Discrete Structures

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Lecture – 09
Predicate Logic (Contd.)

We have read the Predicate Logic and the two quantifiers the universal quantifiers and the existential quantifiers. And mainly the, how these universally quantified statement and the existentially quantifier statements are represented logically. Another thing that how De Morgan's law is applied on them; that means, what are the relationships between the existentially quantifier statement and the universally quantified statement or how to negate, simply I can tell that how to negate them.

Now, in real life there are complex statements are there, most of the problem they involve complex statements and if we try to represent them logically or if we try to represent them with our predicate logic then more than one quantifier are normally involved. So, we must read or we have to learn that how the more than one propositions or the predicates are connected with more than one existential quantifiers or the universal quantifiers. How they are represented or how they are converted from one to another; how De Morgan's law can be applied on this type of complex statements involving more than one quantifiers.

We generalize these topics as the nested quantifiers which involves more than one universal or existential quantifiers, right, nested quantifiers.

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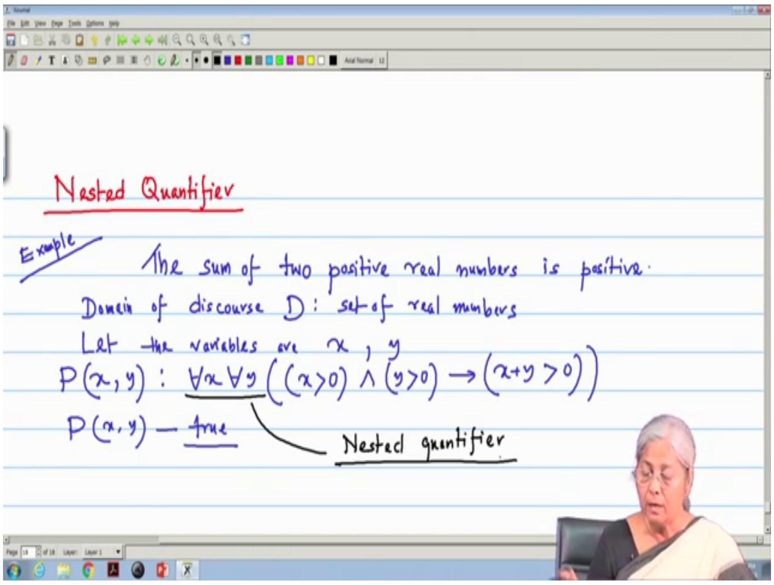

We take one simple example, ok. The sum of, right, the sum of two positive real numbers is positive. So, first thing is here the domain of discourse D is real number set of real numbers. See at two positive real numbers, so let the variables involved are x and y.

So, how we can write these statements or the proposition? Since it involves two variables, so instead of P x or P y, I write P x y which tells that for all x and for all y x greater than 0 and y greater than 0 implies x plus y greater than 0. Now, we have to check whether P x y is true or P x y is false. So, we can represent since there are two variables, so the bound variables are x and y, for all x for all y and we see whether this is true or false. Now, since x is set of real numbers we know that for all values of x and y this is always true. So, we tell that P x y, P x y is true.

Now, see this representation for all x for all y, so this we call the nested quantifier here both are of same type, ok. Now, this can be a different type quantifier also. So, another example if we take another example.

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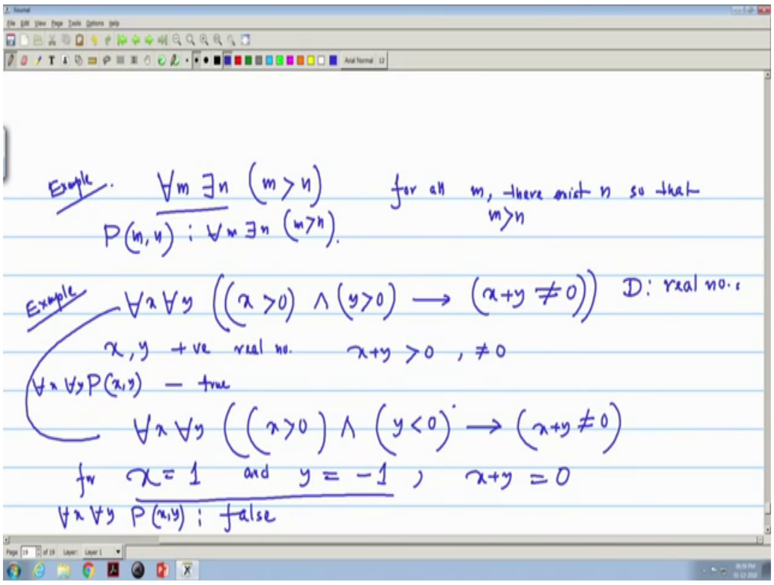

Say for all m there exist n, m greater than n. We write, this is nested so I can write P m n, actually we should write for all m there exist n so that m greater than n.

Now, how to find out the truth values? True or false. So, the previous example that for all x, for all y, x greater than 0 and y greater than 0, so implies x plus y equal to 0 say if I take x plus y not equal to 0. Then how to check? So, if we have to show that this is false then we have to find out at least one value of x and y so that this statement becomes false. So, we take these set of real number. So, x greater than 0 y greater than 0 x plus y not equal to 0; and this is always true. Since they are positive x y are positive real numbers, so x plus y always greater than 0; that means, which is not equal to 0. So, I can write that P x y is true or for all x for all y.

Now, slightly if we change this thing say for all x, for all y x greater than 0 and y less than 0, implies x plus y not equal to 0. Same D is real set of real numbers. Then we get at least one value of x say for x equal to 1 greater than 0 and y equal to minus 1 we get x plus y equal to 0. So, we can get one value of x and y for which the statement is false. So, we can write that for all x, for all y, this P x y this is our P x y is a false. So, the similar way that only one universal quantifier we have checked that whether it is true or false, now we can check or we can find out the truth values of the nested quantifier, ok.

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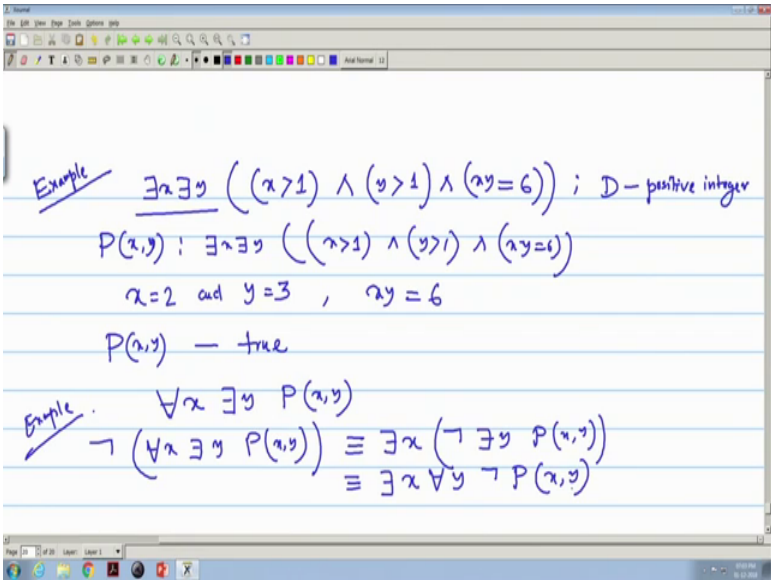

Now, we take one example of existential quantifier, example of existential quantifier. So, there exist x, there exist y, x greater than 1 and y greater than 1 and x y greater than or x y, I take x y equal to 6 or some value I take. You take the D is domain of discourse is the positive integer, D is positive set of positive integer. Since though here it is existential quantifier that means, if it is true for one value of x and one value of y; that means, if we can find out at least one value of x and at least one value of y for which this is true then the predicate is true.

So, P x y I can write there exist x there exist y x greater than 1, y greater than 1 and x y equal to 6. So, for this simple example we get that for x equal to 2 or 3 and y equal to 3 or x equal to 3, y equal to 2, we get x y equal to 6. So, this statement is P x y P x y is true.

Now, how to apply De Morgan’s law? We see how we can apply de Morgan’s law on that. We take one example. So, if we take the negation of this statement for all x there exist y, P x y which is equivalent to negation of since it is nested. So, no negation of for all x is our there exist, the negation there exist y, P x y, this is equivalent to there exist x; negation there exist y means this is for all y negation P x y. So, the De Morgan’s law when it is applied through our predicates or the more than one quantified statement we can apply and we know that the negation of for all x P x is equivalent to there exist x negation P x and there exist x P x if we take the negation of that we get for all x negation P x.

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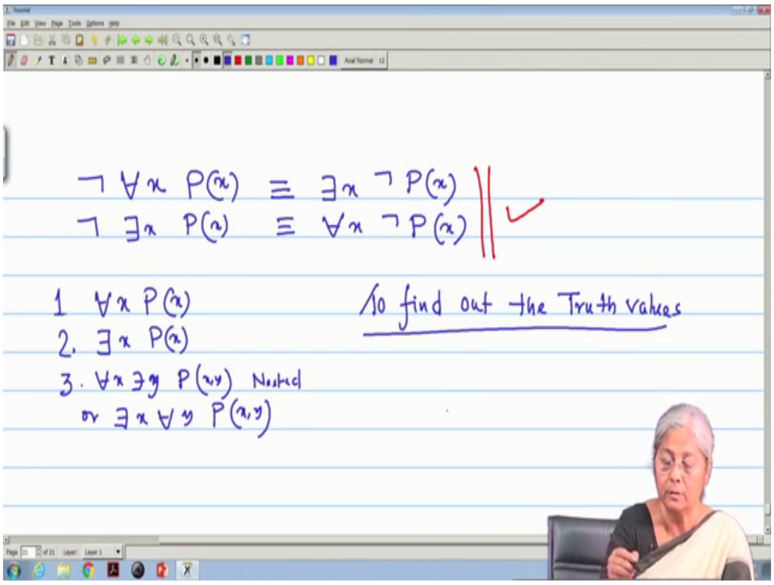

So, given one complex statements having more than one quantifiers the negation if we do from the left hand side we will start converting or applying the de Morgan’s law and we will apply these two De Morgan’s law on that.

Now, we try to find out the pseudo code to find to get the truth values of the quantifier, ok. So, we have three quantifiers, one is for all x 2 and 1 I am telling that when it is we have considered one we had take taking that nested, nested either for all x there exist x, for all x there exist y some, P x y or there exist x for all y P x y. Now, how we can find out the pseudo code? Sorry, find out the truth values, ok. We will see that thing.

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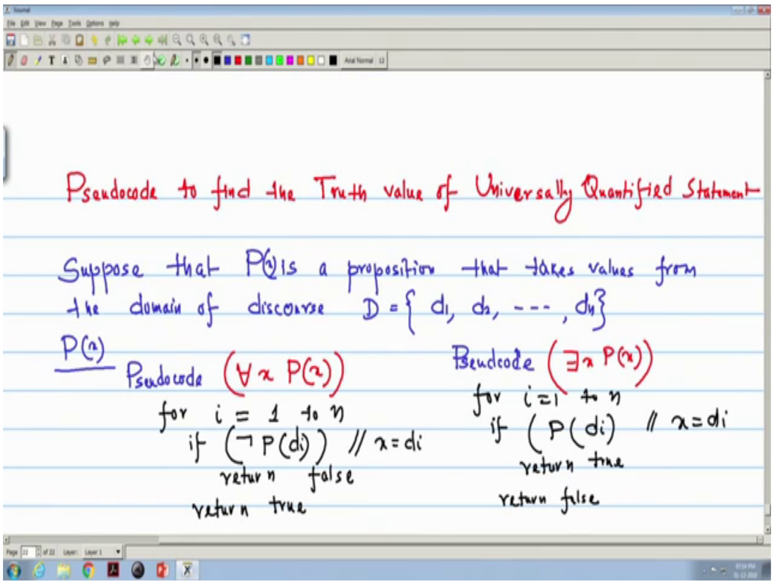

First we see the pseudo code for the universal quantifier for all x. Suppose that P is a proposition that takes values from the domain of discourse D, which is a set of D values d 1, d 2 up to d n. That means, when I consider P x actually x takes the value of d 1, d 2, d n and P x is my proposition, P x is the proposition.

Now, the pseudo code we can write, because already we know the, how to we know how to find out the truth values when it is when it involves a universal quantifier for all x or the existential quantifier. So, I write the pseudo code say for all x. We can write for i equal to 1 to n, because for all x it is for at least one value it should be false. So, if negation P x, then return false else return true. So, better I write this thing pseudo code for all x P x. See if it is only for i equal to 1 to n if negation P x; that means, for one value of i or 1 x can take one value for which this becomes false then it is false no need to check others. But for all values of x for i equal to 1 to n that it is true then my P x is true.

So now, I can easily write that the pseudo code for the existentially quantified statement we take that now P is the existentially quantified statement; that means, there exist x P x and similarly we can write for i equal to 1 to n. If this time it will be totally reverse. So, if P di, because one value of x P di, then it is return true, return false. So, here we can take one value, we can take one value of di represent that is di; that means, x is taking the value of; so this is I am writing x equal to di I take x equal to di. So, for nested quantifier just extend this idea of writing the pseudo code we can write the pseudo code.

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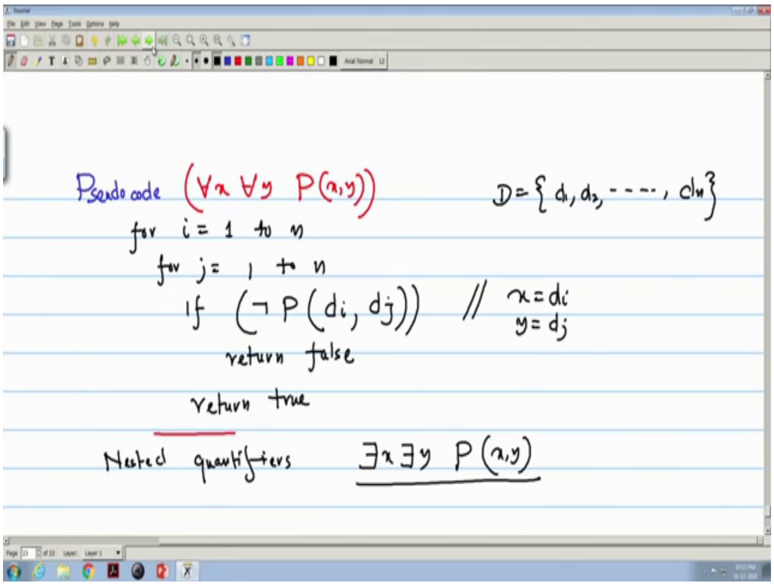

Say for all x, for all y since it is nested I take same. So, for i equal to 1 to n for j equal to 1 to n our nested loop. If negation P this time it will take two values of x and y, so did j. So, if negation P is true; that means, return false, return true. So, here x equal to di, y equal to dj, for the domain of discourse is the set of values of d 1, d 2 up to d n. And for all nested quantifiers the other cases say for all x, there exist y or there exist y and there exist x, we can write similarly the pseudo code, we can find out the values.

Now, quickly we see one example how we can write this thing.

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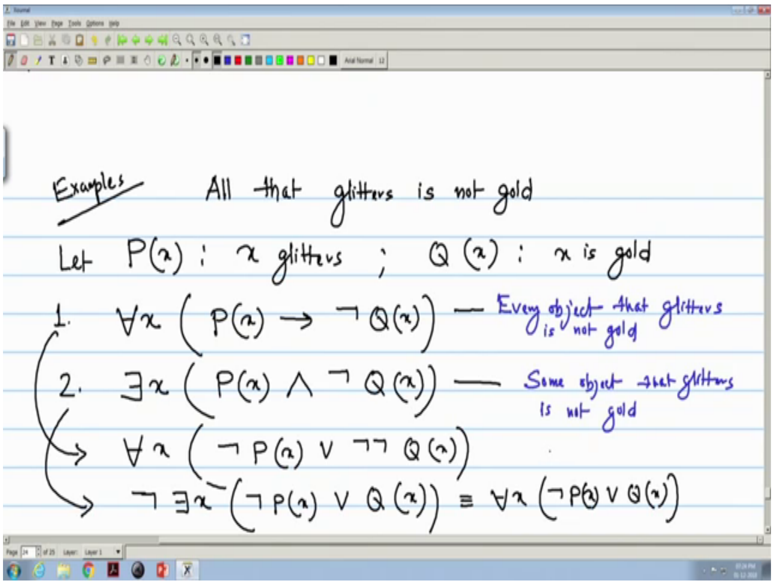

So, we take one example that one statement we take all that glitters is not gold. So, here there are two propositions we can define, we can write one is P x, let P x is defined as P x that x glitters and Q x is x is gold. So, we can write for all x P x implies negation Q x that means, for all x P x is x glitters all that glitters or we can write that there is a exist x, P x and negation Q x. Here every object I can tell that the first one tells every object that glitters are not gold.

Here we can write the, second one we can write some object that glitters is not gold. And the second one that the there exist x; that means, some object there exists some object for which we can tell that if it is glitters and that is not gold. So, we can actually you can write that there exist x see though or see we get from we can apply the our all the logic here, we can apply the logic that say from one we can tell that for all x see P x implies negation Q x.

So, you know that this is negation P x or negation of negation Q x and similarly from 2 also we can write that there exist x negation, negation P x or Q x. So, 1 and 2 are the same. Here also is same negation of, double negation means Q x only. So, this is from 1 and this is from 2 that if I take the negation that negation P x or Q x. So, this is negation of this is for all x, ok. If I take negation of P x double negation if we take, so which is same as set of equivalent to for all x negation P x or Q x. So, we see that both one and two are both one and two are same.

So, with this example what we see that whatever we have applied in our propositional logic the rules, that is applicable in predicate logic and we see that with the application of de Morgan's on predicates we get the same results. So, for complex statements with one quantifier or more than one quantifier that we learn how to represent or how logically they can be written or how they can be converted from one statement to another statement using De Morgan's law.