Introduction to Automata, Languages and Computation

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Lecture – 10
Subset Construction

So we are talking about how we can construct the DFA from NFA the equivalency between the DFA and NFA; that means, given NFA how we can get a corresponding DFA, which is accepting the same language.

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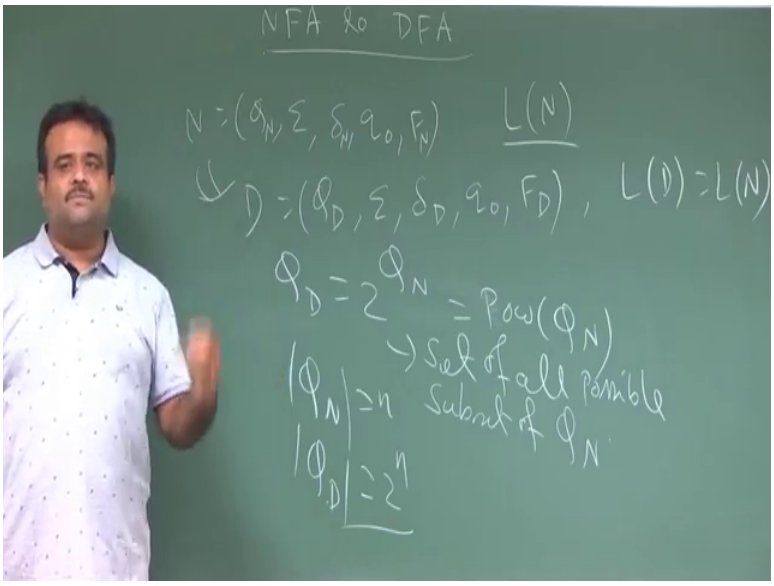

So, this is the NFA. So, this is the language of this NFA is this now from here you have to construct a DFA D such. So, this is NFA. So, these are the state for the NFA. So, we have to construct a DFA. So, Q D say sigma is same the input alphabets is same for both the cases.

But rules will be different q 0 and F D. So, this is the DFA such that the language of the DFA will be same as language of the NFA. So, this is our goal. So, in the last class we have seen some way to construct that that is the subset, to subset construction method. So, how can do that? So, basically we have to take the given a NFA we have to take the all the DFA Q is nothing, but set of all possible states, subset of this Q D is two to the power.

So, this is the power set of this. So, this is the set of, set of all possible subset of subset of the states in NFA. So, this is huge number, if there are n state in the NFA if q n is equal to n then Q D will be the cardinality of this set this is two to the power, but we do not really need to consider all the states. So, we will take the states which are accessible from q. So, we consider all the subsets which are containing the sorry q 0.

So, which are accessible from q 0 the starting state because we are talking about in terms of the accept the language. So, language means we start from a starting state and we should able to reach to the finite state. So, that is the string accepted by that automata. So, since we are talking in terms of the language or string accepted by the automata. So, we will only considered those states in the Q D those subsets which is which is containing q 0; that means, those are which are access which are accessible from q 0 only those set will be considered. So, for example, so if you take the example from the last class.

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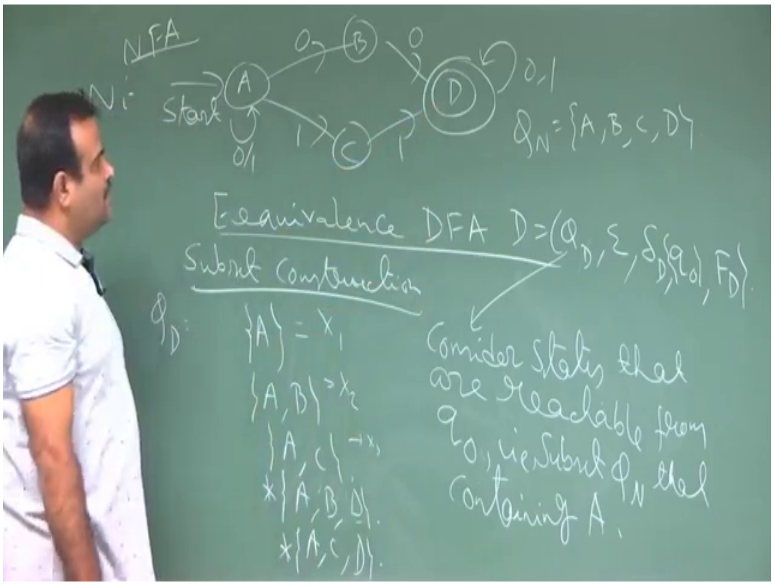

So, suppose we have given this given this NFA this is the starting state or the initial state and we have these are the states and you have a finite state which is D. So, and this is 0 1. So, this is NFA. So, we can have non deterministic move 0 1 0 1 now we have to consider this is the NFA now we have to consider equivalence DFA D Q. So, Q D sigma will be same for both the cases and the rule, starting state and that F D ok.

So, F D F D. So, these will do using the subset construction method, construction. So, here we just consider the states the states these are the states that can, these are the all subsets. So, we do not need to consider all the subsets we consider only those subset states which are that are reachable the reachable form from the starting state q 0 that is the subset of these which contained q 0 subset of this q n this is the Q N. So, Q N is nothing, but A B C D.

So, the subset of subsets of Q N that contain, all the subsets of Q N that contains containing the starting state the starting state here a so that containing A. So, who are the. So, A these are the possible state of this DFA. So, these are the Q D A A B then A C then A, B, C then A, C, D; A, C, D. So, these are the possible these are the all possible subset of Q N which are containing A. So, and these are the set subset which are reachable from A. So, among these since says this is the finite state. So, these two are the; so these are our this side is our Q D.

So, these we can referred as X 1 X 2 X 3 like this. So, there are five states in the new DFA and among these, these two are the finite state because these are this is the A B, A C, A B oh A B D because C A C A B D is the state which is reachable from A. So, we consider only those subsets which are reachable from A ok. So, this is the state. So, now, let us construct the and these two are the finite state because these to contain the finite state of Q N.

So, now let us consider the construct this rules, transition room for the DFA ok.

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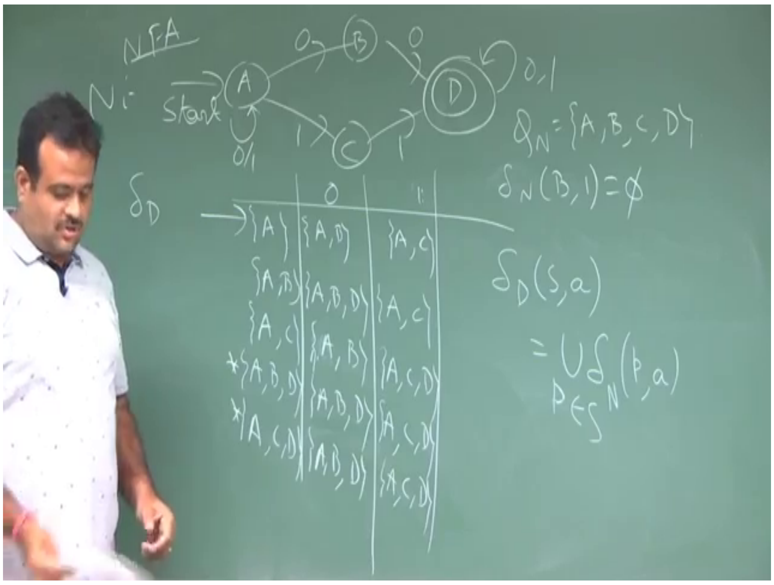

Now, let us consider the tangential rule. So, so we are constructing delta D. So, we are constructing a Q, I mean this is our Q D. So, this is the DFA were constructing Q D is A then A B and A is the starting state then A C. So, these are the substrate subsets which are visible for A under that rule and these two is that because these two are containing the D, D is the finite state of the automata.

So, these two are the finite state of the DFA also now the rule. So, if you have a 0 1. So, 0 1 so if we have 0. So, 0 we can go to here. So, 0 means we can go to B or we can go to A. So, this is going to A B. So, rules are same basically delta of D delta of D. So, we are considering a subset that subset is S comma a is nothing, but this is delta N of p comma a. So, p is belongs to s. So, this is the way we construct this so; that means. So, delta of a here we have only one set. So, delta A A is basically from a where we are going by the 0 move.

So, we are going to B and A so A B. Now with one with one from a where we are going we can go to A or we can go to C. So, this is the A C ok, now from A B. So, from A B with the 0 move we can go to from A we can go to A B and from B we can go to d. So, if you take the union by this rule by this formula if you take the union it is A B D; A B D and then similarly from a A C from A B with the one move from a we can go to A C and from B there is no move form with the one move. So, from B delta of delta N of B comma one this is m. So, empty along with A C is A C. Now from A C if we take the 0 move from A if you take the 0 move we are going to A B and from C if you take the 0 move there is nothing. So, this is going to A B now from 1 I mean with the one input. So, from A with one we are going to see A C and a A C and with a from C we are going to D. So, this is basically A C A C D.

Now, come here A B C from A B C with 0 move from a we can go to B or B and A, A B and from B for 0 move we can go to D and from D we can go to D. So, this is basically A B D. So, you are using this formula to get this similarly this will be A C D and this will be again A B D and this will be again A C D. So, this is the corresponding DFA, rules of the stagnation rules of the corresponding DFA. So, if you construct this DFA it will be like this.

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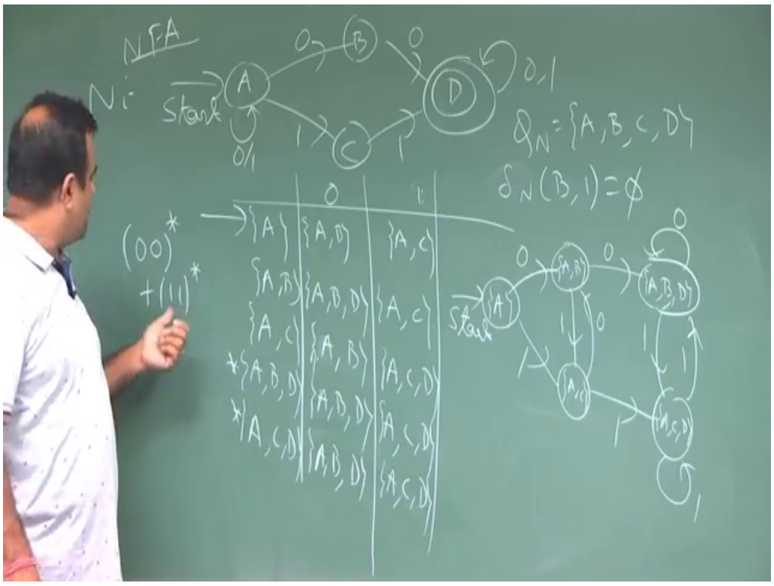

So, you have five states. So, you can rename this say this is a then we have A B then we have A C then we have two more steps A A A B D and A C D; A B D and A C D. So, these are the states and then we have the rule. So, from A this is the starting state, from A we go to with the 0 move we go here, with all we go here. You know one and from A B with the 0 move we go here with the one move we go A C and from A C we go A B with the 0 move and we go to a C D with the 1 move and from A B, A B D; A B D if we take a 0 move will half here and if we take your one move well go to a C D and from a C D if you take a 0 move we go here and with 1 move we go here.

So, this is the corresponding DFA rules we can just rename these as other seem more like p q r s t. So, this are just a notation. So, so this is the equivalent DFA and both that both these are accepting the same language and the language is the ending with the 0 0 or 1 1. So, any string ending with 0 0, I mean any string containing 0 0 x y. So, x y are belongs to this is one option and like it is in regular expression is 0 0 star last 1 1 star. We will discuss this when you talk about regular expression this is the union of two language this is one language which is having the 0 0 as a substring and this language which is having 1 1 as a substring.

So, this is the, and the finite states at these two, these and this. So, so this is the equivalence DFA and we can just as I said I can just rename this DFA like this.

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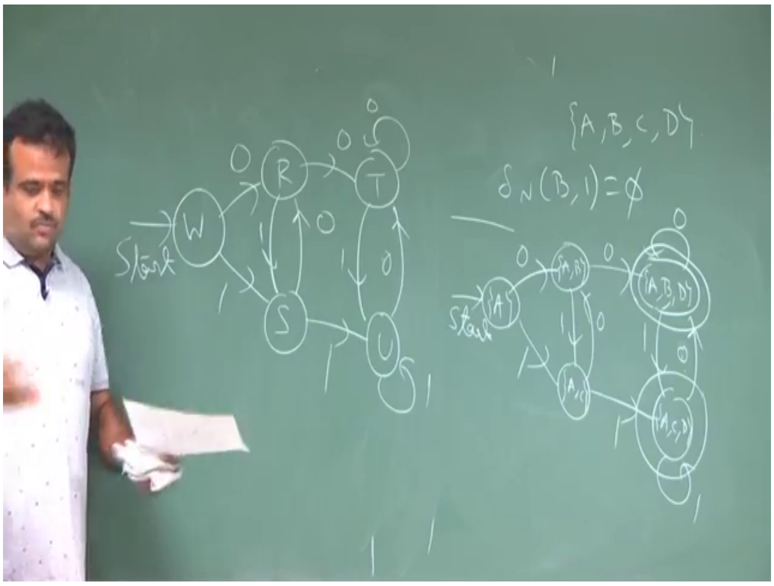

So, W, R, S, T, U so, this is 0 move this is the starting this is 1 move, this is 1 move, this is 0 move this is 0 move this is 1 move and this is 0 move and this is 1 move, this is this should be 0 move this is 0 move this is and this is. So, this is the corresponding DFA of that NFA, and these two are accepting the same language will prove that.

So, let us take another example one more example then we go to the result that this two are accepting the same language.

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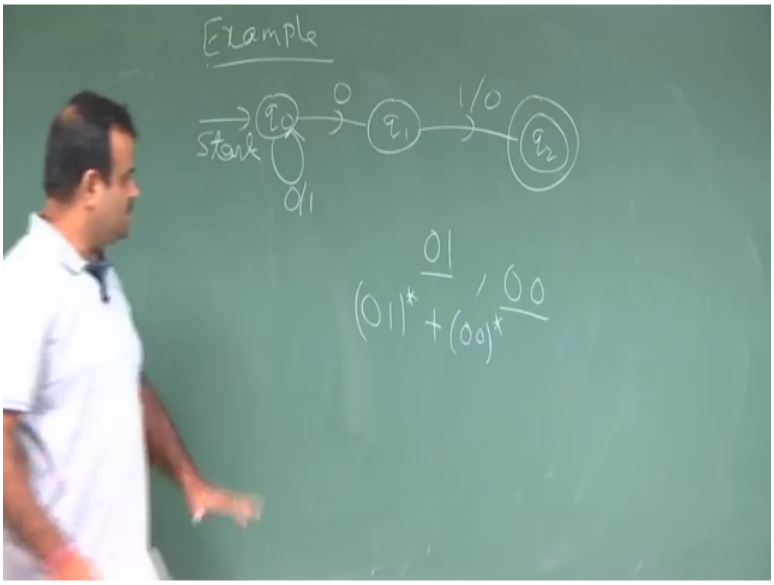

Let us take another one. So, we have q 0 this is the start and q 1 q 2. So, we have 0 1 0 and this is the finite state and here we have 0 or 1. So, this is NFA simple in NFA. So, what is the language accepting it? So, it is accepting like 0 1, 0 1 or 0 0 or we can have. So, the string containing the end of this string containing 0 1 or 0 0.

So, this is basically this is the way we write in regular expression. So, the string containing 0 1 or 1 0 say any D sub. So, now, you want to have a corresponding DFA. So, as you said there are three states now the DFA state will be now all the possible subsets, but among these we considered only those subsets will containing the this starting state that q 0.

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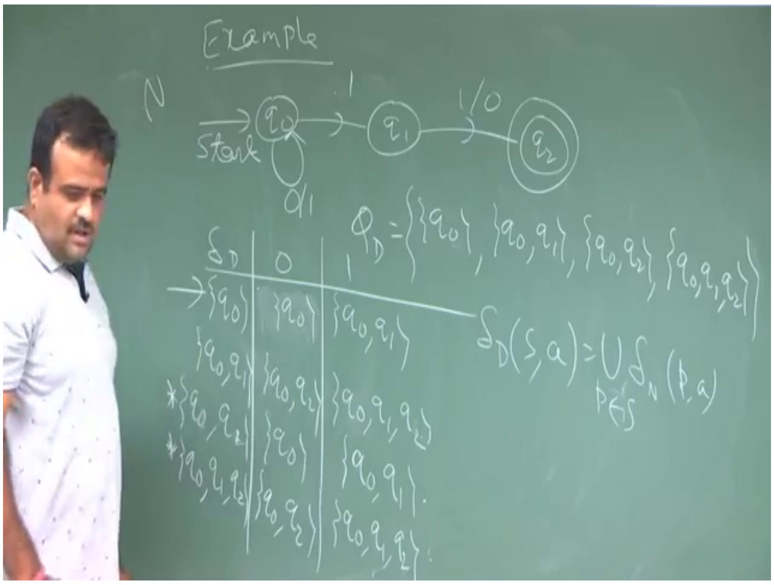

So, who are there they are basically q 0 q 0 then q 0 q 1 then q 0 q 2 and then we have q 0 r q 1 q 2. So, this is our. So, this is our N this is our Q D, now regarding the rules.

So, you can just write this state. So, this is the delta D we are we are getting. So, this is q 0 which is the starting state and we have on the other state you know q 1 and q 0 q 2 sorry q 0 q 2 and we have q 0, q 1, q 2 and among this which is which are the final state. So, these two continue q 2 which is the finite state. So, these are the two finite states for the corresponding DFA now we put in 0, if you have 1 now the rule is same that that q D of because this is subset S comma a is nothing, but union of I sorry delta D of S comma S p a were p is a S.

So, from q 0 where we can go with the 0 move from q 0 we can go to q 1 and q 0. So, this is going to q 0 q 1 and then we can only go to sorry this is 1 mistake over here if this is 1, then this is accepting the string 0 1 1 or 1 0 sorry this is 1 this is 1 anyway. So, this is q then it will be for 0, it is only q 0 and with one it is q 0 q 1 because with one we have two option now q 0 q 1 with q 0 1 0 we have q 0 and one q 1 we have q 2.

So, this is basically q 0 q 2 and similarly here with q 0 1 this is the case and with q 1 1, q 2. So, this is q 0, q 1, q 2 by this union rule and from q 0 q 2; q 2 there is nothing. So, from q 0 if it is 1 we are going to q 0 already I saw 0 we are going to q 0 only and from q 0 q 2 with one move from q 2 there is no move that is the empty set you need only the empty set is the same.

So, with the q 0 with the q 0 with one we have q 0 q 1 and with this we have. So, we have q 0 q 2 and then we have with one what we have from q 0 with one we have q 0 q 1 and q 1 with one is q 2 and q 2 with one is nothing. So, this is basically q 0 q 1 q 2. So, this is the rules now if you draw that picture it will be like this.

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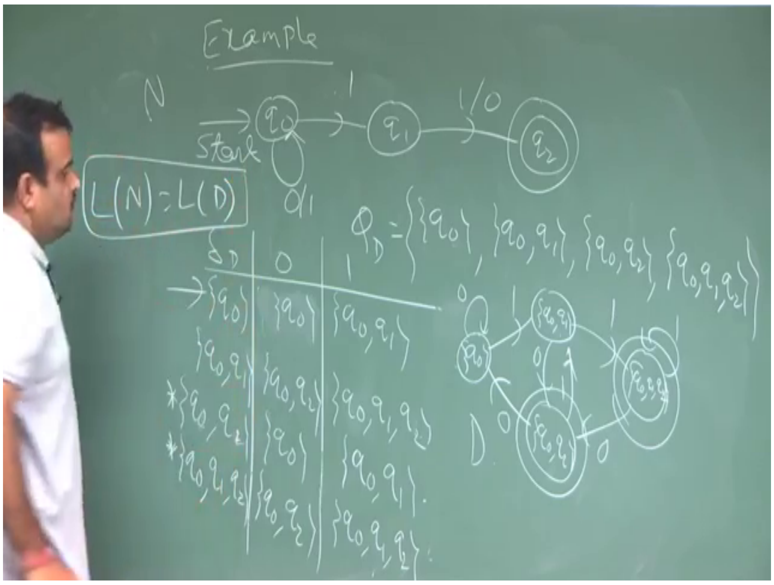

So, this is q 0 we have q 0 q 1 then you have q 0 q 2 and you have q 0 q 1 q 2 these are the states and from q 0 we have with the 0 move we are here with one move we are going here. And from this one q 0 q 1 with 0 move we are going here and it one move we are going here.

So, this is 0 move, this is 1 move and from q q 0 q 2 with 0 move we are coming back here and with one move we are going here and from q 0, q 1, q 2 with 0 move we are going here and with one move we are here and these are the what are the finite state, finite state are basically this subset with containing q 2. So, these two are the finite states. So, this is the corresponding D DFA from this given NFA. So, this way will construct the DFA, now we will have a theorem to show that. So, this is the NFA and this is the DFA now the language of this NFA same as language of this DFA. So, these we have to formally prove this ok.

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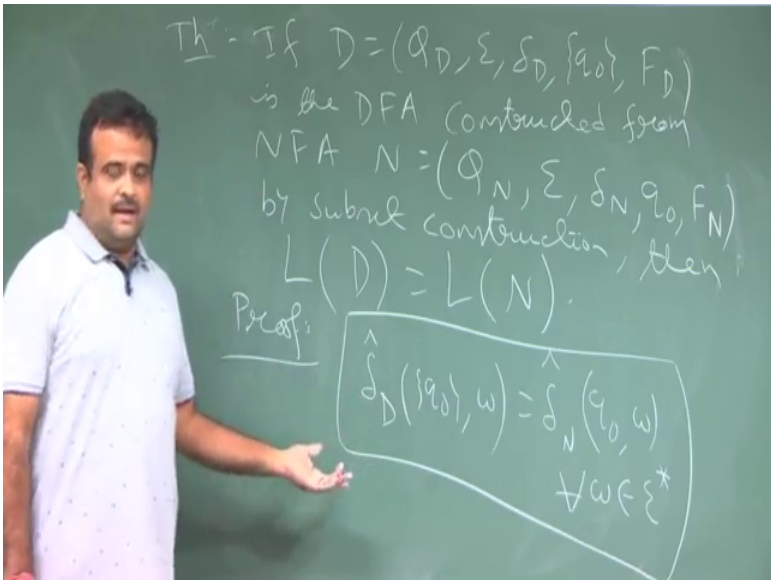

So, this just we take a theorem over here, if B is the corresponding DFA delta D q 0 F D is the DFA constructed subset construction constructed from the NFA Q N sigma delta N q 0 F N. So, this is the given NFA from this given NFA you have constructing DFA by the substrate construction method, then the language of this NFA is same as language of the DFA. So, that is the goal I mean given a NFA we can have a equivalence DFA so that because we know the DFA NFA is easy to construct if you have a language which is regular language.

So, from a given language you can easily construct a NFA than DFA. So, once you have NFA then we can construct the DFA. So, how to prove that? To prove that you have to use a result which is on the extended transition rules delta D of q 0 w is same as delta N of q 0 w for all. So, this is the, this we can prove by induction. So, what is the meaning of this meaning of this is meaning of this is.

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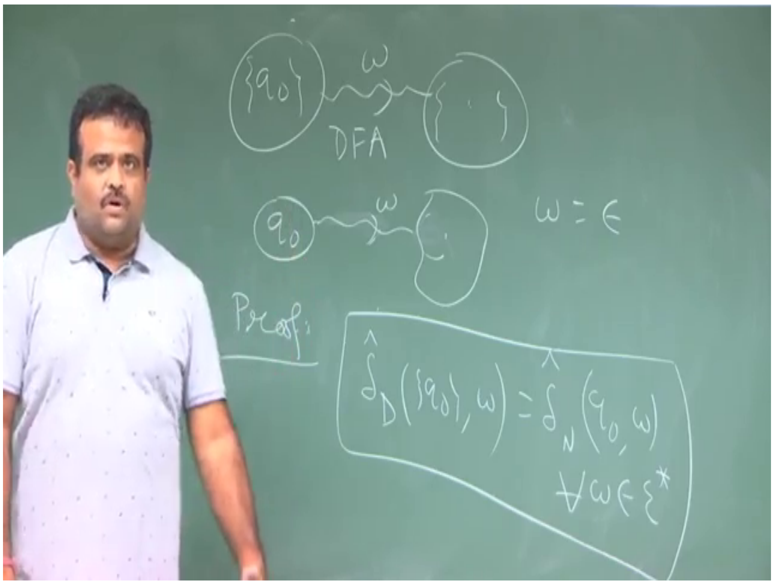

So, we start with q 0 now we have constructed the DFA. So, we start with q 0 we read a w this is the string and it will end up to a subset this is the DFA move and the way the DFA is constructed this is same as in the NFA we start with q 0 will reach to this. So, with that this w will reach to those sets. So, this is a subset also these two sets are same and this is true for every w. Because for w is equal to epsilon this is true because we will be remain that q 0, but for w is not equal to epsilon we you have the inductive proof of that.

So, this will give us that conclusion that they are accepting the same language because this needs a proof, but this proof is not tough this is the inductive proof is there.

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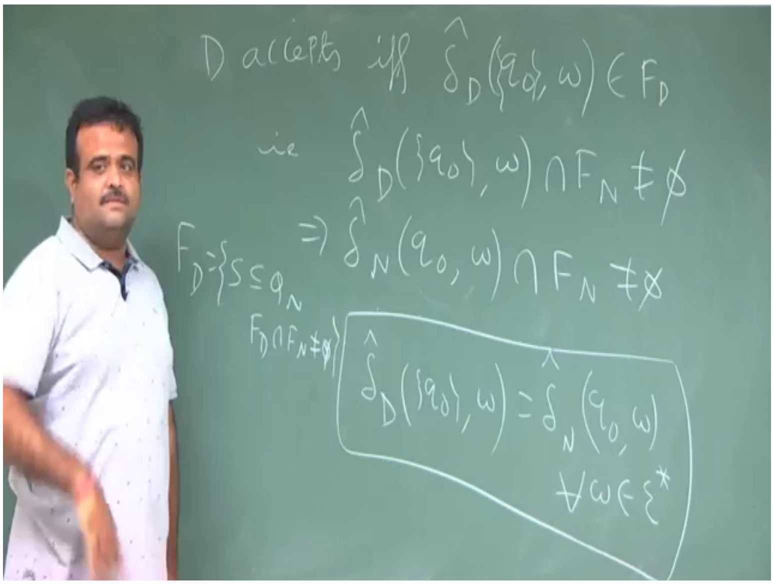

So, if we assume this is then the DFA accepts a string if and only if delta D of q 0 and w is belongs to F D and f D is what F D is the F D content one of this finite state of q. So, that is delta hat of q 0 w intersection fn is not null. So, this implies delta hat of N q 0 w intersection F N is not null so; that means, if it is if a string is accepted by this DFA then that seem same string will be accepted by this NFA because the way we have constructed F, F D.

So, F D is the accepted state of DFA. So, if it is the set of all subsets are such that. So, it is containing all subset of this Q N sorry. So, it is containing all the subset of Q N such that the F D and F A naught not null. So, at least it should have one finite state so; that means, if it is reaching to a finite state; that means, the same rule will give us to the finite state by the NFA that is the way we have constructed by the substitution. So, this is the equivalency. So, if a language is accepted by a DFA then the same language will be accept, if a string is accepted by a DFA the same string will be accepted by the corresponding NFA.

So, this is the equivalence. So, so to show a language is regular or not we just need to have a NFA which is accepting that because you know once we have a NFA we can have corresponding DFA which is accepting that same language. So, will talk about more in details.

Thank you.