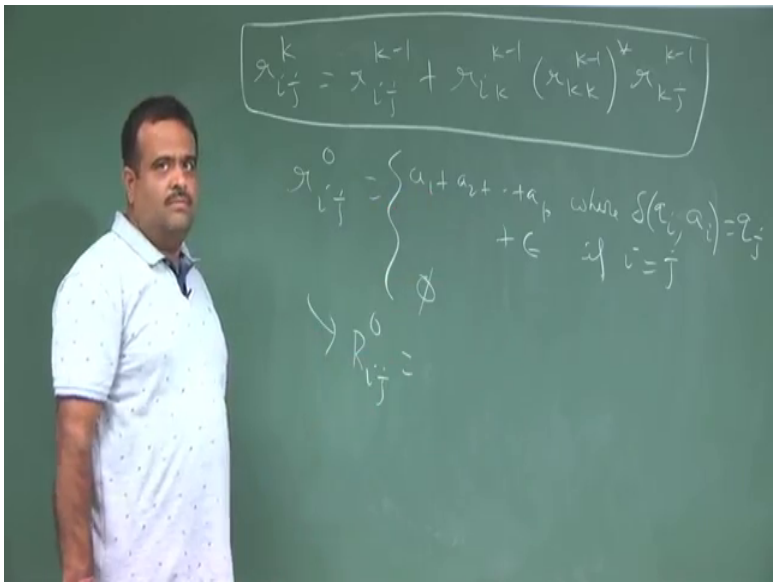

So, this part you have seen and from using this recurrence we have constructed the regular expression like this r ij k is nothing, but r ij k minus 1 plus r ik k minus 1 r kk k minus 1 star r kj k minus 1. So, this is the relation see, this is the regular expression for R ij k and this is true for all i j and k. And r ij 0 we defined as this is a 1 plus a 2 plus a p where this delta of q i, a i are going to q j ok.

And it is plus epsilon with this if these rules are there if i is equal to j and if no such a i exists then it is empty, it will corresponding to the empty set ok, because this will corresponding to the language R ij 0 and R ij 0 is same as this in a union form, this part we have seen. Now today we will now we take an example which will give us a regular expression from a given DFA.

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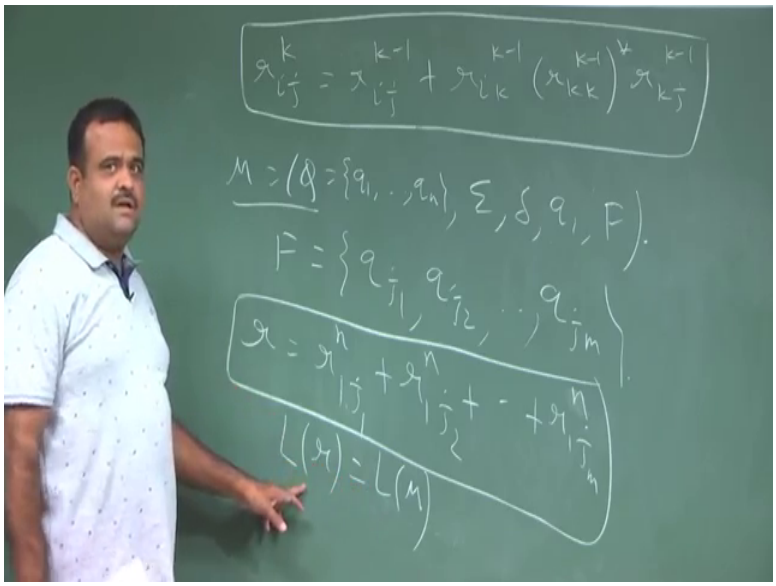

So, let us take an example to construct this, now once we have this now for the given DFA suppose this is a DFA q n sigma delta q 0 q 1 F, now F is the final state suppose F is F consists of say q j1, q j2 up to q jm. There m's final states are there then basically the regular expression which is accepting this language of this DFA is nothing, but r 1 because 1 is the starting state j 1 n plus r 1 j 2 n dot dot dot r 1jm n sorry.

So, this is the regular expression we are looking for, such that L of r is equal to L of m the language accepting by this DFA is same as the language of this regular expression. So, we will take some example. So, we will keep this.

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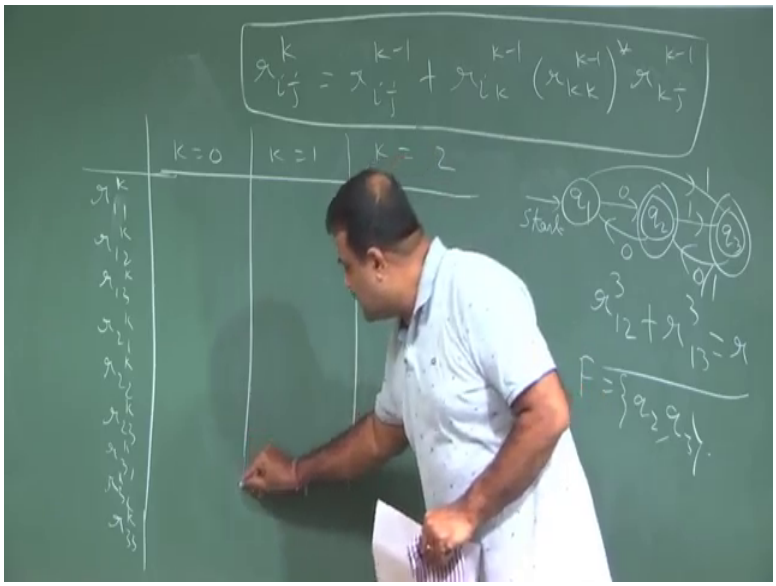

Suppose we have given this. So, we use this let us write this here, we have given this DFA this q 1, q 2 and q 3 we have 3 states and this is the starting state. So, we have with 0 you go here, with 1 we go here and with 1 will go here, with 0 you go here, i with 0 or 1 both we come here and these 2 are the final state of this DFA so, this is the given DFA.

Now, we want to construct a regular expression for this DFA. So, these 2 are final states. So, what is the regular expression? Regular expression is nothing, but r 1 is the starting state r 12 3 there are 3 states plus r 13 3. So, this is our, this we have to find out, this r we have to find out because F consists of q 2 q 3 ok.

So, let us start computing this. So, we use this formula to get this, but for start with. So, like we will put a table over here for k is equal to 0 then for k is equal to 1 and then for k is equal to 2 and this is our r ij r ij k. So, first we will compute r 11 k, then r 12 k, r 13 k then r 21 k, r 22 k, r 23 k then r 31 k, r 32 k and r 33 k. So, we compute all these terms in terms of with the k value now for k is equal to 0 this is r 11 k. So, just we can have here; what is r?

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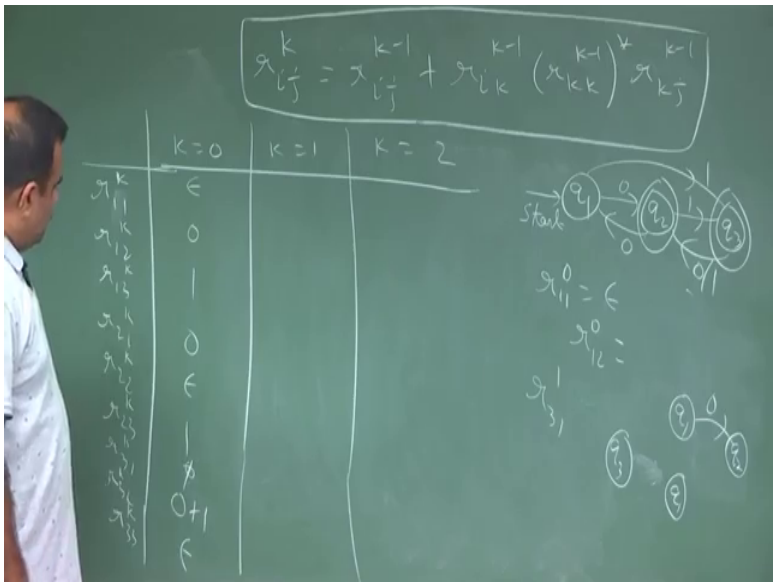

So, like r 11 0 so, r 11 0 means so, we are at q 1 will be at q 1 is there in the self loop over here. So, there is no self loop so, it will be just epsilon so, this is just epsilon. Now r 12 k so, r 12 0 so, from q 1 to q 2 is there any move yeah we have a 0 r so, it is 0 this is 0. Similarly this is 1, this is 0, this is epsilon, this is 1; now r 31 0 r 31 0. So, that means, from q 3 to q 1 is there any move from q 2 through q 1 no so; that means, this will be empty then this is 0 plus 1 and this is epsilon ok. Now, we can find out the next k.

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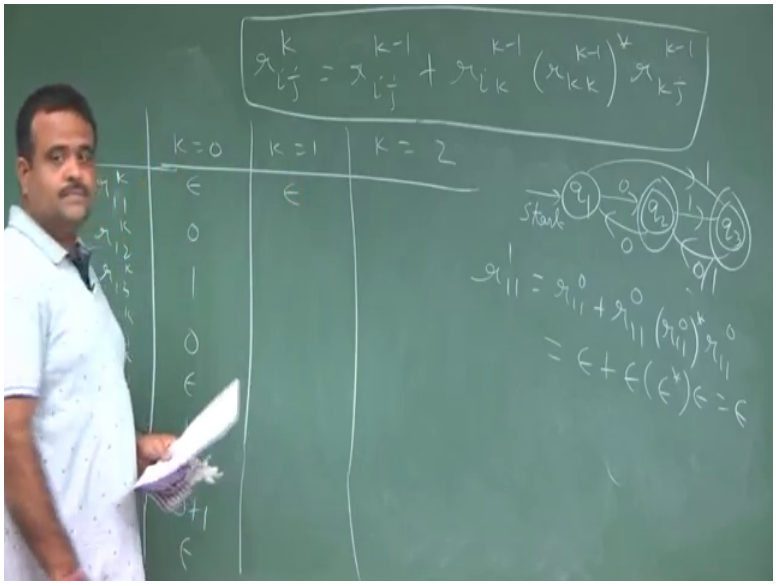

For the finding the next k, what we need to do say for example, if we want to find r 11 1. So, to find r 11 1 this 1 we need to use this formula this recurrence. So, r 11 1 is nothing, but r 11 0. So, we put k is equal to 1 over here r 11 0 plus r 11 0 r 11 0 star then we have r 11 0. So, if we use this value r 11 0, epsilon all are basically epsilon over here. So, epsilon plus epsilons epsilon star then epsilon. So, this will give us epsilon so, this is epsilon ok.

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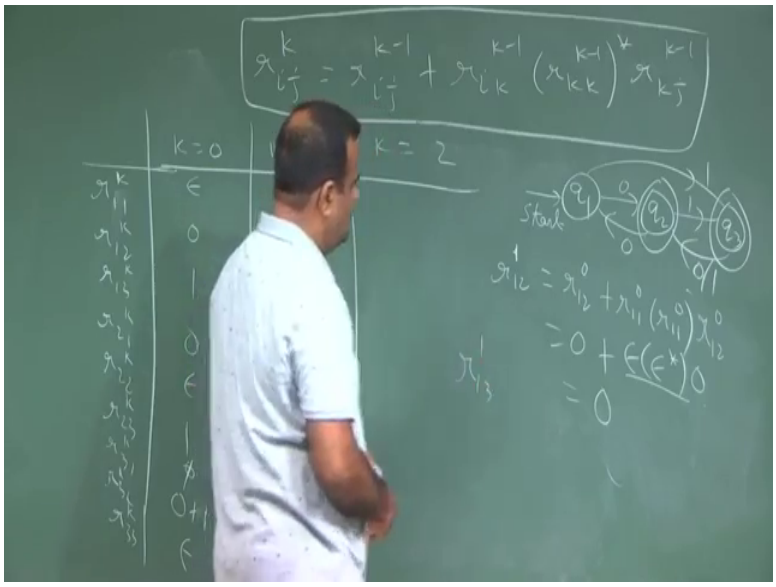

Similarly, if we have say r 12 1 this one, r 12 1 is nothing, but if you use this formula r 12 0 plus r 11 0 r 11 0 the k is 1 so, r 11 0 then r 12 0. So, if you put this value this is 0 plus epsilon epsilon star and r 12 0 r 12 0 is 0. So, this will give us a 0 because this is epsilon this is 0 so, 0 plus 0 union 0. So, 0 plus 0 is 0 we are using that those rules. So, this is basically 0. Now, similarly r 13 we can compute, r 13 1 yeah we can compute here.

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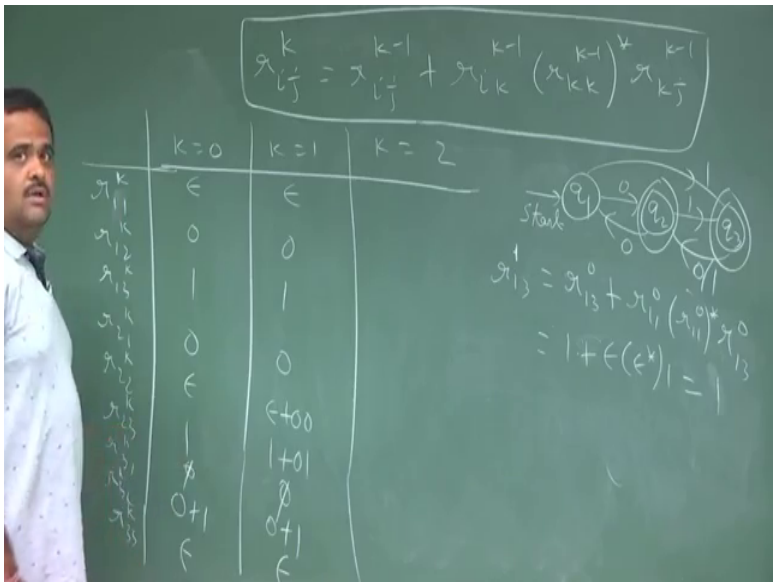

So, r 13 we can compute like this. So, r 13 0 plus so, k is 1. So, r 11 0 then k is 1 r 11 0 star then r 13 0. So, if we put this value will be getting 1 plus 1. So, 1 plus epsilon epsilon star 1 this is 1. So, this is 1, similarly we can continue this and implicating this value. So, 0 0 0 0 0 1 plus 0 1 you can easily verify this 0 plus 1 epsilon so, this part is done. Now we will use the will do it for k is equal to 2. So, once we do it for k is equal to 2 we need to use the formula recursion formula.

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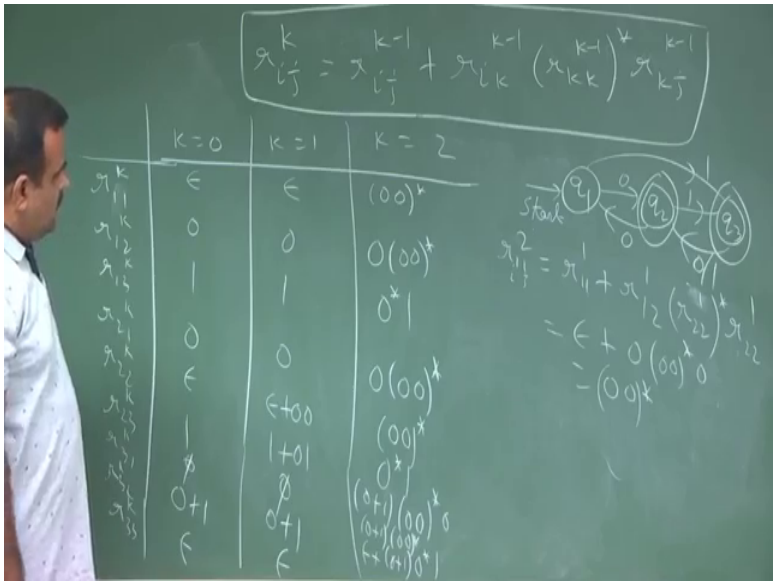

So, for example, r 11 2 so, r 11 2 again we can have say yeah r 11 2 do it. So, this is again epsilon because this is again if we use the formula r 11 1 plus. So, this is I, this is j, this is k. So, r 11 1, then r 12 1, then r 22 star sorry r 22 1 star, then r 22 r 22 1, if you use this table r 11 this is epsilon plus r 12 1 it is 0, then r 22 r 22 is 0 plus 0. So, it is 0 0 star then 0. So, this will give us 0 0 star ok. So, this is nothing, but this table you can fill this is 0 0 star.

Similarly, here also you can compute 0 0 star, this we have to work out 0 0 star 1, 0 0 0 star, then 0 0 star, 0 star 1 and 0 plus 1 0 0 star 0, then 0 plus 1 0 0 star, the last one is epsilon plus 0 plus 1 0 star 1 ok. So, this is the up to k is equal to 2. Now, what is our regular expression, our for regular expression we need k is equal to 3. So, for that we need to calculate this 3 for k is equal to 3. So, finally, so, this is after completing all these.

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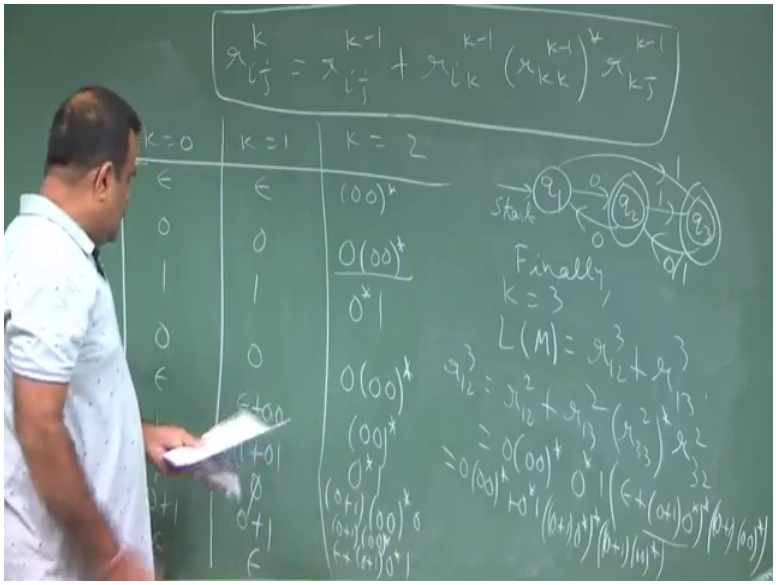

Finally, our answer is for k is equal to 3 L M equal to r 12 3 plus r 13 3. So, we need to find out these two. So, what is r 12 3. So, this you have to get. So, just use this so, r 12 3, r 12 3 is nothing, but if you use this formula. So, r 12 2 plus then k is equal to 3. So, r 13 2 we are just using this formula then r k is equal to 3 r 33 2 start then r k is equal to 3 j is equal to 2 r 32 2.

Now, if you put the value over here r 12 3 is this one r 12 2 is this 1 like this. So, if you put the value over here we are getting 0 0 0 star then 1 3 2, 1 3 2 is this one, 0 star 1 and

then r 33 r 33 2 this one this big expression. So, this is nothing, but epsilon plus 0 plus 1 then 0 star then whole star, then r 32 2 r 32 2 is this one it is in the bracket 0 plus 1 and then 0 0 star

So, if you simplify this we will be getting like we can do little simplification 0 0 0 star will be remain same and then plus 0 star 1 then this epsilon this will come, then 0 plus 1 0 star star and then 0 plus 1 0 0 star ok.

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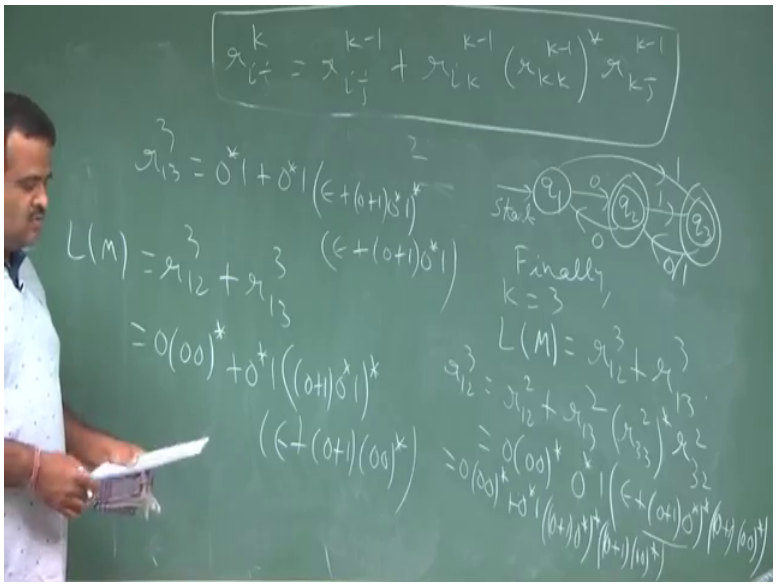

Similarly, we can find out r 13 similarly we can find out r 13 3, using that formula it is nothing, but 0 star 1 plus 0 star 1 epsilon plus 0 plus 1 0 star 1 star then into epsilon plus 0 plus 1 0 star 1 ok. So, this plus this, our L M, L M is r 12 3 plus r 13 3. So, this plus this our L M.

So, if you simplify these two will be getting 0 0 star 1 0 plus 1 then 0 star 1 star into epsilon plus 0 plus 1 then 0 0 star, maybe you can simplify a little for that. So, this is the final regular expression for this automata so, from a given DFA we can use the recursive formula to get a regular expression. Now, we will take some simple example this is I wanted to show you that if we have 2 state of acceptance to acceptance state then we have that r 1 3 and r 23 3.

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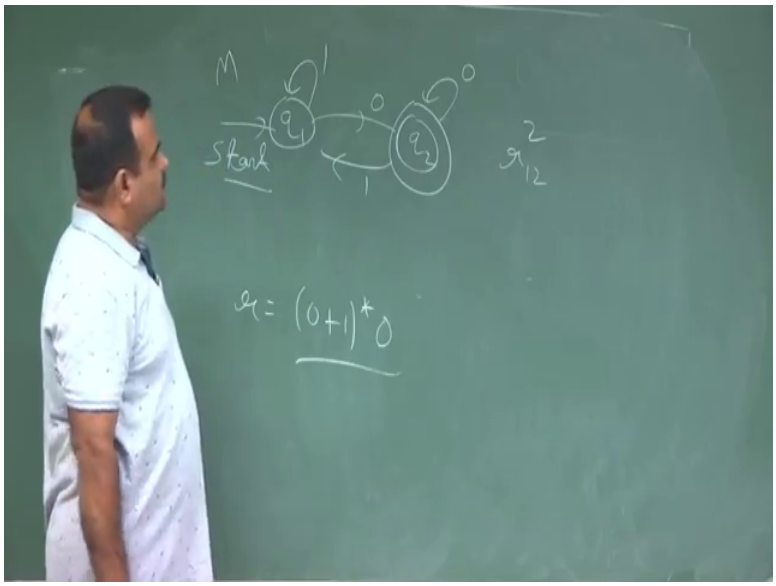

Now, if you have simple one like if you have this q 2. So, if we have a 0 you go here, if you have 1 we remain here and from if you have a 0 you go here, 1 will remain here. So, this is our starting state and we have 1 final state ok.

Now, what is the language accepting this DFA, we know this language accepting this DFA is the ending with 0. So, this is M L of M is nothing, but x 0 when x is belongs to x is any string. So, this is any string ending with 0 that is the language and this we can write in regular expression like this 0 plus 1 star 0 this is our r ok, but now these are we need to get it be using that formula. So, I just give you the outline so, what is. So, if we use that so, this is 1. So, basically r is will be coming from r 12 then there are 2 state r 12 2 ok. So, r 12 2 for getting our 1 to 2 what we need to get we need to get all the r so, this we have to achieve.

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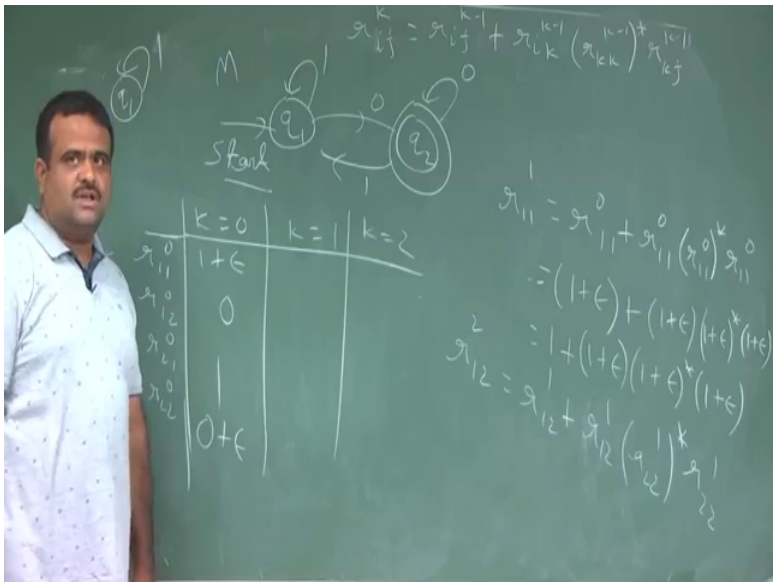

So, this is our k is equal to 0, k is equal to 1, k is equal to 2, k is equal to 2 is the final one and this is r 11 0, r 12 0, r 21 0, r 22 0. Now, what is r 11 0, r 11 0 is so, r so, q 1 so, it is 1 plus epsilon because from q 1 you have to go to q 1 you have a 1 1 over here. So, 1 plus epsilon and r 12 0, r 12 0 is just 0 and r 21 0, r 21 0 is 1 and r 22 0 is 0 plus epsilon now this one you need to fill. So, to fill this one where we need to use the formula like r ij k is equal to r ij k minus 1 plus r ik t minus 1 r kk k minus 1 star r kj k minus 1.

So, then what is r? So, we want to find r 11 1. So, r 11 1 is basically r 11 0 plus k is 1 r 11 0 r 11 0 star then r 11 0. So, if we use this so, this is our 1 plus epsilon r 11 0 then 1 plus epsilon. So, r 1 k is equal to 0 i is equal to both 0. So, 1 plus epsilon star. So, 1 plus epsilon 1 plus epsilon star 1 plus epsilon. So, you can all simplify this. So, this is give you this is this will give us. So, 1 plus epsilon means 1 plus this one. So, we can have 1 plus this. So, 1 plus epsilon like this.

So, we can simplify this and we can compute all the values over here, all the values over here and once we have computed all the values. So, we do not need to compute all the values over here we just come you need to compute r 12 2. So, r 12 2 is nothing, but if you use the formula r 12 1 plus r 12 1 then r 22 1 star then r 22 1. So, this we can use this table to get this value. So, I want you to do this as exercise ok.

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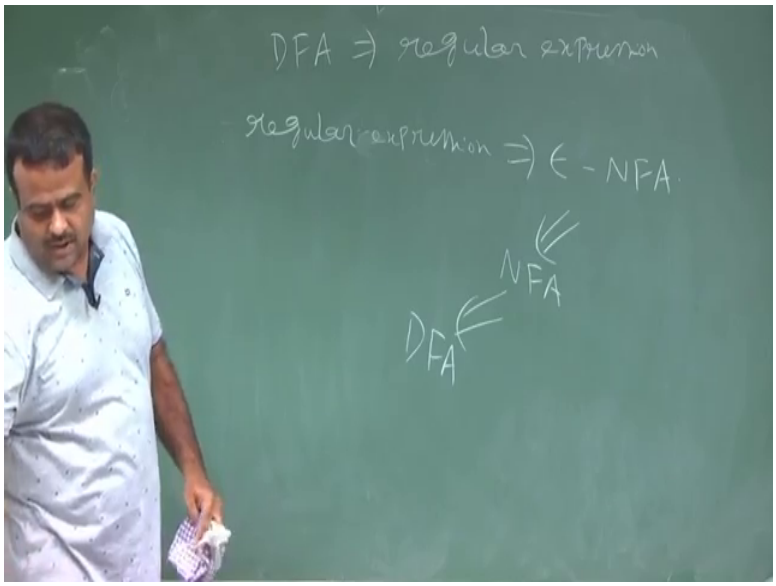

So, for what we have. So, we know that given a DFA we can have a regular expression and we know, given a regular expression, given a regular expression we can construct the epsilon NFA and from this epsilon NFA we can construct a NFA or directly we can construct a DFA, this is some subset construction. So, basically given the regular expression we can have a DFA, given the DFA we can have a regular expression so, this we can write it here.

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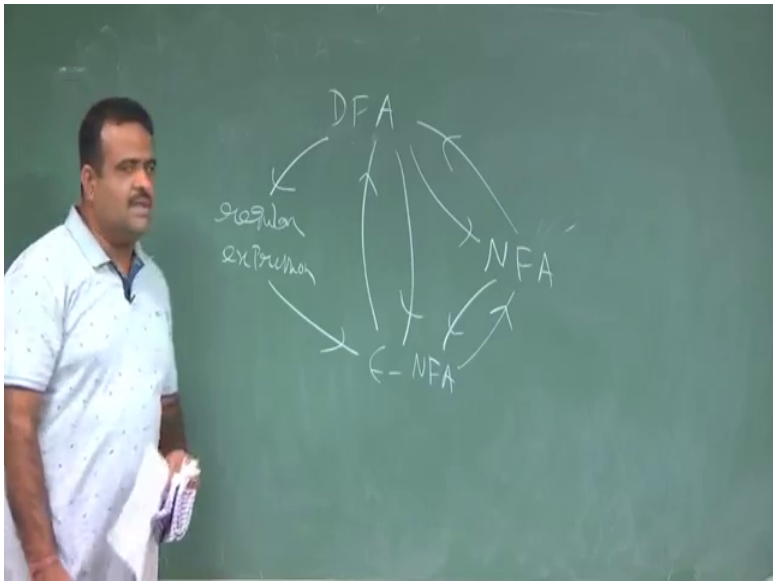

So, this is DFA so, from DFA the regular expression, then NFA and here we have epsilon anything. So, from DFA we can have a regular expression and from regular expression we can have NFA and from NFA we can have epsilon sorry from regular you can have epsilon NFA or from the epsilon if we can directly have a DFA and DFA is can be treated as NFA and DFA can we do this as epsilon if you all these are true and epsilon NFA is basically an epsilon in effect. So, this is the diagraph ok. So, given a DFA we can have a regular expression, from the regular expression we can construct epsilon NFA which is same as construct in the NFA all these things.

Thank you.