Introduction to Automata, Languages and Computation

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Lecture – 12
Extended Transition Function of Epsilon-NFA

So, we are talking about Epsilon NFA.

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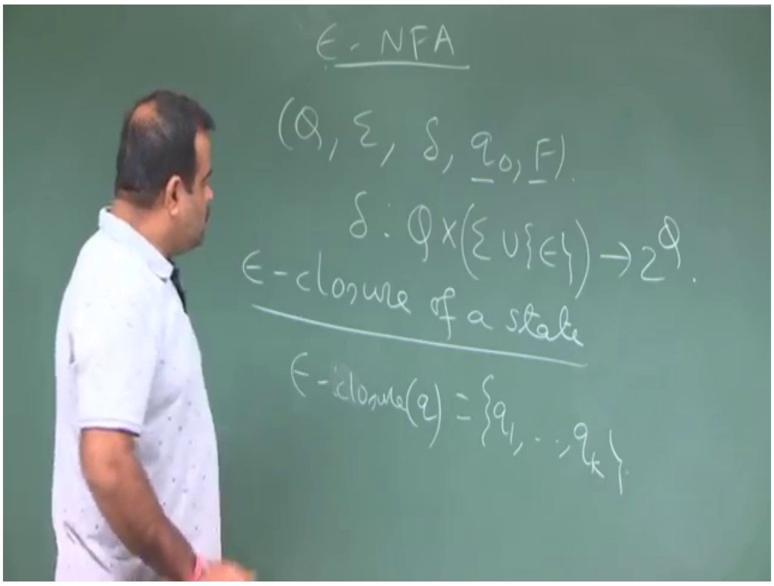

So, just to recap it is a 5 tuple. So, Q is the set of all possible finite state and this sigma is the set of all possible finite alphabet and that the here the transition rule is Q cross Q 2 to the power Q the power set. So, it can take so, here we are allowing to have a epsilon move so; that means, the without seeing any tape in the in without seeing any alphabet in the tape, it can move automatically. So, this is a extra added feature there.

So, and then this is the starting state this is the final state. So, now, we are going to extend this delta to delta hat. So, for that we need to define the epsilon closure; what do mean by epsilon closure of a state of a state. So, here we are allowing epsilon move; epsilon move is just there is no input still the our system is moving from one state to another state.

So, epsilon closure of epsilon closure of q is nothing, but it set of all states q 1 q 2 q k such that, from q we should able to move to this states only with the epsilon move; so, only with the epsilon move.

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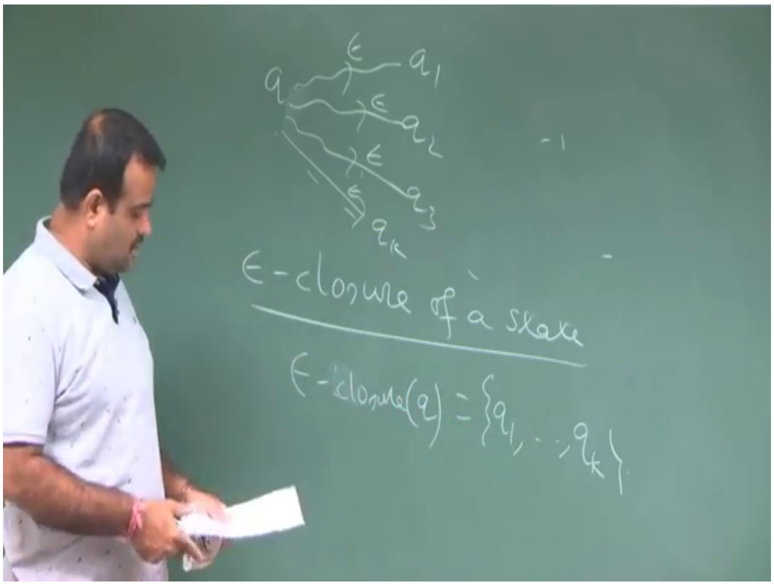

So; that means, here at q. So, we are at q. So, if we q 1 all the epsilons, q 2, q 3 dot dot dot qk. So, all are epsilon move.

So, only by epsilon move we should able to reach to this states, then it is called epsilon closure this set of all state will call epsilon closure of q. So, we can take an example like last example we had in the last class.

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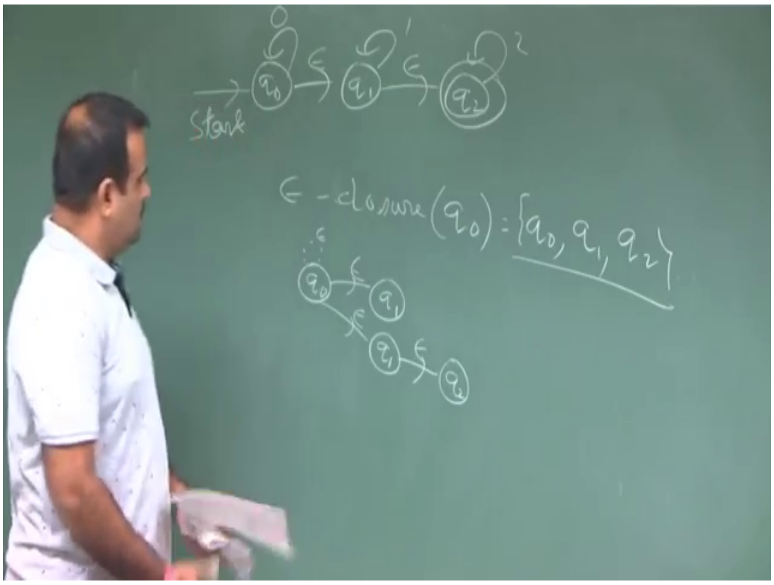

So, if you take this q 0, q 1, q 2 and we have this is our starting state, and this is 1, this is 2 this is epsilon, epsilon move ok. Now, suppose you want to find the epsilon closure of q 0. So, epsilon closure of q 0 means, from q 0 with the epsilon move wherever we can go. So, we cannot from q 0 we cannot go to q 0 with epsilon move, but we can go to yeah for a q 0 yeah epsilon move we can go to q 0, because that is always there. Delta of q 0 epsilon is a always this is always q 0, because if we are at q 0 without seeing any table will be remain at q 0. So, this is by default. So, this set is q 0 and with epsilon move we can go to q 1, and with epsilon move this is epsilon this is another epsilon you can go to q 2. So, this is the epsilon closure of q 0.

Because, from q 0 with epsilon move, we can at q 0 and we done I mean epsilon is no input and q 1 we are going, now to go to q 2 you have to go to q 1 with this epsilon move and then from there q 2 this is q 0. So, this is the set epsilon closure for q 0. Now what is the epsilon closure for q 1? Epsilon closure for q 1.

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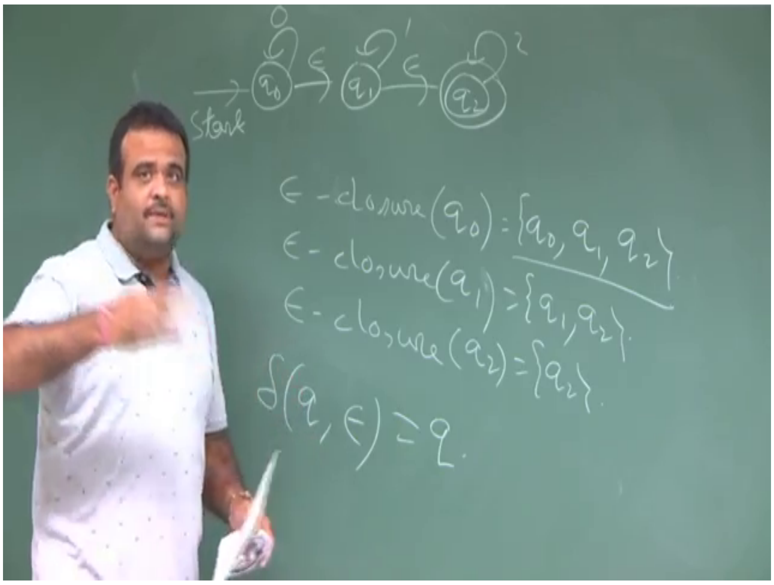

So, epsilon closure for q 1 is, we can q 1 will be there and q 2 and epsilon closure for q 2 is only q 2. Although there is no explicit epsilon move for q 2, but if we are q 2 and if you are not seeing any tape if you are not reading any tape will be remain at q 2. So, that is by default. So, the epsilon if we are q this is always q ok. If you are not seeing any tape any input in the tape will be remain at that state ok.

So, this is the way how we define the epsilon closure. So, epsilon closure is a path. So, it is the I mean from all the epsilon move we can reach from one state to another state. So, that will be the epsilon closure of that is. So, we have recursive construction also for epsilon closure, that is like this. So, the arc will be level only by epsilon.

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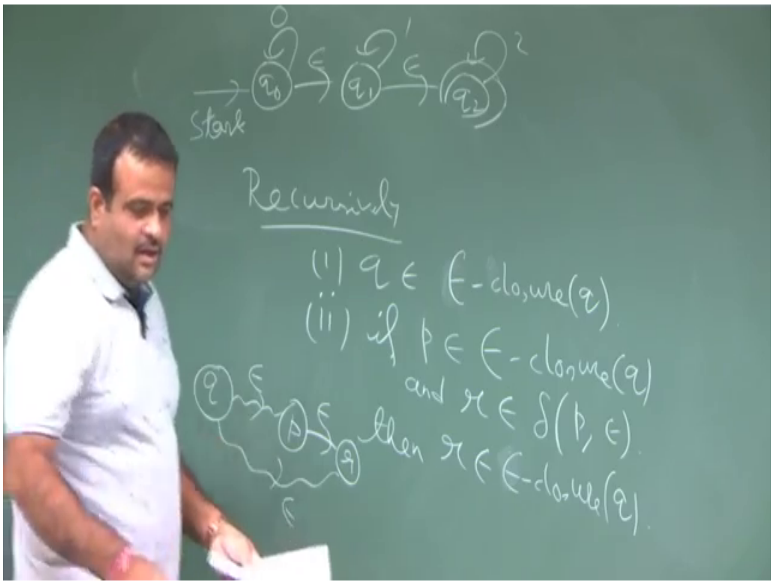

So, this is the recursive construction recursively. So, first one is q belongs to epsilon closure of q. This is by default and if p belongs to this is belongs to and this is epsilon, we can take a big this is belongs to this is epsilon. Epsilon closure of q and r belongs to delta of p comma epsilon, then r belongs to epsilon closure of closure of q. What is the meaning of this? Meaning of this is suppose we are at q.

Now, only by epsilon move, suppose we are reaching to p this is only by epsilon move. I mean this contain all the arc which are leveled by epsilon, then we if we have another epsilon move to r. Then obviously, r will belongs to because by all the epsilon move we can reach to r so; that means, r belongs to closure of epsilon closure of q. So, that is the meaning of this statement ok.

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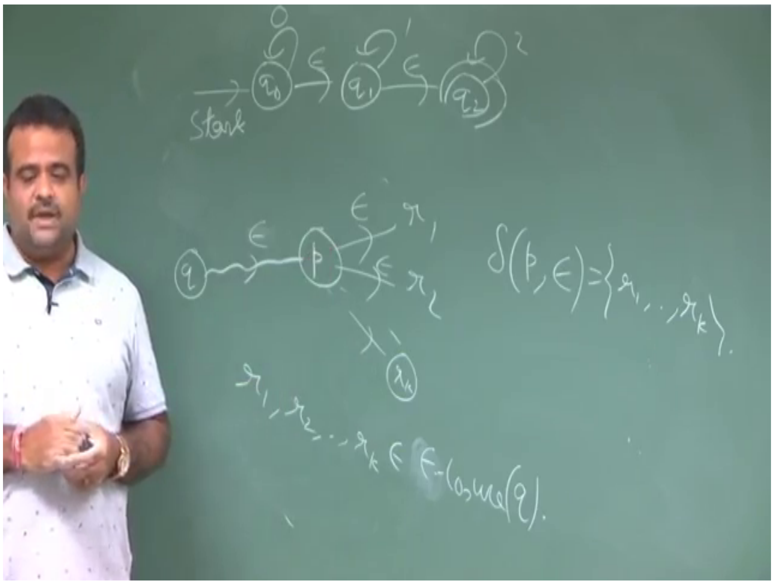

So, see when we can have like if you from q, say if p is going to r 1, r 2 all the states r k so; that means, delta of p comma epsilon if this is the state r 1 r 2 r k, then all this r 1 r 2 r k. So, then we have a p over here, q over here suppose you have only by epsilon move we are going to p. So, all this r 1, r 2 r k will be belongs to epsilon closure of q. So, this is the recursive definition.

So, basic idea is to just we move from one state to another state, just by reading the I mean just by having no input the epsilon move just we see the all the r which is level by epsilon. So, that is the meaning of this ok. So, now with the help of this, we just extend this delta to delta hat.

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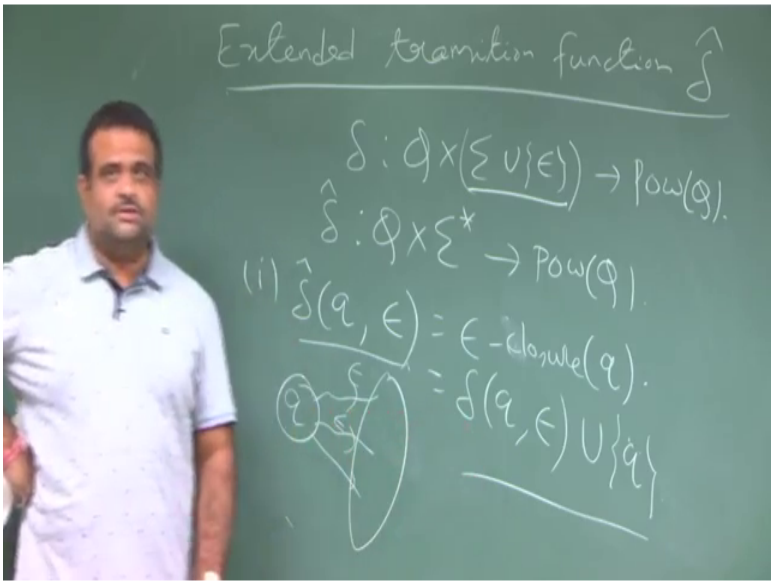

So, with the help of this, we just extend delta to delta hat; extended transition function that is called delta hat ok. So, we know delta; delta is nothing, but Q cross sigma with the epsilon is going to power set of Q it is a sub set. Now we want to have a delta hat which is Q cross, it should take a string instead of a single alphabet or the epsilon move this will also go to the power set of Q.

Now, how to define this? So, to define this the base case is same, delta hat of q epsilon because epsilon is a also belongs to this ok. So, this is basically epsilon closure of q. Because we have a epsilon move here. So, this means we are at q now this is the all the epsilon, all the epsilon move wherever we can reach. So, this side is basically this. This is nothing, but delta of q comma epsilon, but include this includes q ok.

Because if we at q, this these in your q. q is there because if we are at q if you are not seeing any input if you are not reading any input will be remain at q that is one thing and another thing now we have a option for epsilon move, without reading the tape we can move. So, that is the we can go from q to with epsilon move we can go to these are the states. So, these are all will belongs to this, this is the base case. Now, we extend this for other than epsilon.

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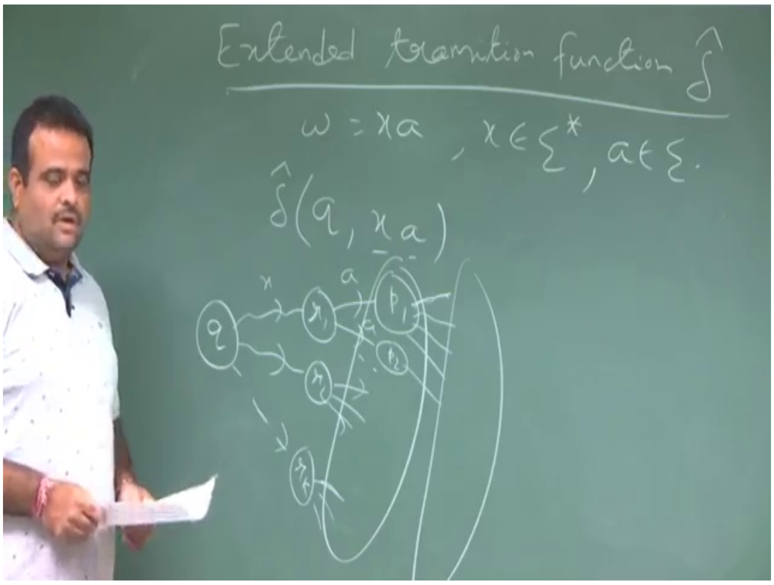

So, general w so, w is equal to x a so, here x is a string and a is an alphabet now how to define this? So, delta hat of q xa so, the idea is we are at q; now we are reading this x and will be going to some states r 1, r 2 r k say or r l ok. Now from here we can have a epsilon. So, from here yeah; so, we can have a epsilon will be before that and after that also. So, what is the convenient?. So, now from here we can have a, we can go to p 1 with a we can go to p 2. So, maybe this includes the epsilon move after r 1 and before r 1. So, dot dot dot dot. So, like this like this now for all of these we can have now epsilon closure. So, this set will be the delta hat of this ok. So, let us write this formulae. So, let me write this formulae.

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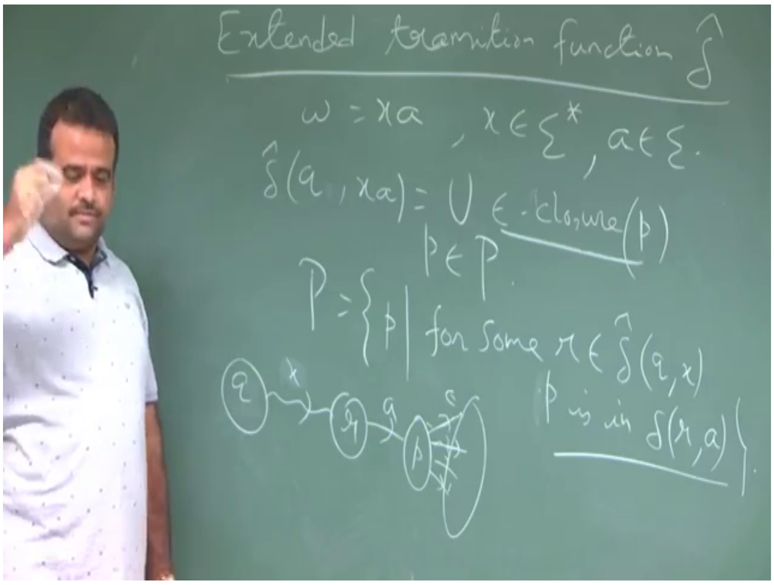

So, we are defining delta hat of q comma xa which is nothing, but epsilon closure of the set P. So, epsilon closure of a set P is nothing, but. So, you we can say that union of the epsilon closure of p, where p belongs to capital P. So, what is capital P? Capital P containing the set p such that for some r belongs to delta hat of q comma x, and p p is in after the r we have take the a p is in this. So, this is the meaning; meaning means we are at q.

Now, we go to some r with the x, this includes epsilon move is here. So, there are many such r s, but at least 1 r. So, it contain all such ps and then from here we have many ps this is one of the p because this is NFA this is one of the p. So, after that we take all such ps and their epsilon closure. So, this is the epsilon closure of at this is for one p. So, we have many such ps over there. So, this is how we defined this. So, basic idea is we just we have a tape.

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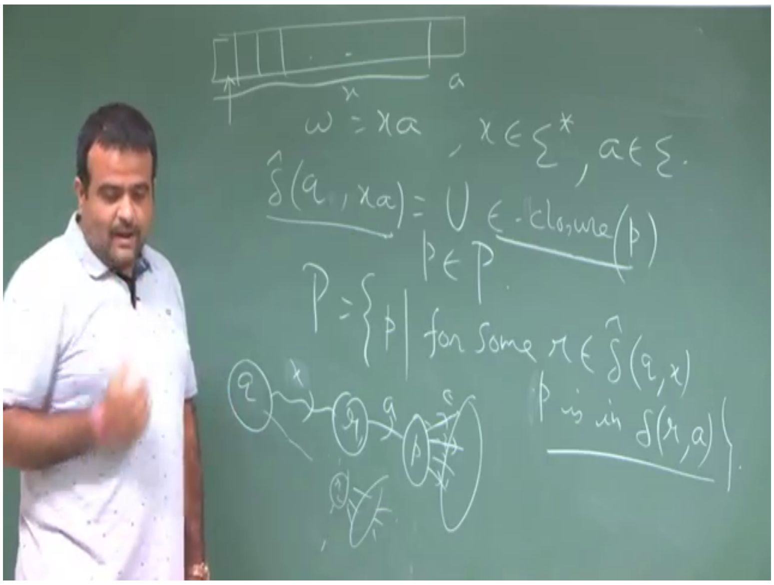

So, we have given a tape. So, of alphabets so, this is our x this is our a. Now we keep on read the read the tape, and sometimes we will see I mean the possibilities where without reading the tape we can move or not that is the epsilon move. So, that is so, you will explore those branches. So, there are many branches over there. So, those branches also you will explore then finally, after exploring all such r s there are many such r s and p is there are many such ps. So, this is r 1 this is r 2 we have many such ps. So, from there we can go to epsilon closure of this. So, this way we can take all the all these epsilon closure of p, those will be this delta hat of this.

So, delta hat of w means, we just read the string sometimes we do not take the input we move and then finally, we reach to the some of the states, which include the epsilon move also in between. Many epsilon moves are going on inside ok. So, this is the way we define the delta hat. So, now we can define the language of this epsilon NFA.

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So, from delta hat we can just extend delta and delta hat to the set of state in set of a given state to the set of states. So, far we have seen we have a set q, and the delta is just a direct move, it could be epsilon move also and this is the set, this set is delta q comma a or a belongs to it could be epsilon also.

And the delta hat also you have defined q, we take a w we are including the epsilon move we are reaching to this set this is delta hat of w. So, these are on a state, now we are going to define this or set of states. Instead of a state if we have set of states from that set of state how your where you can go. So, that is basically just the union. So, that is basically just the union.

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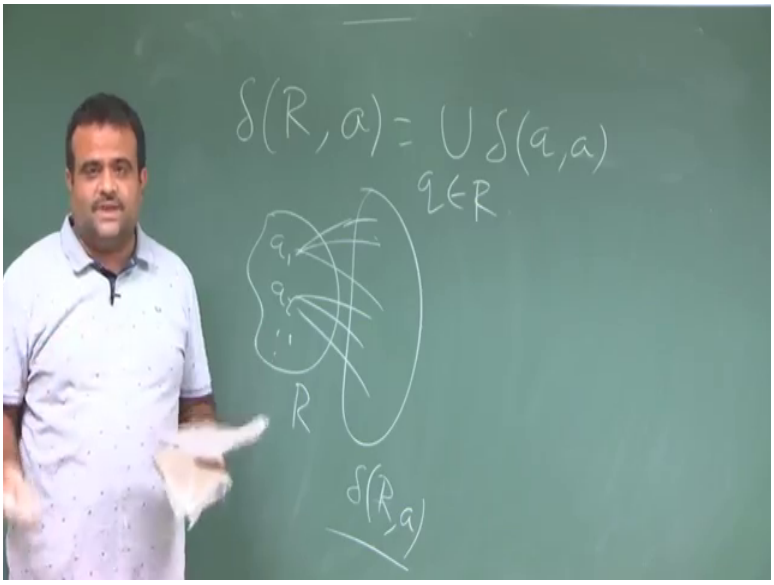

So, delta now R is a set of state of a is nothing, but union of delta of q comma a q belongs to R. So, we have a in set of a single state we have many state say q 1, q 2 like this. So, on this set by delta where we can move so, that is the way we are defining. So, this is the delta of R comma a ok. Just to I mean because here we are having the it is non deterministic. So, we are dealing with the subsets. So, that is why it is easy to have this.

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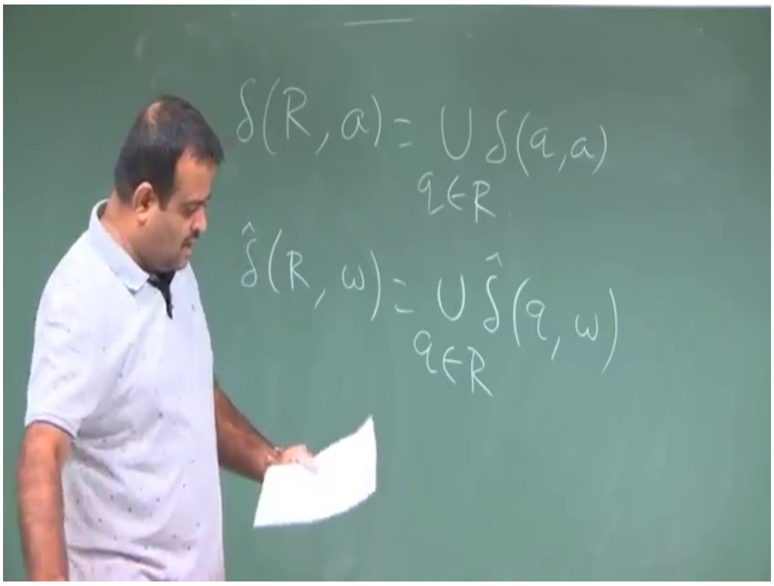

And then delta hat of delta hat of R comma w is nothing, but union of delta hat of q comma w q belongs to R. Similarly I mean in set of alphabet if we read a string, where it can go. So, this is the way we are defining ok. So, now, now we are going to defined a when a string is accepted by the epsilon NFA.

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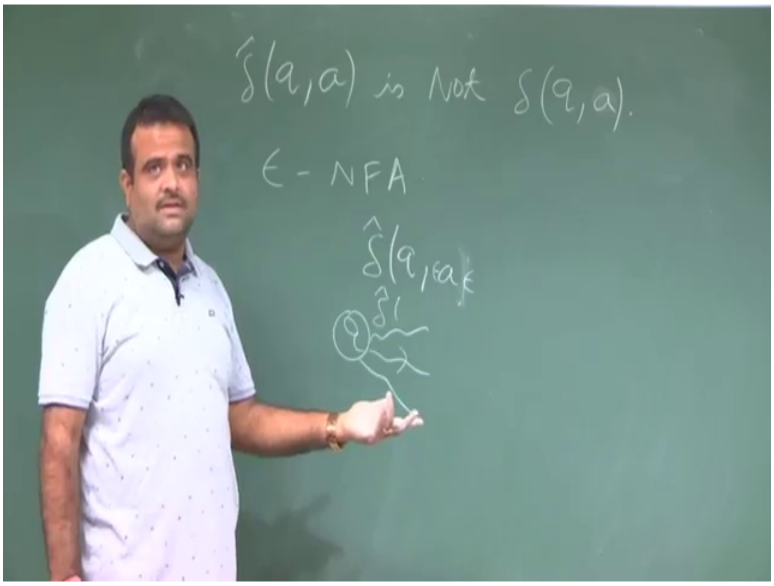

Now, there is a observation, observation is delta hat of q comma a is not necessarily equal to delta of q comma a yeah because this is epsilon NFA. When epsilon NFA what we have? So, delta of ah delta hat of q comma a is the set of all state reachable from q by a path level by a including the path. So, we are at q. So, we are reaching to only a or because a consists of we can write a epsilon over her epsilon over here. So, we can just see epsilon before a we can see a after a. So, this is a string.

Now, once we define this, this is not a this a will be treated as a single alphabet its not a string. So, there is a fundamental difference between these. So, here we are allowing the epsilon move, but they are a could be epsilon that is fine, but other than that it is just a input alphabet. So, this is a fundamental difference between these 2 ok. So, and now we define when a string is accepted by a epsilon NFA.

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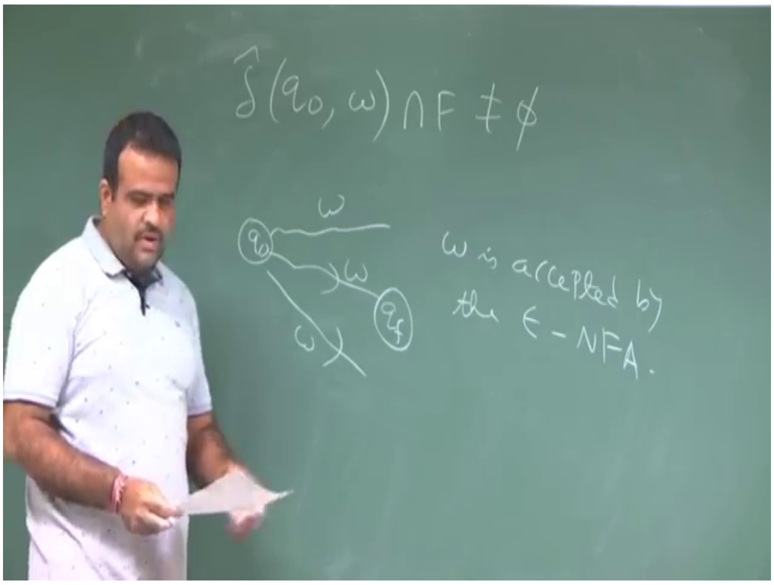

W if delta hat of we are at q 0 if we say w, if this intersection F is not empty; that means, from q 0 there will be many path going, if one of the path reach to a final state, then these are all ws then we call w is a accepted by either epsilon NFA we can go back to the last example.

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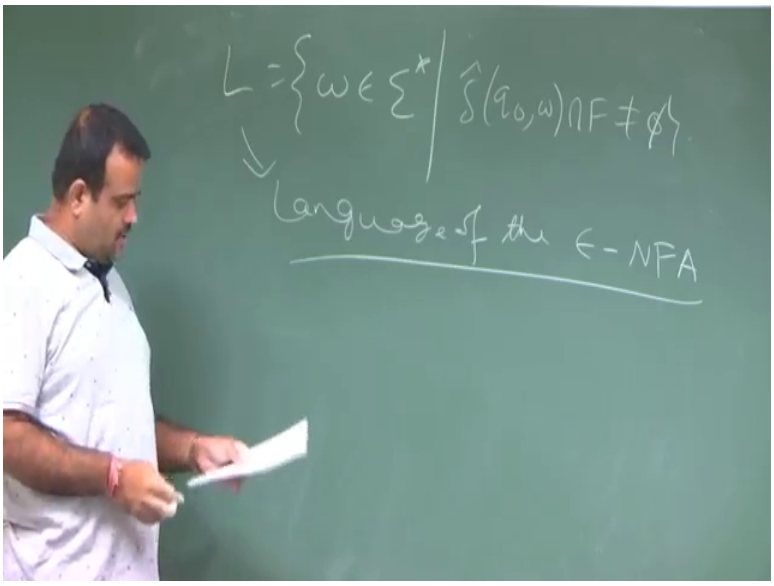

And the language of this is, the language of the epsilon NFA is nothing, but set of all w such that delta hat of q 0 w (Refer Time: 22:44). So, this is the languages language of the epsilon NFA. So, this is the language of epsilon let us take the last example, and later all we will see the from epsilon NFA we can have NFA and then from NFA we can have a DFA.

So; that means, all are basically accepted the accepting the regular language ok, but it is not going beyond the regular language. So, that is the one thing although we are making our life easy by introducing non deterministic by introducing the epsilon move, because with epsilon move is it is easy to construct a finite automata for a given regular language. But it is not we are not going beyond the regular language so, that we will go after that after this topic.

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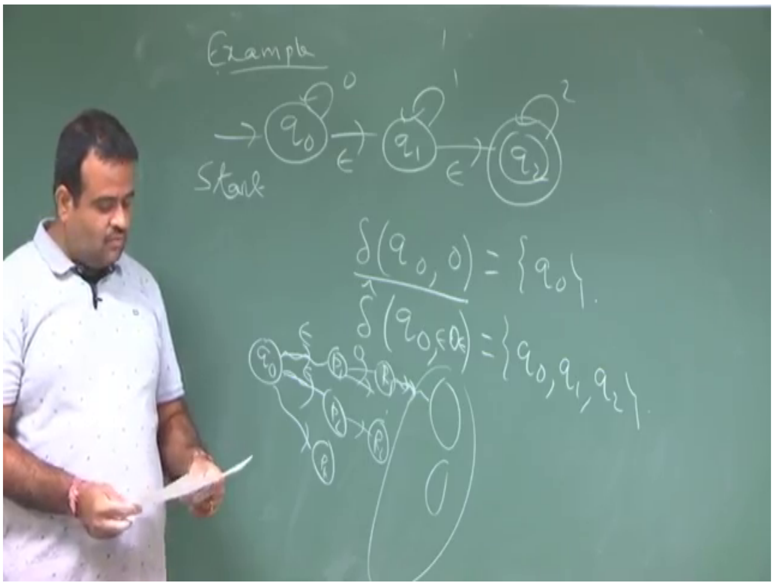

So, let us take the that same example from the last class q 0 q 1 q 2. So, this is the starting state, this is epsilon move ones this is epsilon move 2 when this is the final state ok. Now, here that is why delta of q 0 comma 0; delta of q 0 comma 0 is q 0, but delta hat of q 0 comma 0 if we consider because delta hat consists of when we talk about delta hat this we treat as a string. So, this 0 means epsilon here and epsilon here we can say. So, this is nothing, but epsilon closure of all possibility. So, this is basically q 0, q 1, q 2. So, this is the fundamental difference between these two.

So, from q 0 we can go to by epsilon move we can go to because here in this in this definition we are not allowing the epsilon of this the this that function direct transition rules, but here we are allowing the epsilon move that is the fundamental difference. So, you can go to p 1 p 2 this is just by epsilon move pk then after that we can see a alphabet these are all going to R 1 R 2 like this. This is a set R 1 set this is also going to some say that and again after that we have a epsilon move epsilon closure of this.

So, you can understand. So, we can have a. So, this 0 when you talk about hat, this 0 can be treated an epsilon 0 epsilon. So, this epsilon move so, this will include the epsilon closure of every this state of this set. So, this is the freedom we have a epsilon move; that means, without seeing the input we can move to one state to another state. So, that is the difference between delta and delta hat.

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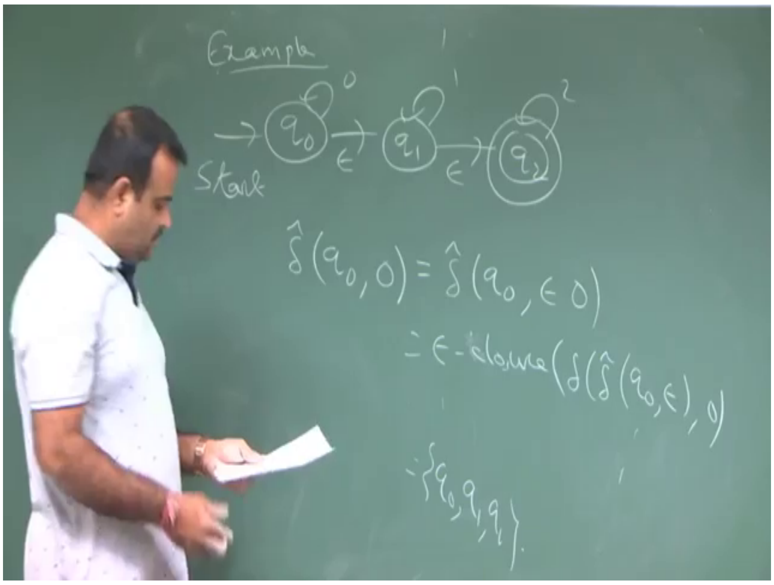

So, if we just take this delta hat of q 0 0, we can go to delta hat of q 0 epsilon 0, because epsilon move we can go to q 1. So, this is the epsilon closure of delta of delta hat of 0 like this. So, this way we will reach to the q 0, q 1, q 2 ok.

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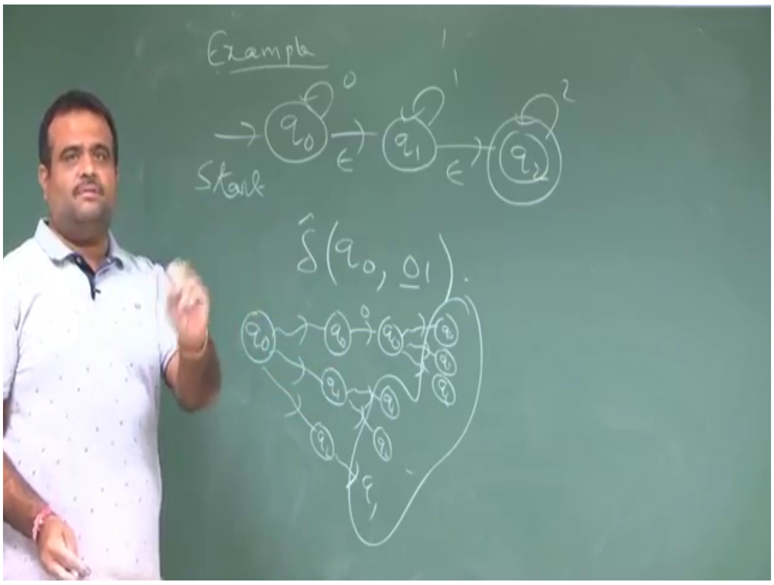

So, similarly if we want to find out the if we want to find out the delta hat of q 0, 01. So, this is by definition we know this is we are at q 0. So, we first apply all the epsilon move on this, epsilon closure of this. So, if we apply all epsilon closure on this. So, you will go to q 0, q 1, q 2 then we can use this 0. So, from here we if we use 0 will be at q 0 again, there is no 0 move here. So, will be remain at this and then again we can go for epsilon 1. So, may if you go for epsilon move this is the epsilon closure of this set.

So, this is q 0, q 1, q 2 and we can have a epsilon move for this. So, this is again q 1, q 2 and this is again q 1. So, these are the set q 0, q 1, q 2 is again the this so, this is the way. So, you have the epsilon option from the both side, before string after string before alphabet after alphabet. So, it is a extra feature you can say ok. So, we will continue this.

Thank you.