

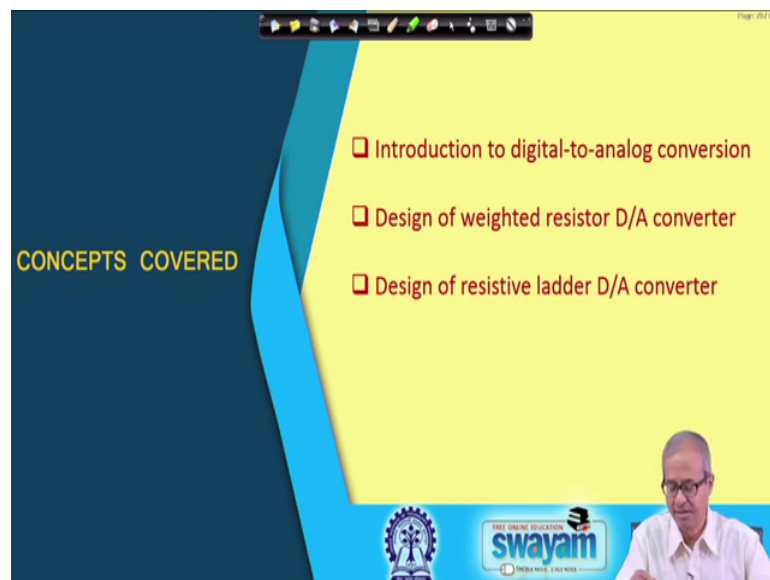
Embedded System Design with ARM
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Lecture – 12
Digital to Analog Conversion

So, when you are trying to design and implement an embedded system as I told you, you need to interface with the outside world; there will be various kinds of sensors, there will be various kinds of actuators. So, whatever voltages are coming from outside they are very rarely digital in nature, many of them are analog. Similarly when you are activating the actuators, you often need to provide an analog control signal for those. So, for that reason you need to have devices called digital to analog converter and also analog to digital converter to interface with your microcontroller processing unit

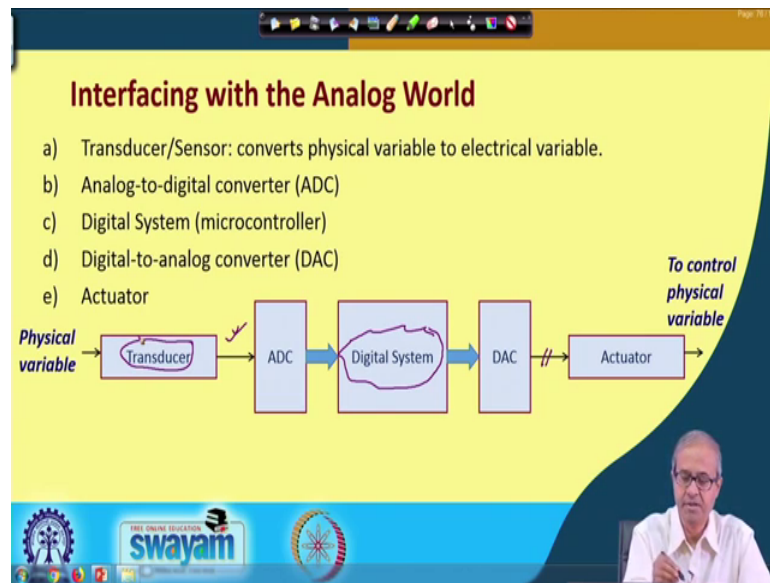
So, the topic of this lecture is Digital to Analog Conversion. So, here we shall study the different ways in which digital to analog conversion can be carried out.

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Now, in this lecture we shall be talking about some of the basics of digital to analog conversion and we shall be discussing two different alternate design styles to implement such D A converters weighted register and resistive ladder types.

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So, just to repeat to interface with the analog world so, I am showing a schematic diagram here. So, your microcontroller can be sitting here, this is the actual processing unit I am calling it the digital system. So, the physical variable that you are sensing from outside you need to use some kind of a sensor, it is sometimes called a transducer to convert it into a voltage. And an analog to digital converter will convert this voltage into a digital input and this microcontroller can read the digital input directly because, it is a digital circuit.

And similarly when it wants to activate the actuator it provides with a digital output which through a digital to analog converter it is converted into a voltage; it is an analog voltage that is fed to an actuator. So, these are the basic components of a typical embedded system right. So, you see here, you have analog to digital conversion and digital to analog conversion both. So, we first talk about digital to analog converter here.

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Digital-to-Analog Converter (DAC)

- Converts a given digital word D to a proportional analog voltage V_{OUT} .

$V_{OUT} \propto D$

D_3	D_2	D_1	D_0	V_{OUT}
0	0	0	0	0V
0	0	0	1	1V
0	0	1	0	2V
0	0	1	1	3V
0	1	0	0	4V
0	1	0	1	5V
0	1	1	0	6V
0	1	1	1	7V
1	0	0	0	8V
1	0	0	1	9V
1	0	1	0	10V
1	0	1	1	11V
1	1	0	0	12V
1	1	0	1	13V
1	1	1	0	14V
1	1	1	1	15V

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So, what is a digital to analog converter or a DA converter let us see. DA converter, you see here you look at this block diagram. We will understand what we are trying to do in the input we apply a digital word. In this example, I am showing it is a 4 bit word $D_0 D_1 D_2 D_3$. Let us call this as capital D it is the decimal equivalent of this number and in the output, we generate an analog voltage. Analog voltage means a continuous voltage which will be proportional to this digital value I am applying to the input. So, this V_{OUT} will be proportional to this D and often in this DA converter, we apply some reference voltage which will denote or tell what will be the maximum value of V_{OUT} that is possible.

Now, in this table I have just given a very small example you see here I have assumed the reference voltage to be 15 volts; just like this it is a 4 bit DA converter. So, there can be 0 0 0 0 up to 1 1 1 1, there can be 15 combinations 16 combinations in fact, and the output voltage can be like this 0 0 0 0 means output is 0 volts 1 volt 2 volt 3 volt 4 volt up to 15 volts. So, the output volt is clearly proportional to the decimal equivalent of the input digital word ok. So, in this case for every increase in D , there is a 1 volt increase in the output for this example.

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Resolution or Step Size

- Smallest change that can occur in V_{OUT} as a result of a change in input D .
- Equal to the weight of the LSB, also called step size.
- Same as the constant of proportionality in $V_{OUT} \propto D$

$V_{out} = k \cdot D$

$D: \bar{i} \text{ to } \bar{i}+1$

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So, this increase in the output for every one value increase in D this is sometimes called resolution or the step size of a DA converter. So, how do you define the resolution? This is the smallest change that can occur in the output voltage as a result of a change in D . Now D is a digital input right. So, D we are applying 0 1 2 3 4. So, whenever the value of D changes from let us say any value i to the next value $i + 1$. So, whatever is the change in the output analog voltage V_{OUT} that is my step size or resolution this is how you define. Sometimes we refer to this as the weight of the least significant bit because, when the least significant bit changes that also means an increase of 1 like I had a value 0 1 0 0 this means 4, the last bit changes to 1 0 1 0 1 it means 5 there is a increase of 1. So, least significant bit change is also the same as the resolution or the step size.

So, whenever you write it like this V_{OUT} proportional to D so, V_{OUT} will be some constant into the value of D this constant is nothing, but this step size. So, whatever value you are applying a D . So, that step size multiplied by that value that will be the actual voltage you will be getting.

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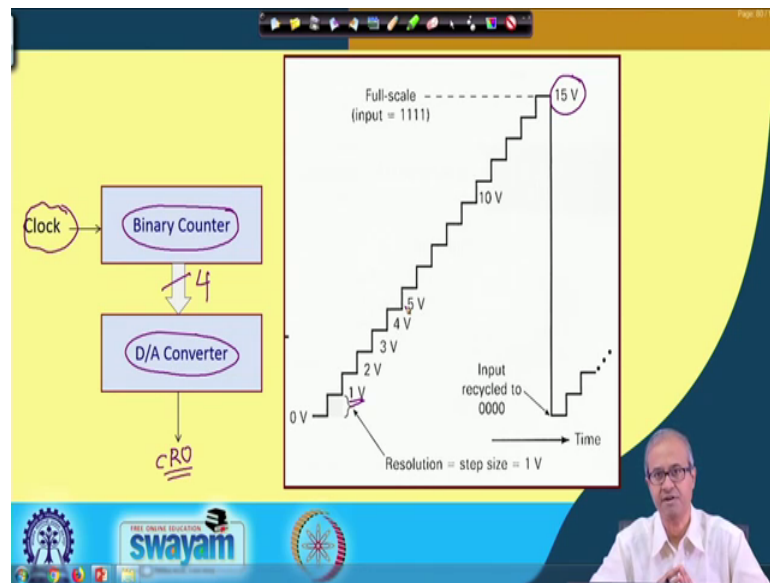
Resolution or Step Size

- Smallest change that can occur in V_{OUT} as a result of a change in input D .
 - Equal to the weight of the LSB, also called *step size*.
 - Same as the constant of proportionality in $V_{OUT} \propto D$
- Can also be defined as a percentage of the full-scale voltage:
Step-size or resolution $\Delta = \frac{V_{REF}}{2^N - 1}$, for an N -bit DAC
Percentage resolution = $\Delta / (\text{full-scale voltage}) \times 100\%$
 $= \frac{1}{2^N - 1} \times 100$

Now, this resolution can also be defined as a percentage of the full scale voltage. Like you see the step size or resolution if you call it delta, this can be defined as the full scale voltage V reference divided by the total number of steps of the DA converter like for a 4 bit converter you could go from 0 0 0 0 up to 1 1 1 which is 15. For an N bit converter you can go up to 2 to the power N minus 1. So, many steps so, the total voltage divided by the number of steps that will be the height of every step resolution. So, this is how you can define the resolution or step size for an N bit DA converter.

Now, if we want to express it as a percentage, what do you do? We divided this delta by full scale voltage which is V_{REF} . So, this divided by V_{REF} will be only 1 by this V_{REF} will cancel out. It will a 1 divided by 2 to the power N minus 1 into 100; this is the expression for percentage resolution ok. So, if you are asked to compute the percentage resolution of an N bit DA converter, you will be using this formula well it ok.

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So, let us look at one thing. Suppose we have a DA converter like this and to the input of the DA converter let us say, we have connected the binary counter which counts up and we apply a clock continuously the clock is coming. So, the counter will be counting up 0 1 2 3 4 like that, let us say this is a 4 bit DA converter. So, there are 4 number of lines here and it is a 4 bit counter.

So, as the clock counts the counter will continuously go from 0 1 2 3 from 15 and then again it will go back to 0. So, if I observe the waveform that is being observed at the output of the DA converter, suppose I connect a cathode ray oscilloscope in the output. I will see an same waveform like this, that the output will go step by step from 0 to 15 and then again it will go back to 0 again 1 2 3 4 like this.

Now, depending on V_{REF} , the step height will be adjusted. So, if V_{REF} equal to 15; that means, the full scale voltage is 15 then your step height will be 1 volt. So, it will be 1 2 3 4. So, using oscilloscope you can also see that, what is the voltages this is the kind of waveform you are expected to see in the output of the DA converter alright.

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Types of D/A Converter

- We shall discuss two different designs of digital-to-analog converters.
 - a) Weighted resistor type DAC
 - b) Resistive ladder type DAC
- The first type is easier to analyze, while the second type is more practical from the point of view of implementation.

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Now, talking about the types of D A convertor, I had mentioned in the beginning that we shall be talking about broadly 2 types of D A converter which is most common; one is called weighted resistor type, other is called resistive ladder type. Now the point is that the first type is easier to build and understand how it works. But there are some problems and issues in the design we shall see, but the second type is easier to build, but it is a little more complex in terms of the circuitry. Let us see.

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Weighted Resistor Type DAC

- For an n -bit DAC, it consists of n different resistance values of magnitudes $R, 2R, 4R, \dots, 2^{n-1}R$ respectively.
- The resistances help in generating currents inversely proportional to their magnitudes.
- The total current is added up by an operational amplifier, and is converted to the voltage output V_{OUT} .

V I $\frac{V}{R}$

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So, we first start with the weighted register type D A converter. Now let us consider design of an n bit D A converter, in general there are n number of bits which are coming in the input. The first thing is that to construct this kind of a D A converter, we need n different resistance values and this is one of the main problems in manufacturing this kind of D A converter. You see you can very accurately manufacture 1, 2 or 3 resistance values, but if I tell you require 16 different resistance values; you have to manufacture all of them very accurately, it becomes that much more difficult. So, this method requires n different resistance values to be used and the magnitudes will be some base value R, 2R, 4R; that means, multiples of 2; 4R, 8R and this will go up to 2 to the power n minus 1 into R.

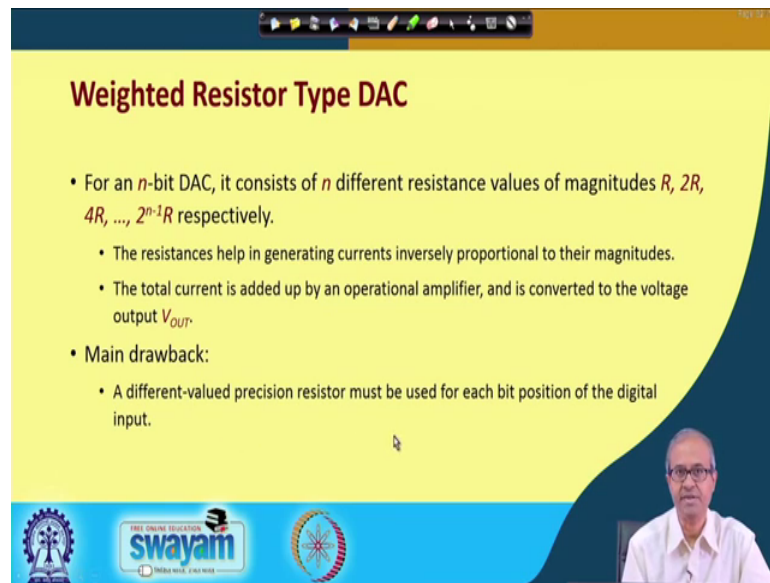
So, many different resistance values and this resistance values are actually responsible for generating some currents like let us say you have a resistance here and we apply a voltage let us say the resistance is small r. And at this end suppose I ground this point so, the current that will be flowing will be given by V by r. So, if I use n different resistance values, the current will be determined by the value of the resistance, the r resistance will be having some value, 2R will be having half the current, 4R will be having half of that and so on. The current values will be changing in again multiples of 2 ok.

Now, if I have a mechanism to calculate the total current that is finally, coming out I shall see the circuit we shall show, then that will give you a final DAC output, if you can convert it into a voltage.

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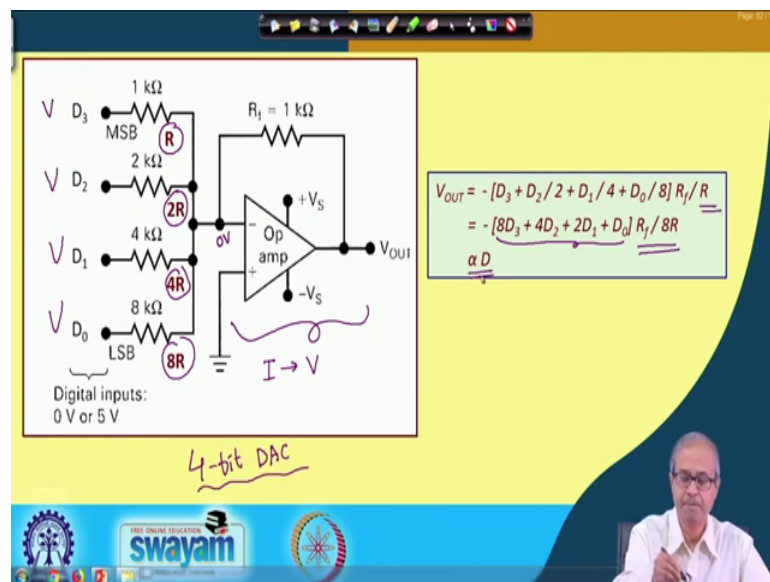
Weighted Resistor Type DAC

- For an n -bit DAC, it consists of n different resistance values of magnitudes $R, 2R, 4R, \dots, 2^{n-1}R$ respectively.
 - The resistances help in generating currents inversely proportional to their magnitudes.
 - The total current is added up by an operational amplifier, and is converted to the voltage output V_{OUT} .
- Main drawback:
 - A different-valued precision resistor must be used for each bit position of the digital input.



So, we shall see that. So, main drawback I already mentioned. So, you need to use so many resistances.

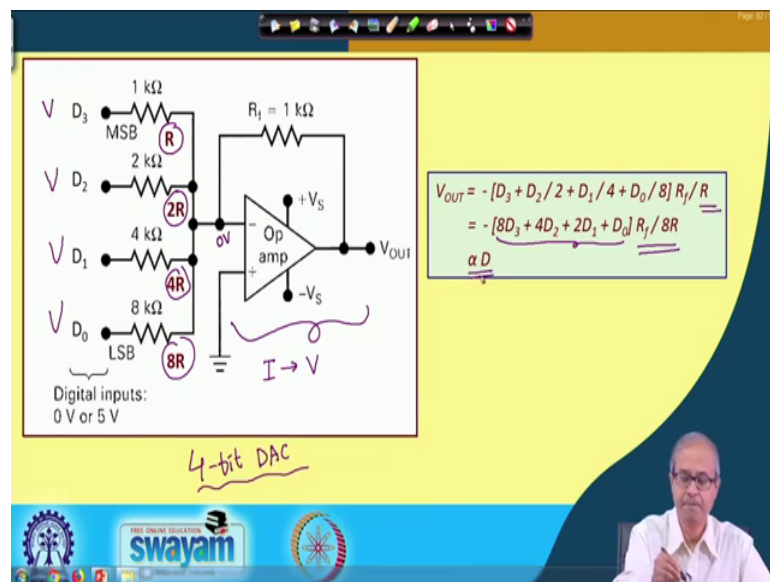
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$V_{OUT} = -[D_3 + D_2/2 + D_1/4 + D_0/8] R_f/R$
 $= -[8D_3 + 4D_2 + 2D_1 + D_0] R_f/8R$
 $\propto D$

4-bit DAC

Digital inputs:
0 V or 5 V



This is how this circuit looks like. So, here I am showing a 4 bit D A converter of weighted register type. So, you see the values where using are R $2R$ $4R$ and $8R$ just as an example I am showing it to be 1 kilo ohm, 2 kilo ohm, 4 kilo ohm and 8 kilo ohm; 4 values and the digital inputs are applied like this D_0 D_1 D_2 D_3 least significant bit is connected to the highest resistance most significant bit is connected to the lowest

resistance. On the output side, we are using an operational amplifier. So, I am not going to the detail how an operation amplifier works, but basically this operation amplifier converts the current into a proportional voltage whatever current is flowing and another point is that this point of the operation amplifier where you connecting this is at a ground potential; this is called virtual ground.

So, the currents that are flowing if you just look at it the first one will be let us say I am applying some voltage V here. V means logic 1 V by R V by $2R$ V by $4R$ V by $8R$ like that. So, if you finally, try to find out what will be the value of V_{OUT} so, it will be D_3 divided the R if you take this R outside plus D_2 by 2 divided by $2R$ this 2 will be there plus D_1 by 4 plus D_0 by 8 . And R_f by R this is a characteristic of the op-amp when the current is converted to voltage this will get multiplied by R_f and if you do a little simplification if you take this 8 out R_f by $8R$ this becomes $8D_3$ plus $4D_2$ plus $2D_1$ plus D_0 . This is the decimal equivalent of the binary number $D_3D_2D_1D_0$ which means V_{OUT} is proportional to D .

So, this is how the weighted register DA converter works. This is actually quite simple to understand. So, each of the Resistances is generating a current that is proportional to the weight of that particular bit and all the currents are added up by the operation amplifier and it is converting into a voltage V_{OUT} ok; this is how it works.

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Resistive Ladder Type DAC

- Most widely used, and requires only two different values of precision resistances (R and $2R$).

$V \propto D$

4-bit DAC

Voltage Follower

V_{out}

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Let us now come to the other type resistive ladder type. See here also I am showing you this schematic diagram of a 4 bit D A converter. The first thing you notice here is that the number of resistances required are much more. In the earlier method, we are requiring only 4 resistances, but here if you count 1 2 3 4 5 6 7 8. So, you are needing 8 resistances. But the good thing is that the values of the resistances are only of 2 types R and $2R$, you do not need to use many different values of the resistances right. Now this circuit, we shall just analyze this circuit this circuit that I am showing you on this side; this directly generates a voltage here.

So, it is not that it generates a current which has to be converted to a voltage it directly produces a voltage which will be proportional to this digital input that you are applying. And in the output again, we are using an operational amplifier circuit, but we are connecting this op-amp in a different way as you can see. This is called a voltage follower circuit. So, it does not convert anything to anything, it is voltage it remains a voltage, but it actually creates like a buffer so, that when the output is connecting to some other circuit that circuit should not disturb the operation of this D A converter circuit ok. Now, let us see how this voltage V which is generated here, this gets proportional to the digital input D . This is something we need to be convinced first right. Let us see that.

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Calculation

- Let us calculate the voltages at the opamp input when exactly one of the D_i inputs is at 1 (say, $+V$ volts).
- Case 1: Input is 1000

The diagram illustrates a resistive ladder network for a 4-bit DAC. It features four digital inputs at the top, labeled D3, D2, D1, and D0. D3 is connected to a positive voltage source $+V$, while D2, D1, and D0 are connected to ground (GND). The ladder is composed of four vertical resistors, each with a value of $2R$, and four horizontal resistors, each with a value of R . The output voltage V_x is measured at the right end of the ladder. A handwritten note on the right shows the equivalent circuit for the first two stages, where two $2R$ resistors in parallel are equivalent to a single R resistor.

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So, we calculate the voltages at the op-amp input let us say we assume that at a time one of the input is at 1 other 3 inputs are at 0. Let us start with this assumption. The first case let us assume that the input is 1 0 0 0; that means, just one bit is 1 and the all other bits are 0. Let us see what happens here. So, the circuit looks like this. This 1 is the most significant bit. Now in this circuit the rightmost input is the most significant bit and the other 3 are the lower significant these are all 0's. So, we have connecting this to ground, ground and ground ok.

Now, let us try to analyze this circuit. You see here these 2 Resistances 2 R and 2 R they are connected in parallel to ground. So, 2 resistances connected in parallel both to ground both are 2 R and 2 R. This is equivalent to a single resistance R connected to ground 2 R 2 R in parallel right. So, in this way let us do some simplification.

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Calculation

- Let us calculate the voltages at the opamp input when exactly one of the D_i inputs is at 1 (say, +V volts).
- Case 1: Input is 1000.

$$V_x = \frac{V \cdot 2R}{2R + 2R}$$

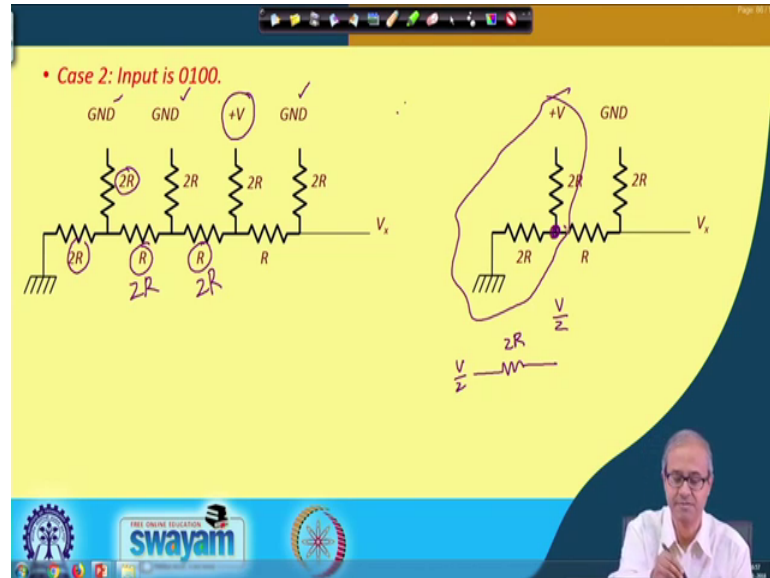
$$V_x = V/2$$

This becomes R then that R and this R will come in series this R and this R will come in series. So, the net resistance will again become 2 R. So, again this 2 R and this 2 R will come in parallel that will become R that R and this R again in series this will become 2 R this 2 R and 2 R in parallel. So, again this will become R this R and this R will become 2 R. So, finally, you will be having a circuit like this; this is the equivalent circuit 2 R connected to plus V and this side equivalently 2 R to ground.

And this is a voltage divider circuit V. So, the output voltage VX will be V into 2 R divided by 2 R plus 2 R. So, it is nothing, but V by 2. So, if you calculate this VX, VX

will be just V by 2 . So, the first thing is that when I apply one 000 only the MSB is 1 , the output voltage is V by 2 just remember this ok.

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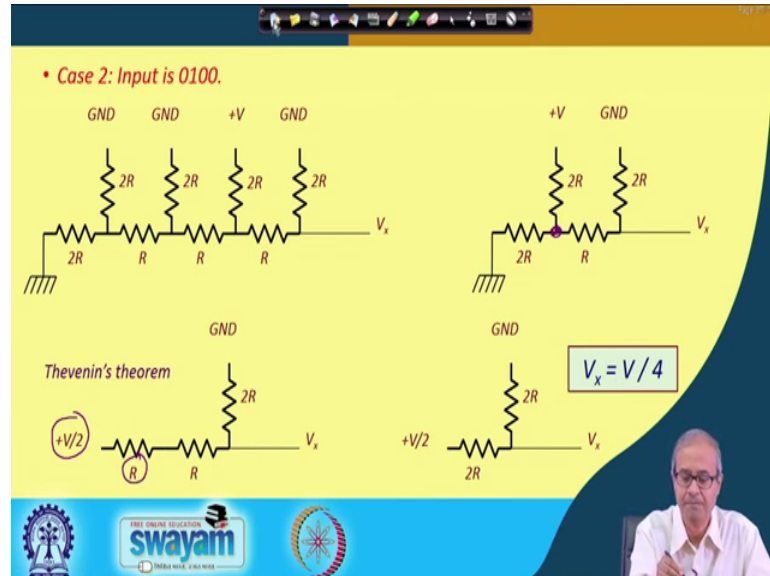


Next let us assume that this second bit is 1 . This second MSB, most significant bit is 1 . So, this scenario is this second MSB is 1 ; that means, I am applying a plus V here all other 3 are ground 000 . So, following our same philosophy or argument what we did for the previous case, these 2 resistances are coming in series. They become R that R and this R in series becomes $2R$. Again this $2R$ and this $2R$ in parallel becomes R . So, R and R in series becomes $2R$. So, we come to a point where our equivalent circuit looks like this. So, I have plus V here $2R$ here around the left my equivalent resistance is $2R$; so, it is $2R$. See here I cannot combine them in parallel because one of them is connecting to ground other is connected to plus V in the earlier cases both were connected to ground that is why you could use the parallel combination formula.

Now, here let us see what is happening here. I have a circuit like this we apply something called Thevenin's theorem. Here Thevenin's theorem says that if we have a circuit like this let us look at this point. This whole circuit can be replaced by a voltage source which is equal to the voltage that is coming here; you forget the remainder of the circuit, forget this part of the circuit only this part so, $2R$ $2R$ V . So, the voltage will be V by 2 and assuming this voltage is 0 , you connect it to ground and look at the equivalent it is $2R$.

So, V by $2 \cdot 2R$ you assume that there is a voltage V by 2 with a resistance $2R$ in series. You can connect it to this equivalently this is what Thevenin's theorem says.

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So, you see so if we apply Thevenin's theorem so, this point this point let us say here you replace it by a voltage source V by 2 and an equivalent resistance R . So, again this R and R comes in series becomes $2R$ and $2R$ $2R$ in parallel not an unparallel; this is V by 2 here. So, it will become something like this V by 2 here $2R$ here and $2R$ here to ground. So, again this is a voltage divider circuit one side V by 2 one side ground. So, the output will be V by 2 into $2R$ by 2 plus $2R$ plus $2R$. So, it becomes V by 4 . So, if you just calculate it becomes V by 4 . So, for the earlier case when the MSB is 1 , it was V by 2 when the second one is 1 , it is V by 4 ok; so, it is becoming half.

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• Case 3: Input is 0010.

Thevenin's theorem

$V_x = V/8$

Let us look at the next bit. So, what happens when I apply 0 0 1 0? It is like this 0 0 1 0. So, again similarly the first 2 resistances here, this you can connect in parallel and then do a series you get an equivalent circuit like this. So, you have V by 2 here and you combine this again apply Thevenin's theorem here. So, at this point so, you get V by 2 here and $2R$ here and again you apply Thevenin's theorem.

Here you get V by 4 here and $2R$ here. So, at the end you have a scenario where you have a circuit like this; again a resistance divider V by 4 $2R$ $2R$ to ground. So, the resistance will become V by 8. So, you see V by 2 V by 4 V by 8; it is progressively becoming half and half. Let us see whether the same thing happens for the last one where the input is 0 0 0 1.

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• Case 4: Input is 0001.

Thevenin's theorem

Thevenin's theorem

Thevenin's theorem

Thevenin's theorem

$V_x = V/16$

So, it is like this 0 0 0 0. So, here from the first step it itself you have to apply Thevenin's theorem because there is ground on one side and V on other side. So, you apply V by 2 here and 2 R to replace this you get a circuit like this. So, we apply Thevenin's theorem here again, you get this V by 4 and 2 R. You apply Thevenin's theorem again you get V by 8 and 2 R and finally, you will see that VX becomes V by 16. So, effectively you see as the bits are changing MSB to LSB the weight of each of the bit is becoming V by 2 V by 4 V by 8 and V by 16.

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• When all the four inputs $D = D_3D_2D_1D_0$ are applied (where $D_i = GND$ or $+V$ volts), we can apply the principle of superposition to compute the final output voltage V_A .

D_0 D_1 D_2 D_3

V_A V_{out}

Now, the question is when all the 4 bits are applied together, what will happen? When the bits are applied together, then there is a well known theorem in circuit theory that is called principle of superposition. It says that when multiple inputs are applied to a linear circuit resistance is linear, then the output will be the same as the sum of the outputs where you are applying the inputs one at a time. So, whatever you are applying you may imagine that one input is being applied at a time, you calculate add all the voltages up you will be getting the final voltage. So, using principle of superposition you can use this circuit to carry out the final calculation that will give you the final output V A or V X whatever you call that value.

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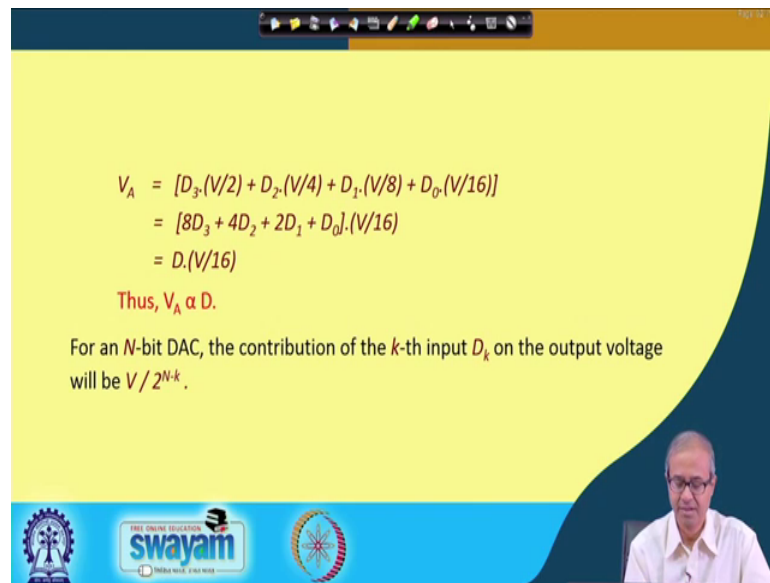
$$\begin{aligned}
 V_A &= [D_3 \cdot (V/2) + D_2 \cdot (V/4) + D_1 \cdot (V/8) + D_0 \cdot (V/16)] \\
 &= [8D_3 + 4D_2 + 2D_1 + D_0] \cdot (V/16) \\
 &= \underline{\underline{D \cdot (V/16)}}
 \end{aligned}$$

$D_i :: \begin{matrix} 0 - 0V \\ 1 - 5V \end{matrix}$

So, you will be getting some expression like this. So, where D_3 will be having a weight in V by 2 we said D_2 into V by 4 D_1 into V by 8 D_0 into V by 16 where this D_i is are the digital inputs ok. So, when you are applying 0, this will be 0 volt; when you are applying 1, let us say this will be 5 volts.

So, these voltages will be applied here. So, if you take this V by 16 out you get $8 D_3 + 4 D_2 + 2 D_1 + D_0$ which is again the decimal equivalent of the binary number. So, D into V by 16 means V A is proportional to D.

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The slide displays the derivation of the output voltage V_A for a 4-bit DAC. The equations are:

$$\begin{aligned}V_A &= [D_3(V/2) + D_2(V/4) + D_1(V/8) + D_0(V/16)] \\ &= [8D_3 + 4D_2 + 2D_1 + D_0] \cdot (V/16) \\ &= D \cdot (V/16)\end{aligned}$$

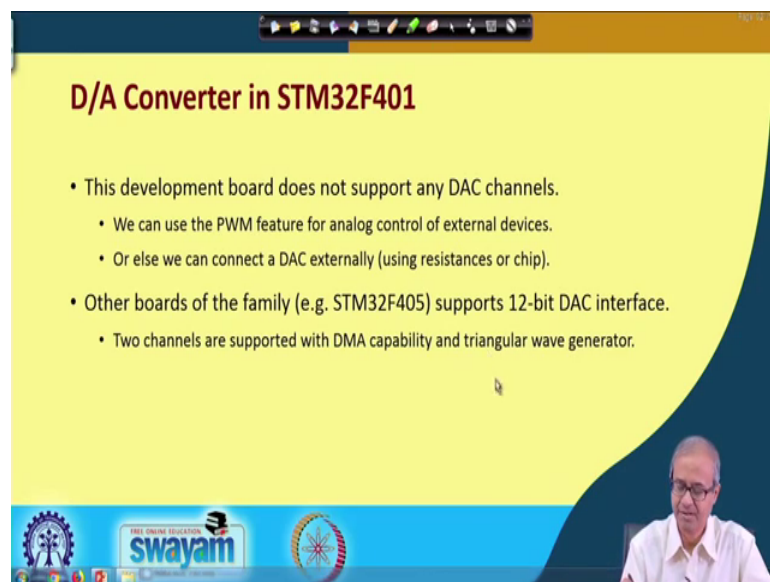
Thus, $V_A \propto D$.

For an N -bit DAC, the contribution of the k -th input D_k on the output voltage will be $V / 2^{N-k}$.

The slide also features the Swayam logo and a small video feed of the presenter in the bottom right corner.

V_A is proportional to D . So, for N bit D A converter in general if you look at the k -th bit, the contribution will be V by 2 to the power N minus k in general. So, we checked it for a 4 bit D A converter ok.

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The slide is titled "D/A Converter in STM32F401" and contains the following bullet points:

- This development board does not support any DAC channels.
 - We can use the PWM feature for analog control of external devices.
 - Or else we can connect a DAC externally (using resistances or chip).
- Other boards of the family (e.g. STM32F405) supports 12-bit DAC interface.
 - Two channels are supported with DMA capability and triangular wave generator.

The slide also features the Swayam logo and a small video feed of the presenter in the bottom right corner.

Now, the point to notice that for the STM32F01 board that we are using, this board does not have any support for a D A converter, but I told you the board already has PWM capability pulse width modulation and using PWM also we can do something which is

very much similar to D A conversion. We can convert this duty cycle into an equivalent average value, average voltage which is like the output of a D A converter.

So, we can have something very similar to a D A converter without using a D A converter using PWM alone ok, but there are some other family like F405 that is another family of those boards where a 12 bit D A converter is built into the board. So, where you can actually use D A conversion to generate an analog output ok. So, with this we come to the end of this lecture where we had discussed the process of digital to analog conversion. In the next lecture, we shall be starting our discussion on the reverse analog to digital conversion.

Thank you.