Introduction to Automata, Languages and Computation
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Lecture - 14
Epsilon-NFA to NFA

So, we are talking about epsilon NFA to NFA.

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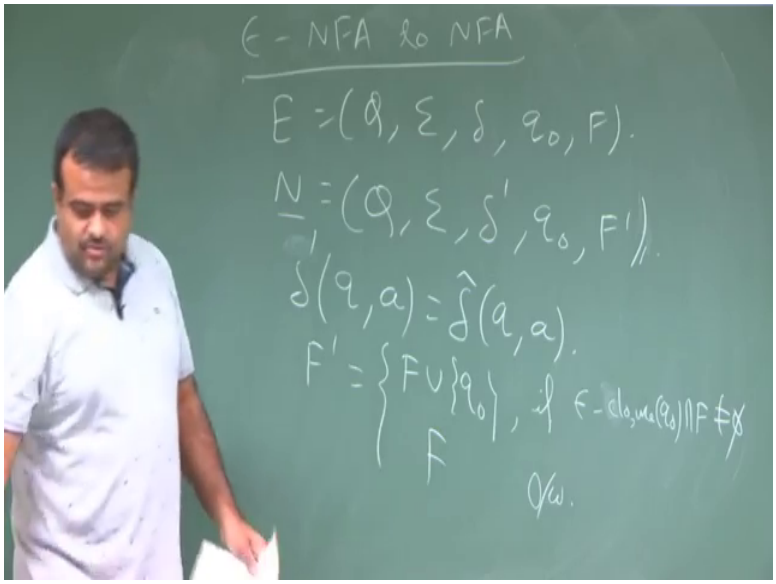

So, we take this, we will discuss two examples. So, just to recap we have given a epsilon NFA, now you want to construct a NFA which will accept the same string delta prime, q 0 and F prime. So, delta prime we are defining as sorry delta prime we are defining as q comma a.

So, here there is no epsilon move for this N. So, this is nothing, but delta hat, this delta hat of q comma a. So, we are just clubbing the all the epsilon move before a, after a in this deltas. So, that is why we use the delta hat and what is F? F is F prime, F prime is F union q 0 if, if from q 0 we can go to any state of the F just by epsilon move. If the epsilon closure of q 0 and F is not empty, otherwise it is just a F otherwise ok.

So, this we have seen now we will take some example on such construction ok. So, epsilon NFA is having the epsilon move and NFA is not having any epsilon move so that is the difference ok.

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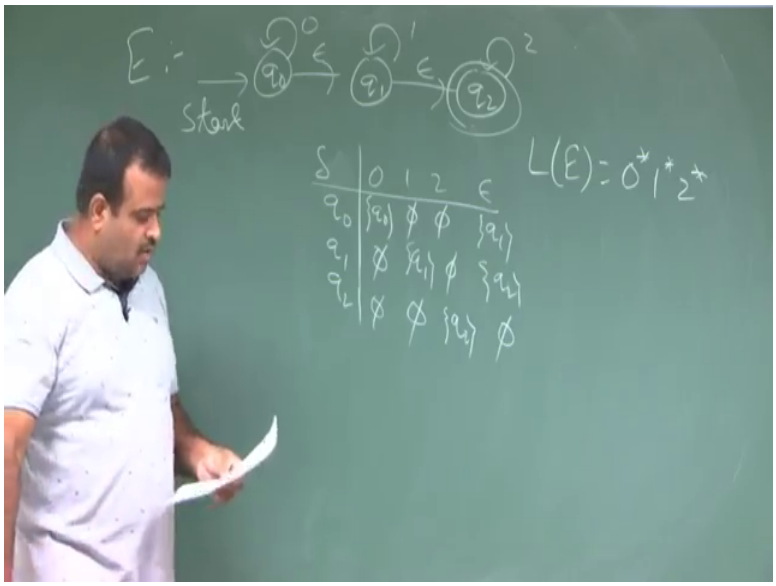

Let us go back to that same example; q 0, q 1 and q 2 so this is our 0, 1, 2 and this is a epsilon move, epsilon move this is your starting state and this is the final state. So, this is our E and we know that L of E is 0 star, 1 star, 2 star.

So, any string of any number of 0's followed by any string of any number of 0's followed by any stream of any number of 2 so that is the language by this epsilon NFA. Now, we want to construct the equivalent NFA. So, to construct so this is just we can write the rules. So, this is 0, 1, 2 epsilon is here and these are the states q 0, q 1, q 2 and this is our delta. Now from q 0 with 0 you have this, from q 1 the, we have from q 0 there is no move for one there is no move for this.

But there is a move for epsilon null string similarly this is empty, this is q 1, this is empty and this is q 2 and this is empty, empty and from q 2 with 2 we can go to q 2 and with epsilon there is no move. So, this is the transition rule for this epsilon NFA. Now we will construct the corresponding NFA. So, for corresponding NFA the state will be same.

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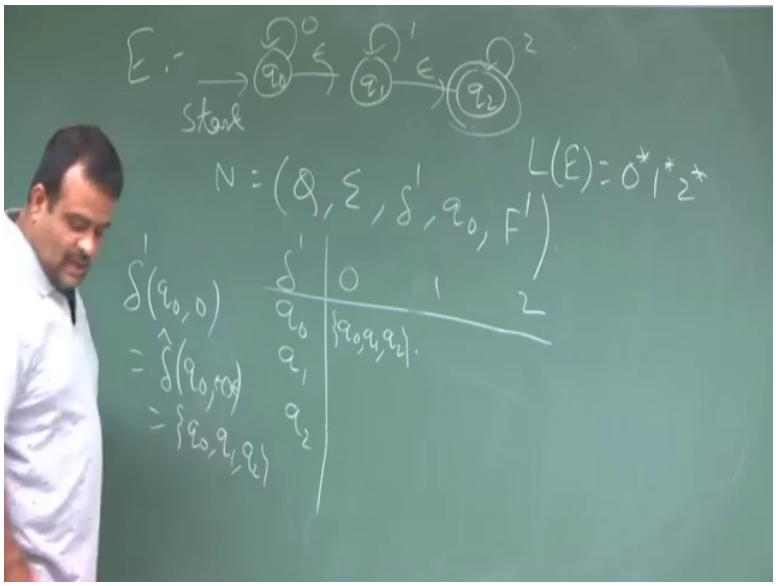

And this sigma will be same alphabet set here we are not having epsilon move delta prime this is prime and we have q 0 and F F will be changing because here with the epsilon move we can go to q 0 to q 2.

So; that means, q 0 has to be added in the this final state ok. So, how to write this? So, basically delta hat we have to define. So, here we have moved 1, 2 now here state are same q 0. So, we will write a space 0, 1 2 q 0 q 1 q 2. Now suppose we are at q 0, now with 0 so delta delta prime of q 0 0 you want to find. So, this is nothing, but delta hat of q 0 0 delta hat, I mean this delta hat for this delta for the epsilon NFA delta.

So, for delta hat of q 0 0 what we do we treat this as a epsilon and this also epsilon. So, this is nothing, but we can just from q 0, we will see where from where we can go with the epsilon move. So, we can go to q 1 we can go to q 2 and then from each of that whether we can take a 0 move. So, from q 0 we can go to again q 0 and then we can take a epsilon move. So, this will be basically the whole set q 0, q 1, q 2 because each of the state is reachable with the delta hat of q 0 0 because that includes because we have to include the epsilon closure inside this because we do not have a epsilon move in the NFA normal NFA. So, this is q 0, q 1, q 2 ok.

So, now delta hat of q 0 1; so, q 0 1. So, these we want to find out. So, this is nothing, but a delta prime delta hat of q 0 1.

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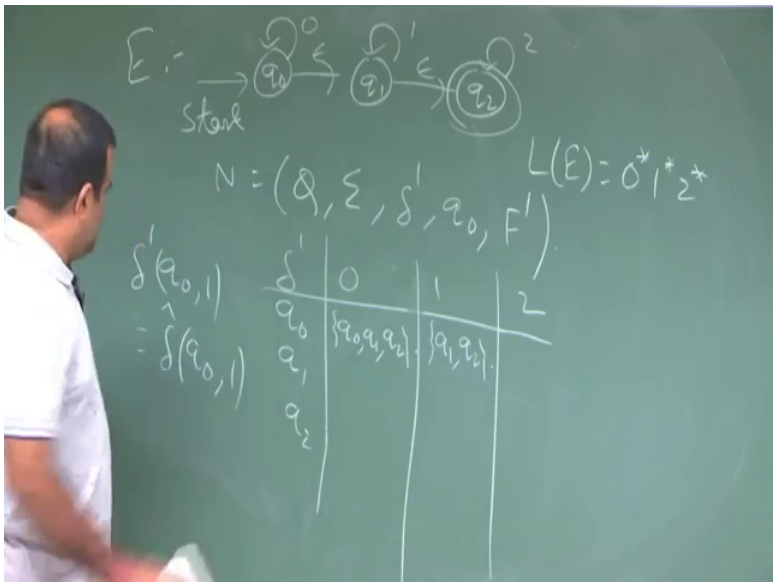

So, from q 0 again we can go to with epsilon move we can go to q 0, a q 0 we can go to q 1 and q 0 from q 1 and q 0 with the 1 we can remain at q 1. So, again with epsilon move we can go to q 1.

So, it is basically q 1 and q 2 ok. So, this is the, and this one is with q q.

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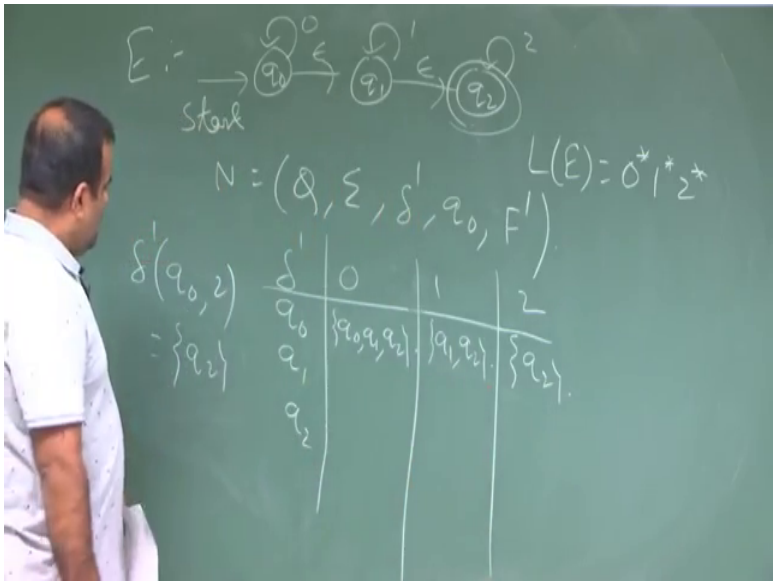

Delta prime of q 0 two where we can go with q 0 2 we can go to from with epsilon we can go to q 1, but after that there is no move with 2. So, this will be empty this will be

yeah, but with epsilon move again we can go to q 2. So, this will be only q 2. So, this will be q 2, now from q 1 to q 1 with 0.

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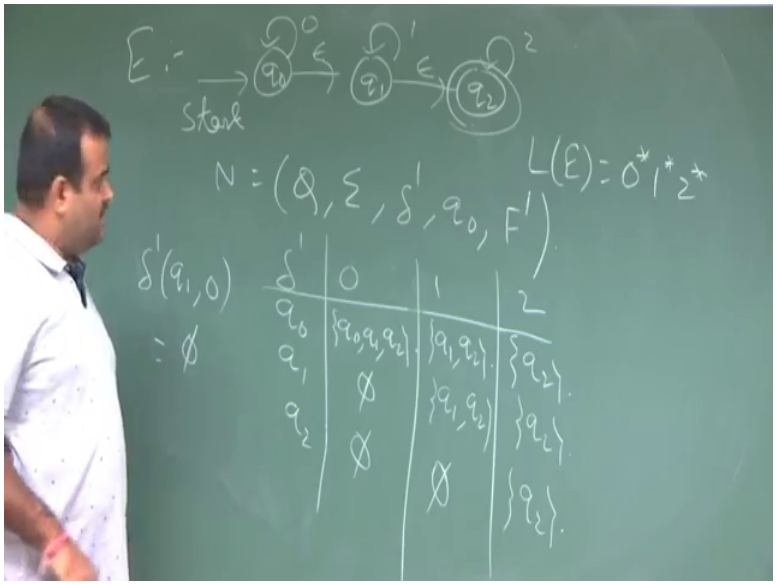

So, delta prime of q 1 with 0 it will be basically. So, from q 1 if we have a epsilon move we can go to q 2, but from there we have no move for 0 and then so this will be basically empty. So, this is empty again similarly we can find this is q 1, q 2 and this is q 2 and from q 2 this is empty this is empty and this is q 2 ok.

So, this is the corresponding NFA. So, if you draw this, this will be like this.

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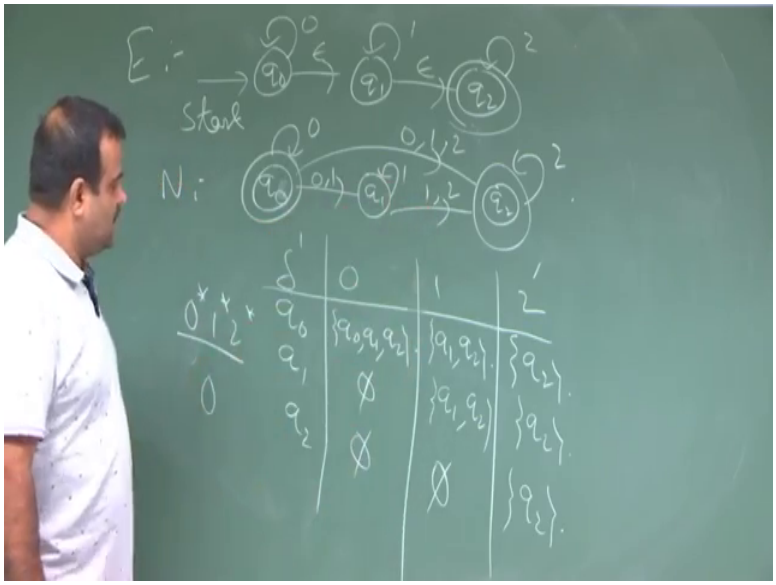

So, we start with q 0 q 0, then we can go to q 1 and q 2 here the final state is both q 0 and q 2 ok. With 0 we can go from here also here also here, with 1 we can go to q 1 so this is 0 or 1 we can go to q 1 or q 2 0 1 with 2 we can go to q 2, 0 1 2 and from q 1 from q 1 with 0 there is no move.

With 1 we can be at this or we can go to q 2 and with two we can go to q 2 and from q 2 we have no moves for 0, no move for one only we have moved with 2. So, this is the corresponding NFA, this is our corresponding and which is a accepting the same; same language because if you take 0 star, 1 star, 2 star so any number of 0's. So, any number of 0's it will reach to the final state, even only 0 if we have only 0 input it is already final state. So, it is remain there.

Because here though we have not a having epsilon move epsilon closer we have included in the delta hat of that this. So, this is accepting the same language even for 0 followed by 1, if we have a 1 we can just half here or we can go there like this. So, this is the equivalence NFA. So, from epsilon NFA we can construct a equivalence NFA. So, we will take another example, then we will see how we can go back to the DFA.

So, from epsilon NFA we can construct a NFA and then from NFA again we can construct a corresponding DFA so, that we will see now ok. So, let us construct this, we take a another example to convert the this epsilon NFA to NFA.

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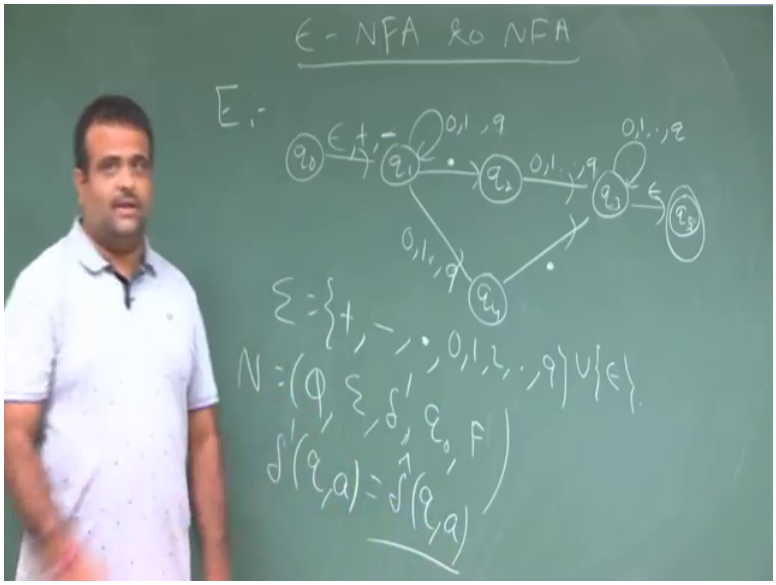

So, this example we have seen. So, this is basically the q 0 then we have q 1 you have epsilon plus dot sorry this is dot dot this is minus and if we have a digit will be hopping there 0 to 9 we have seen this example and we have a dot, will go to q 2 and from q 2 will go to q 3.

If we have a digit input 0 to 9 and from q 3 we hop here if we have a digit input and finally, we will go for q 4 with a epsilon move from here and from sorry this is q 5 this is q 5 and we have q 4 over here. So, q 4 from q 1 if we have a digit 0, 1 to 9 we will go there and if we have a dot over here will go there.

So, our sigma is plus dot, also epsilon is there 0, 1, 2 up to 9 so along with the epsilon move. So, this is our E epsilon NFA and we have seen the some of the string accepting these epsilon NFA like 5 dot 6; now we have constructed those even. So, now, we want to see how we can construct a NFA from this epsilon NFA. So, for NFA what we need to do? So, the suppose this is the NFA q, F prime F prime is only different is if we can from q 0 if with the epsilon move, we can reach to the final state. So, this is our final state.

So no, we cannot reach to the final state only with the. So, f prime is same as f over here ok. Now we can construct this delta delta prime delta prime of q comma a is nothing, but delta hat of q comma a to take care of the epsilon move before a after a. So, that way we will just get the corresponding NFA. So, let me just ok; so, let me just draw this picture over here.

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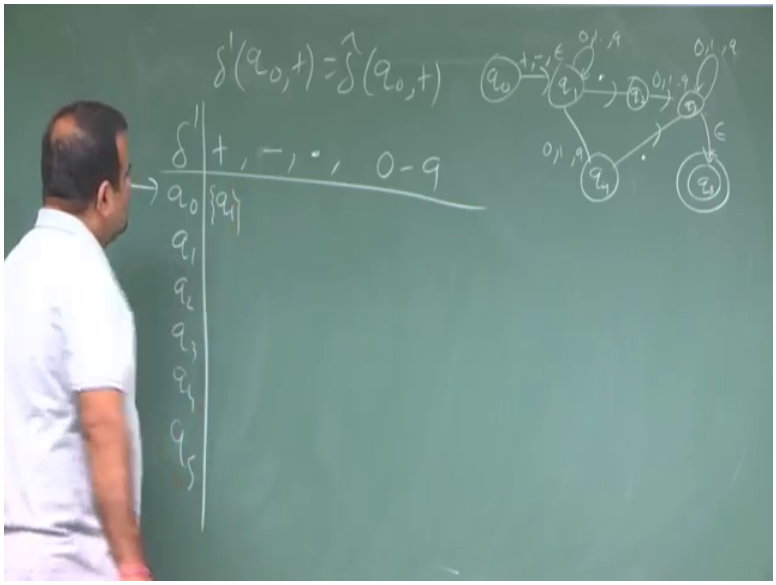

Q 0 in a smaller way q 1. So, plus minus epsilon and then we have 0, 1 up to n up to 9 we have a dot q 2 and then 0, 1, 9 we have q 3.

And then from here we can go to q 4 with a dot you know with 0, 1, 9 and with dot we are going to q 5 and with q 3 we will hop here with the same input and then finally, from here we are reaching to q 5 which is the final state ok. So, now, you have to construct the corresponding NFA, basically the delta so delta prime we will just define now.

So, it has all the states q 0, q 1, q 2, q 3, q 4, q 5 and we have all the alphabet plus dot plus minus dot and the all the digits we can say 0 to 9 ok. So, this is the starting state, now from q 0 with plus where you can go from q 0 with plus. So, basically delta of delta of q 0 plus this is nothing, but delta hat of q 0 plus we need to find out there ok.

So, delta hat of q 0 plus for plus we can have a epsilon move before that. So, we can go to q 1 from q 1 we cannot go anywhere with the plus or plus we can go to q 1. So, it is remain at q q 1 ok so, then with the minus again the same thing.

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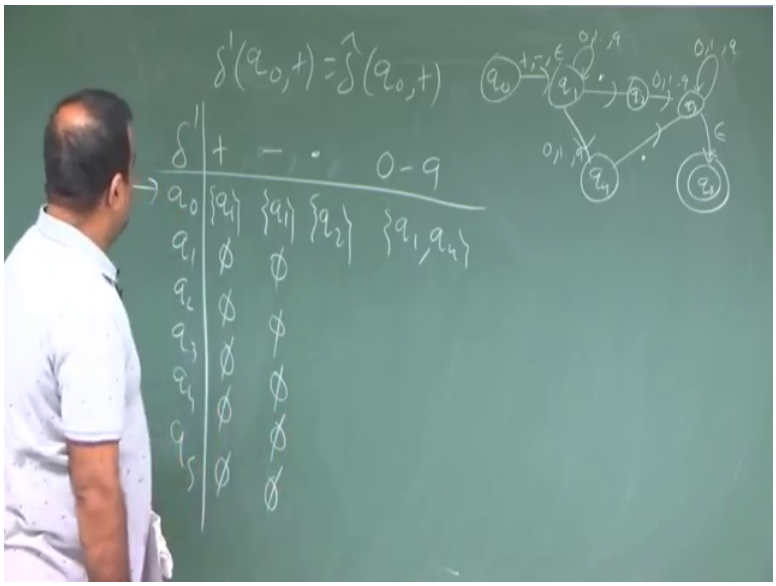

Q 1 with the dot, with the dot where we can go? We can go we can take a epsilon move here we can go to q 1 and from q 1 with dot we can go to q 2 and there is no epsilon move we can remain at there. So, this is q 2 and with the digit what you can go? With the digit we can go to, we can take a epsilon move we go to q 1 and with the digit we can go to yeah with the digit we can go to q 1 and q 4.

So, this is q 1 and q 4, now this will be empty because with q 1 with plus there is no move even no epsilon move also. So, the in fact, these are all empty this is also empty these are all empty you can easily verify and with this delta hat of q 0 dot.

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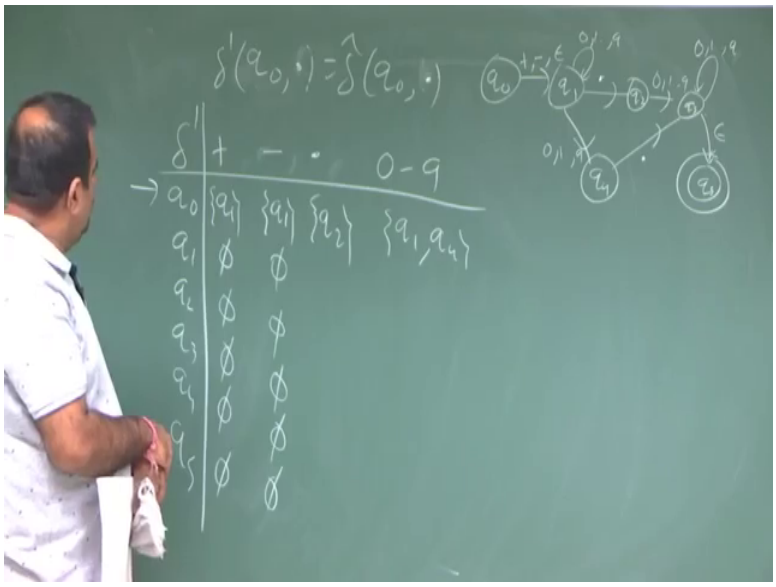

So, this will be q 0 dot, with dot where you can go? So, with dot we can go to oh sorry from q 1 q 1.

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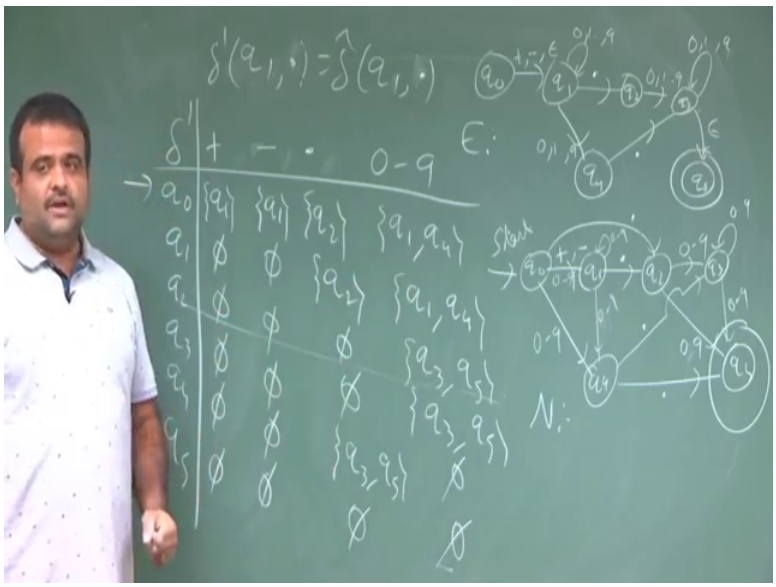

From q 1 with dot there is no epsilon move so we can go to q 2 at most. So, this is q 2 and from q 1 with number where we can go? We can go to either q 1 or q 4, q 1 or q 4 ok.

So, this way we if we complete it so this will be empty this will be again q 3 and q 5, we can easily verify this because with q 2, with q 2 with the digit where we can go we can go to q 3 and from q 3 we can go to q 5 with the epsilon move so q 3 and q 5. So, this is q 3 phi, phi and this is for q 3 we can go to q 3 and q 5, and this will be q 3 q 5 and this is empty you can easily verify this, this is empty.

So, this is the corresponding NFA. So, if you draw this let me just check this yeah we can draw this yeah we can draw this NFA, if you draw this it will be looks like this. So, q 0 and we have q 1, q 2, now we can have q 4 over here and then q 3 and we have q 5. So, where are the collection? This is the starting state this I am drawing the NFA this NFA from q 0 we can go with dot, we can go to q 2 and with plus with q with the plus and minus we can go to q 1, with the plus and minus we can go to q 1 and with this symbol we can go to 0 to 9 we can go to either q 1 or q 4.

Any of the digit and from q q 1 with the dot we can go to q 2 and from q 1 with this digit we can go to q 1. So, this is a self look with 0 to 9, any symbol any this and we can go to q 4 0 to 9 I mean any digit from 0 to 9 and from q 4 sorry q 3. So, this is q 2, q 1, q 2; from q 2 where we can go? We can go to q 3 and q 5.

So, q 3 we can go with the 0 to 9 and from q 2 we can go to q 5 also, from q 2 yeah q 2 we can go to q 5 with the 0 to 9 ok. Now from q q 3 from q 3 what we can go? We can remain at q 3 with 0 9 or we can go to q 5, from q 3 we can go to q 5 with 0 to 9 this is q 5. Now from q 5 q 4 from q 4 where we can go? We have only one move for dot. So, with dot we can go to q 3, from q 4 with dot we can go to q 3 or with the or we can go to q 5 with dot and q 5 is the final state ok.

So, this is the corresponding, this is our epsilon NFA and this is the corresponding NFA, now the question is how we can. So, you can easily check this two are equivalent, the same language is excepting for both the cases only thing we are just absorbing the epsilon closer before the, before the symbol and after the symbol nothing else and this is the final state for this.

Now, the question is how we can convert this to a DFA? So, from epsilon NFA to DFA. So, we first convert epsilon NFA to NFA then, from NFA to DFA so, that we will do by this. So, just we will take the corresponding DFA. So, this is the yeah, this is a epsilon NFA, this is our NFA.

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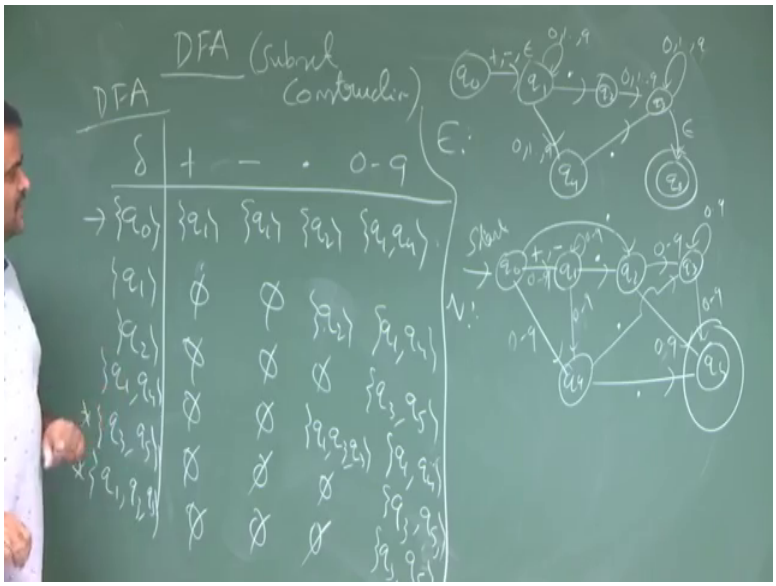

Now what will be the corresponding DFA? So, the, for DFA we know we will do if it from here not directly, there is a method to do that directly we will discuss in the next class.

But we will just convert epsilon NFA to NFA and for NFA we will convert to the DFA by substrate a subset construction we have seen this before. So, for subset construction, whether what we will do? We will consider all the subsets of this state as a possible state of the DFA which is only it is we will not consider all the subsets we will consider those subsets which is reachable from q 0.

So, if we do that we will end up with this delta I leave this as a exercise, I will just write the. So, this is q 0 and we have another states q 1 will come these are the only subset which are reachable from q 0 q 1, q 4 and q 3, q 5 and we have q 1, q 3, q 5 q 5 and this these two are the final state because it contain the this ok. So, if we just quickly write this in the no lecture note it will be given this is q 2 and this is q 1, q 4; q 1, q 4 is there and this is empty, empty, empty sorry this is not empty this is q 2 and q 1 q 4.

And this is empty, this is empty, this is empty, q 3, q 5; empty q 1, q 3, q 5 this is q 1, q 4. And this one will be empty q 3, q 5 which is the final state and the last one is and this one also q 3 q 5 ok. So, we can easily verify this. So, from this NFA we have constructed this DFA by the sub state subset construction rule.

So, we just taking all the subset which are reachable form we are not considering the dead state, dead subset we are considering all the subset which are reachable from q 0. So, these are the possible subset and this is the delta. So, this is the corresponding DFA for this and we can easily check this is accepting the same language. So, from a given epsilon NFA we can go for a NFA from NFA we can go for a DFA which is accepting the same language.

Thank you.