Introduction to Automata, Languages and Computation
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Lecture – 02
Input Alphabet

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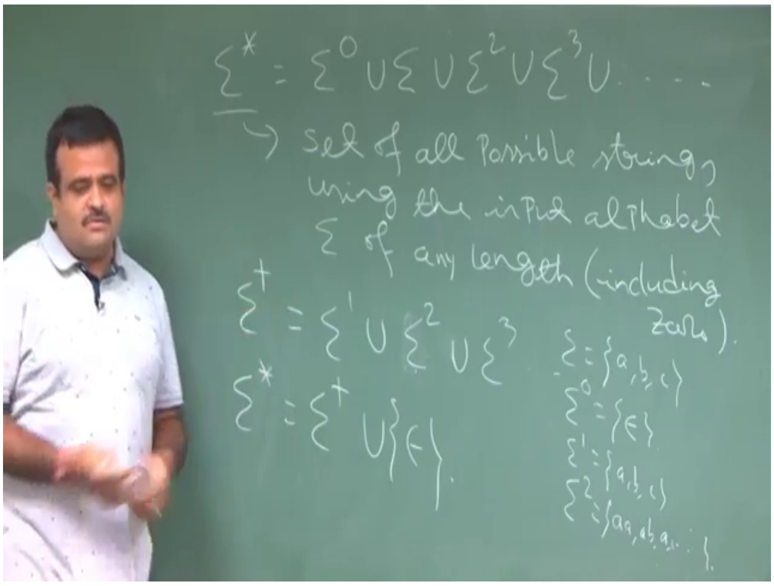

So, we are talking about the string of alpha string, which is a so this is the collection of all string sigma star. And then sigma 0 is basically our epsilon empty string, so no occurrence of the input alphabet. And this is the input alphabet, this is the sigma square. This sigma square is means two times like if sigma is a, b, c, then sigma star sigma 0 is epsilon, then sigma 1 is this one - a, b, c. And sigma 2 is the leave all the string of length 2, so a aa, ab, ac so all the string of length 2, so this is sigma 3, then sigma 2, sigma 3 like this. So, this said sigma star is the set of all possible string of any length including the length 0.

And now we define the language. And this sigma plus is we are just not allowing the empty string. So, this is the set of all possible string, which is having a at least one symbol in that string. And this star is including the empty strings, so star is nothing but we can have a 0 occurrence also. The empty string and the string of any length including 0, so that is our sigma star.

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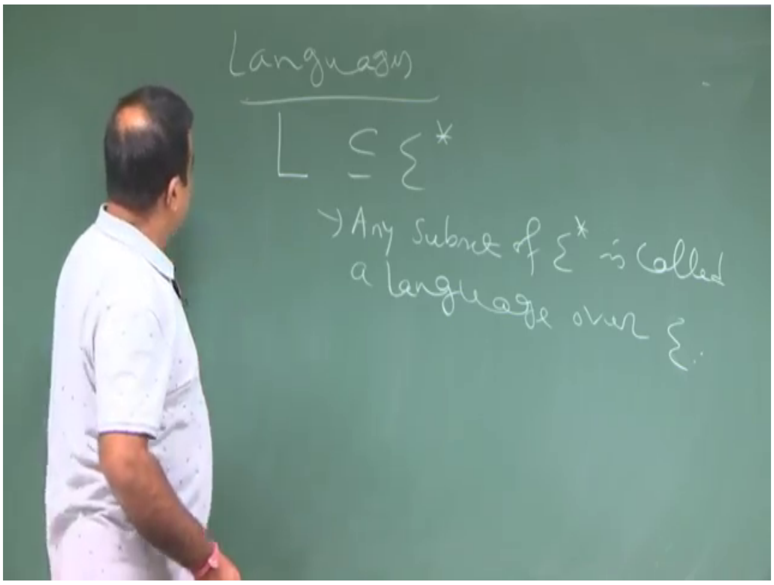

And now we define the language. So, language is any subset of sigma star is called language, it is a basically collection of some strings. So, any subset of sigma is different as a language of the language of languages. So, it is denoted by L, L is a subset of sigma star. Any subset of sigma star is called a is called an language over sigma over the input alphabet sigma. So, it contains the it can be 0 string also I mean null string also.

So, so language is the some set of string, it could be infinite also. But, this is finite the language, this could be infinite number of string, I mean length could be infinite, but it is coming from finite symbol finite input symbol. So, input symbol is finite, but it could be infinite string also.

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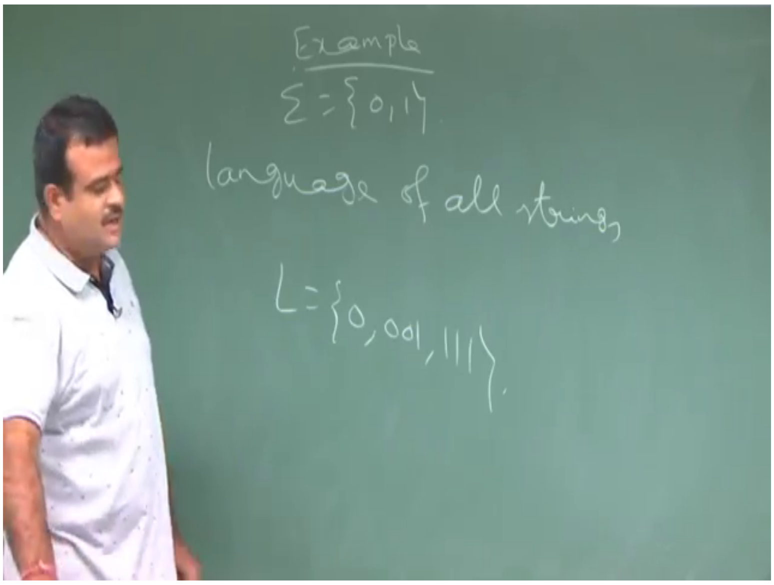

So, now we take some example of languages. Example of (Refer Time: 03:29) sigma is 0, 1 binary alphabet. This set is always finite ok. Our input alphabet is always finite or a string could be we can think of infinite string ok. So, we can define a language like this, language of all string. So, language is a collection of strings basically, so here alphabet is 0, 1 is the collection of string. Anything could be a language, so this is a language 0, 001, 111. This is a language, this is subset of sigma star, so any collection of string is a language.

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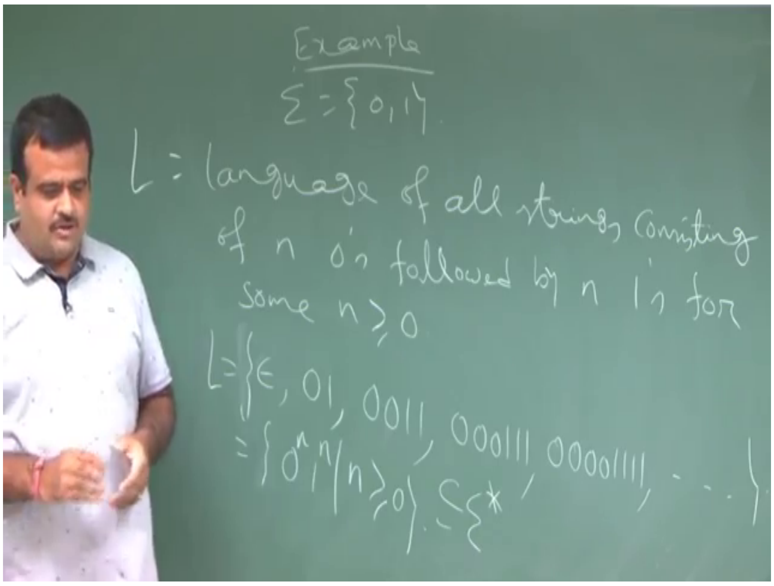

So, now you have we will take some example of a language, which is having some properties like language of all strings consisting of n 0's followed by n 1's for some n greater than equal to 0 ok, so this we have to understand. So, we defined a language like this.

So, we know the language is a collection of the string. Now, what is the property of the string, the property is it should followed by it should have a n 0's, then the followed by n 1. So, number of 0's, number of 1's are same, I mean not so like it should say 1 0 1 1, this is belongs to L, 2 0 then 2 1 this is also belongs to L, three 0's three 1's this is also belongs to L, four 0's four 1's this is also belongs to like this. And no 0's, no 1's that is the that is our empty string, so this is our L this is our L.

So, these things what we can write, it is 0 to the power n 1 to the power n n greater than equal to 0. So, this is the string of this form. 0 to the power n, we have n 0's followed by n 1's, and n could be 0 also that means, no occurrence of this. So, this is also subset of this is a subset of sigma star. So, any subset of sigma star is a language, this is one example of a language.

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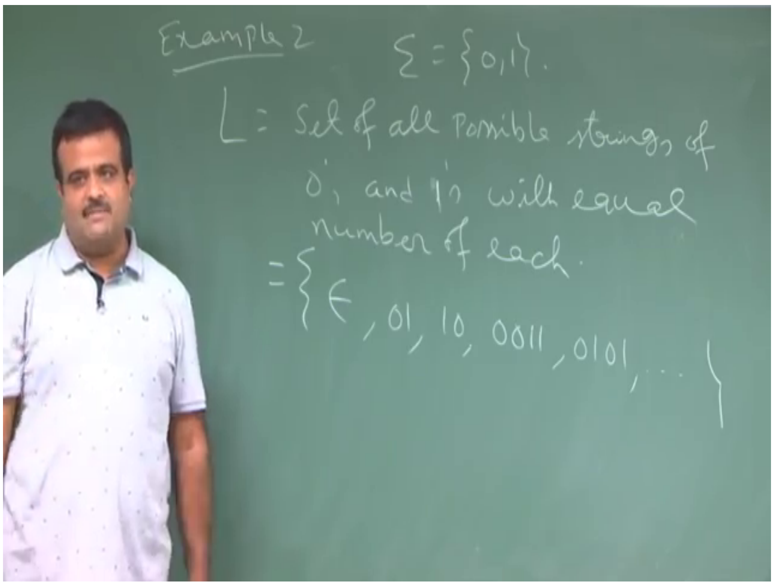

Now, we will take make other example like set of all string, we defined a language such that it is a string, when number of 0's and number of 1's are same. We count the number of 0's and we count the number of 1's, these are same.

So, this is second example two of language is another example. So, our language is or sigma is 0, 1 binary input. So, binary input and our language is a set of string, set of all possible strings, set of all possible strings of 0's and 1's, I mean 0's this is coming from 0 as 1 0's and 1's with equal number of each equal number of each that means, we have equal number of 0's and equal numbers of 1's. So, those strings if we put in a back, that bag is our L the language. This is the way we are defining a language. So, this is consist of so null that means, no number of 0's, number of 1's are both 0's. No 0's, no 1's, so that is the null string.

And we have 0 1, 1 0, then 0, 0, 1, 1 in any of this places it could be like 0 1 0 1, so number of 0's, number of 1's must be same, we do not bother about. So, any combination of, so there are two 0's, two 1's any linear permutation of this will be the 1 1 could be here 1 1 could be there like this so this way so this is the another example of this. Another example could be like this, we can have a string, which is prime number. So, we have these are binary num binary we convert into decimal, so that should be a prime number.

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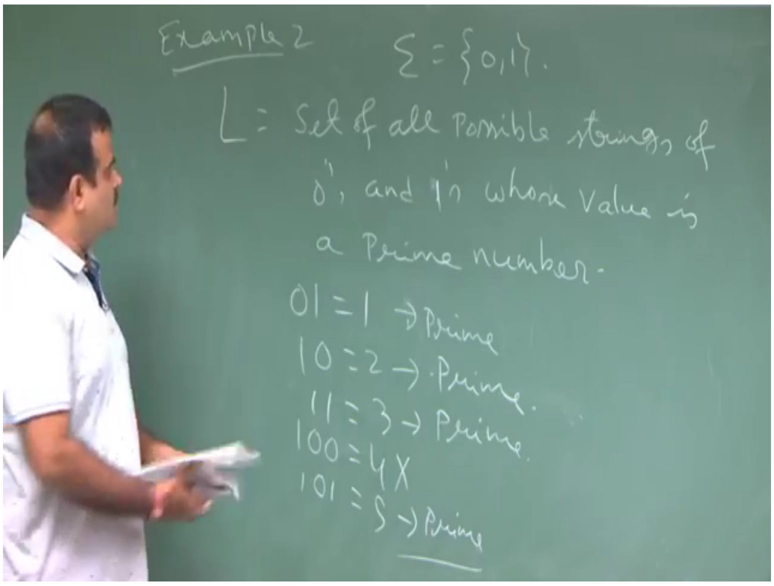

Set of all possible string of 0 1's, 0's and 1's whose value is a prime number 0 and 1 whose value is a prime number. So, here the null string will not come, [laugher] because it has to be a number. So, for example if you take 0 1 0 1 is value is 1, so 1 is a prime, so this is a prime.

If you take 1 0, this is 2, this is not a prime. Sometimes people say 2 is a the even prime, but here we are defining the prime as the which is divisible by 1 and itself, but sometimes 2. But, yeah 0 1 1 0 1 0 2 2 2 is to treated be a prime, because 2 is also 2 is the only even prime ok.

And then say 1 0 1 1 0 1 is nothing but or 1 1 if we have 1 1 1 1 is 2 plus 1 3, this is also a prime. Then 1 then 1 0 0 1 0 0 is 4, 4 is not a prime, because 4 as a factor 2. Then 1 0 1 1 0 1 is 4 plus 1 5, it is a prime number. So, this is an example, I mean the value should be a prime. So, this is nothing but consist of so all the sequence of 0 1's, where value is a prime.

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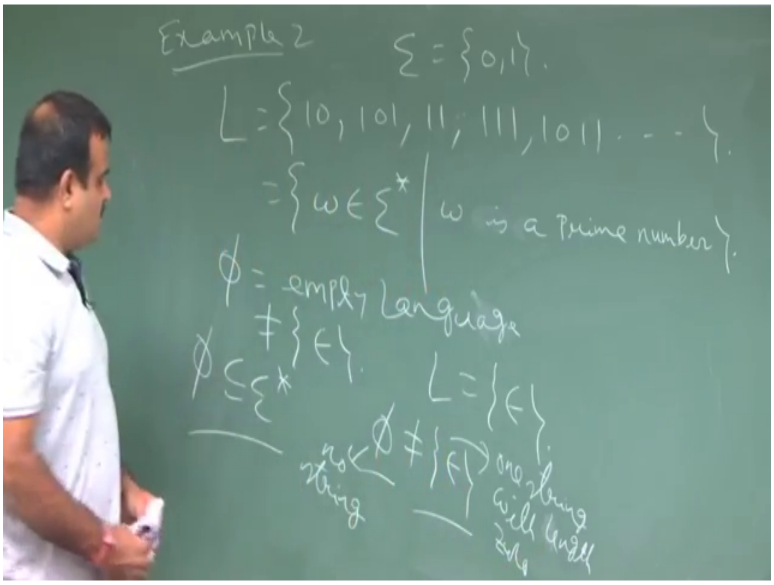

Like we have 1 0, 1 0 1, 1 1, 1 1 1, 1 1 means what this is 4 plus 2 plus 1 7, 7 is a prime number, 1 0 1 1, 1 0 1 1 means, this is 8 plus 2 10 plus 1 11, 11 is a prime number. Prime number means, which is having a factor which is divided by itself or one; so, this set is all language. So, this is a set of all string, which is so this we can write like this set of all string coming from this such that w is a when you convert w in binary, I mean the integer, then it is a prime number. We has w is a prime number in value ok, so that is the meaning of that.

And the phi, phi is the empty language, so it is empty language empty language, but phi is not same as epsilon. Epsilon is a language of length 0, but phi is no input is there. It is a same empty language, because phi is a also a subset of sigma star. Any subset of sigma star is a language, so phi is also a language, phi is language means empty language.

But, if we say L is equal to this, so this is a language this is the lang m this is a language consist of empty string, so these two are not same. So, phi is not equal to this. So, phi is empty, and this is consist of a language, this is consist of a string with 0 occurrence. So, these two are different we have to be little careful about this. So, this means this means the no string, and this means one string with 0 input with length 0. So, empty string one string with length, so these two are not same ok, so this you have to define.

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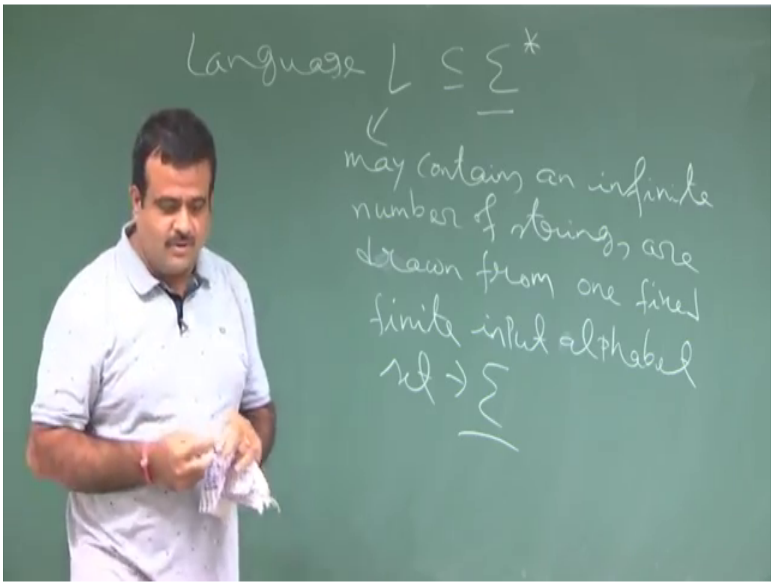

So, now we will talk about something on this some problem based on this, which can be handled by a finite automata. So, before that let us just talk about this language. So, as you said language, which is denoted by L, which is a subset of sigma star.

So, these may contain, because sigma star is a collection of all possible string of any length starting from 0 length to infinite number of length. So, these contained this may contained an infinite number of string infinite number of string, but the string consist of the symbol input symbol that is finite string, which is coming from strings are drawn from, this is important. Our input symbol is finite, but we can make it infinite string of length from one fixed finite input set input alphabet set, and that is our sigma. The sigma is finite, but this is infinite this I mean length could be infinity ok.

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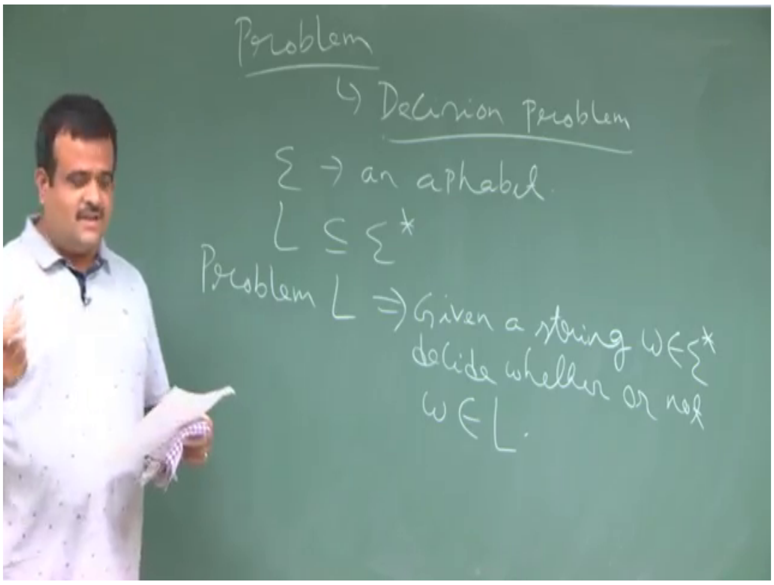

So, now we will talk about some problems like decision problems. Like we have we have given a integer, given is 0 on string, and you want to know whether this is a prime number or not this type of problem. So, this is called decision problem. So, we will talk about the types of problem like decision problem, which can be handled by the deterministic finite automata, so that would be another types of problem. This problem is called decision problems ok. It is basically a membership of a language.

Suppose, we have a sigma which is the input alphabet symbol, and then we defined a language, which is a subset of this ok. So, the problem n means given a string given a string w from these decide whether or not whether or not w belongs to L or not whether or not w belongs to L ok, so this is the that means, we need to decide so given a problem, so this is say for example, if we want to do the primary testing that means, given a given an integer, we have to decide this is prime or not.

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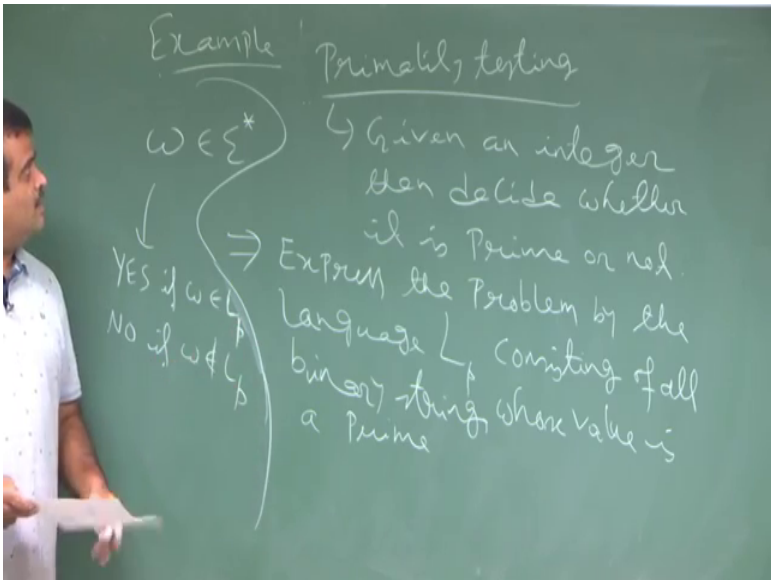

So, we will work on this one example, which is called primary testing yes or no basically primality testing. So, what is the problem, problem is we have given a integer given an integer then decide whether this is prime or not whether it is prime or not ok. So, what we can do? We can express these in terms of we can express the problem by the language express the problem by the language L p, this is prime consist of consisting of all binary string consist of all binary strings whose value is a prime binary string whose value is a prime number, so that we are defining as L p. So, L p is nothing but the set of all string whose value is binary whose value is a prime number.

So, now we have a given a w, so which is a 0 on string, which is coming from sigma star. Now, we have to output our machine should output yes if w belongs to L p, no if w does not belongs to L p. So, this is the decision problem yes or no. So, we just output this so you have to model in such a way such that we have to defer we have to have a finite automata such that it will accept only those strings, which is having which is whose value is w whose value is a prime number, so which is belongs to L p. So, this is a decision problem, so this type of problem will handle by the help of DFA.

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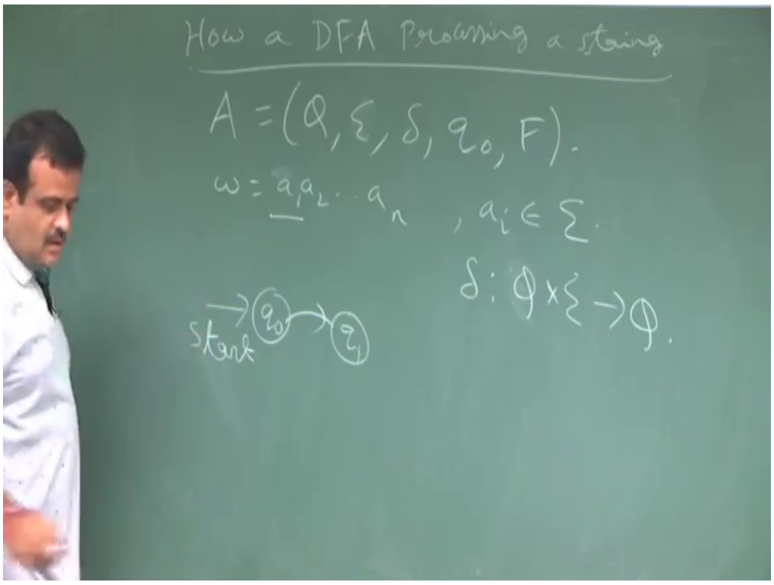

Now, we will we will see how the DFA process a string. So far we know that DFA is processing on a input alphabet. Now, we will try to understand, how the DFA is processing on the string how a deterministic finite automata processing a string ok. So, we have a DFA, which is consist of phi tuple. This is the finite set. This is the sigma, which is the input, delta, q 0, F, this is our DFA.

Now, suppose you have a string, consist of this a 1, a 2, say a n. This is a string, where this a i's are coming from sigma. Now, we will see how we can how a DFA possess on a string. We know the DFA is possessing on a DFA is the so this delta, delta is defined to be a sig so it is a rule, it is a rule. We are in current state, we take a input alphabet, it will be in the next step.

So, now here but if we are having a string, so how will possess? So, basically we have a so we start from a q 0, which is the starting state, then we have the input. We just keep on reading this, we put it into this step tape, and we keep on reading this. We first read a 1. So, a 1 means, so it will be it will be some another state q 1, so say it is q 1. So, q 1 is nothing but I and just erase this ok.

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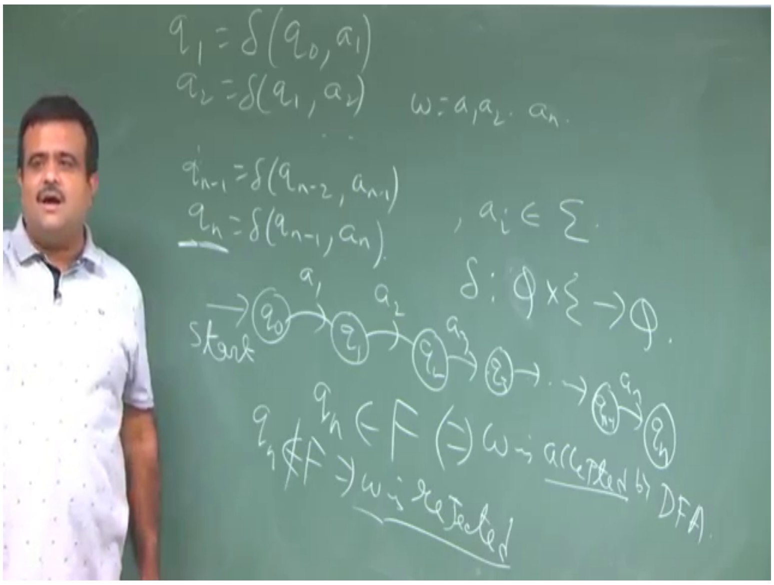

So, q 1 is so we have to start with q 0, and we our first alphabet is so this is q 1, now we are at q 1. Our second alphabet is a 2, so we read a 2, suppose it is going to q 2. So, q 2 is delta of q 1, a 2 like this. So, this way we continue, so dot dot dot dot dot, so at the so we continue like this. So, q 3, it will go to a 3, it will go to q 3 like this. So, finally we are at q say q n minus 1, and then we read a n, and suppose it is going to q n ok. So, this is so q n minus 1 is delta of q n minus 2, a n minus 1, and then at the end q n will come from q n minus 1, after q n. This some of the q may be same we do not know, but to write this as this way ok.

Now, so it is restrested q n. Now, if q n is so after reading this symbol a 1, a 2, a n, so you just read, we start from q 0, we keep on reading this, and we are changing the state based on the rule delta and at the end suppose, it is reaching to the state q n. Now, if it is the final state. If q n is final state, then we say this string is accepted by the DFA.

If this then we say, this w is accepted by the DFA ok. So, this is the meaning of the accepting by a DFA. We have given a string, we start with the starting state, and then we change the state base on based on the input alphabet. And after the end of the string like the last symbol is a n, so after a n we reach to a some state that is q n. Now, if it is happen to be q n to be a final state, then we say this our DFA is accepting this, or if it is not if the q n is not in then we say w is rejected. So, either it will accept or it will reject.

When it will accepting, if it is reaching to a final state up at the end of this execution of this symbols because, our delta is not on the string input, delta is on the alphabet input, but string is consist of the alphabet. So, we keep on changing the state reading the string. And finally if we reach to a final state which is the accepting state, then we say the string is accepted by our DFA. If it is not a final state, then we say the string is not accepted by our DFA, so that is the rejected string. So, we will continue this on more details.

Thank you.