Discrete Structures
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Lecture – 08
Predicate Logic (Contd.)

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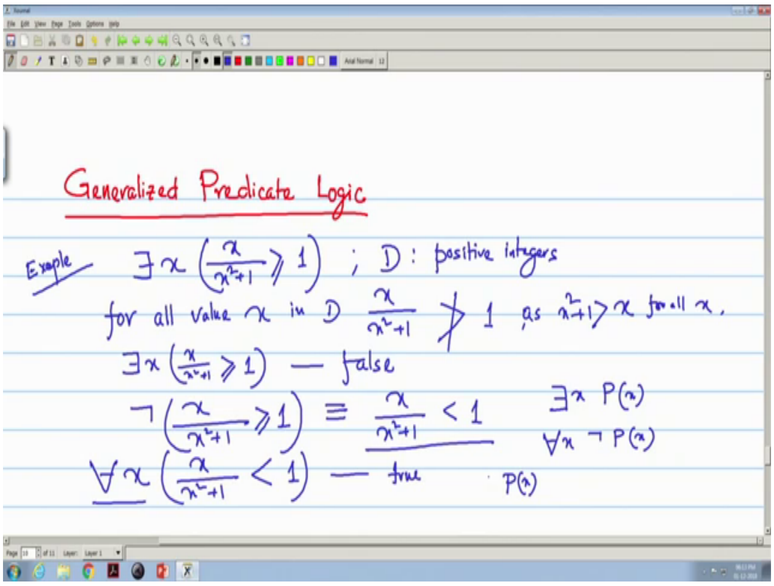

We have read the Predicate Logic and the two quantifiers the universal quantifiers and the existential quantifiers and mainly how we can write, find the truth values of the statements using this two quantifiers. Now, we read the generalized predicate logic and the De Morgan's law. So, today we see that generalized predicate logic. First we will see that how the same statement can be represented using the existential quantifier as well as they using that universal quantifier or whether at all it is possible. So, first we see one simple example we take one simple example like say there exists x, x by x square plus 1 is greater than 1. Say this is one existentially quantified statement and x can take the value from the set D; that means, D is the domain of discourse which you can take some positive integers. So, D are the positive integers.

Now, since it is a existentially quantified statement. So, if we get one such x for which is x by x square plus 1 greater than 1 then the statement is true. Now, for any positive integers since x takes any value; x takes value from D, so, for all values of x in D this x by x square plus 1, we know that never is greater than 1 because x square plus 1 as x square plus 1 is greater than x for positive integers, for all x. So, for all values of x, this is false. So, these existential quantified statement is false, this is false.

Now, I can write if I what we have seen that for all values of x this is not true, this is false. So, then can I if I just invert this thing that or we can take negation x by x square plus 1 greater than 1. So, what is that? That is equivalent to that x by x square plus 1 less than equal to 1.

So, now can we check this thing that for what happened for all values of x, these x by x square plus 1 less than equal to 1. Now, since it is the universal quantifier for all x so, if we have to show that it is true then for all values of x in D this should be true. Now, see that this is the negation value this is the value of negation and we see that for any value of x or for x any value of x in D, this is true x by x square plus 1 is less than equal to 1 actually it is less than 1. Only if we consider that x equal to 0 then it is then also it is less than 1. So, if I take this is change that ok, if I take this thing as the greater than equal to 1 and negation we take, negation we take then we take this is less than this is greater than. So, we take this is less than I think then we can tell that it is true; that means, for x value for x in D this is true.

Now, see that this is nothing, but the negation of the statement and these there exist x what I have earlier we have given there exist x P x and here this is for we have taken for all x negation P x. Now, we can write, then we can write the same p x the propositional function using the existential quantifier or using the universal quantifier, but obviously, that it will be the totally reverse or the negation here it will be the statement it will tell either it is true or it will be false.

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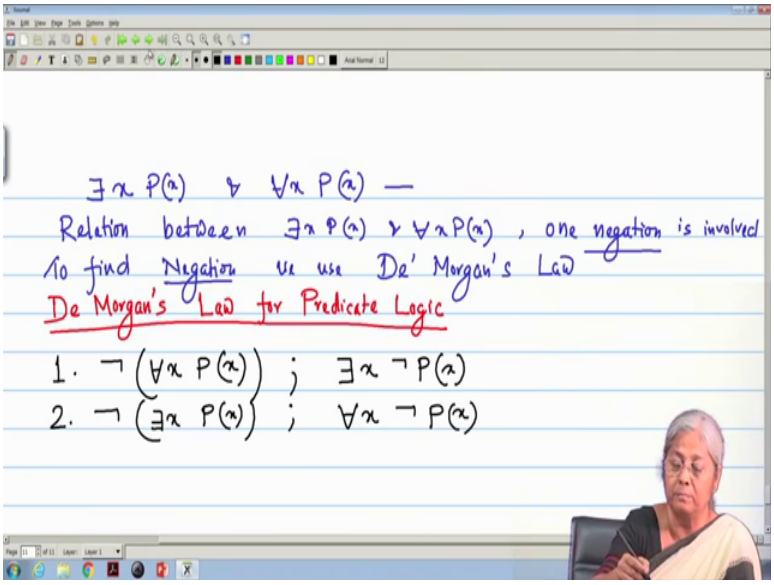

So, what we can tell from these simple example, we can think that there exist x and or I should write there exist x P x and for all x P x the relation or they are connected or the relation between these two relation between these there exist x and for all x, one negation is involved. Since in the last example what we have seen that x by x square plus 1 greater than equal to 1 and the negation is that x by x square plus 1 less than 1 and then we can write if one is written using there exists x then another we can write a using for all x.

Now, how to find negation? Already we have read to find negation we use the De Morgan's law we use De Morgan's law that we have used in our propositional logic. Now, here we see how we can use De Morgan's law for all the predicate logic. So, the Morgan's law for predicate logic. Since we have two quantifiers, so, here we must have two pairs of statements that should be equivalent. So, we can write that one is for universal quantifier. So, how to find that negation for all x P x and this is there exist x negation P x, second one is for negation there exist x P x and for all x negation P x.

So, for both these pairs they are actually equivalent. So, we have to prove this thing. So, this is the relation between the existential quantifier and the odd existentially quantified statement and the universally quantified statement and they are related by a negation what I mentioned earlier that one negation is involved.

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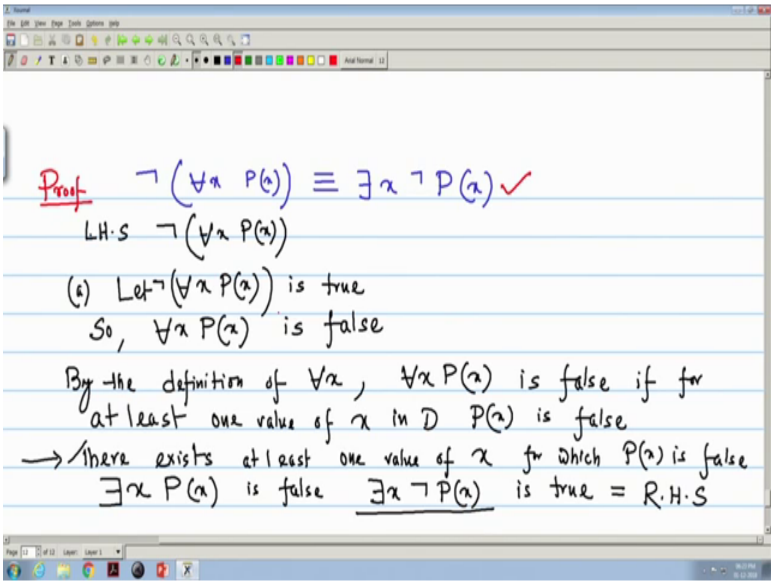

Now, we prove one of the statement or one of the De Morgan's law. So, we prove the negation for all x P x whether they are equivalent there exists x negation P x the first pair we see. So, we start with the left hand side. So, this is one universally negation of some universally quantified statement. So, our LHS is some negation of universally quantified statement. So, the statement can be true or false.

So, first we consider the case that let the statement for all x P x is true or you take the negation take negation for all x P x is true. So, for all x P x if we omit negation so, the reverse is this should be false. Now, according to the definition, the basic definition of the universal quantifier for all x the universally quantified statement is false if for at least one value of x in D P x is false. So, since it is false; that means, there exists at least one value of x for which P x is false.

So, how by using the notation existential quantifier; this statement we can write there exist at least one; that means, there exist x P x is false; that means, negation P x is true P x is false which tells that or we can tell that there exists x negation P x is true. So, negation P x there exists x negation P x is true and we started which is nothing, but our right hand side. So, what we have considered that if the negation for all x P x is true and we see that there exist x negation P x is true which is our these De Morgan's law for all.

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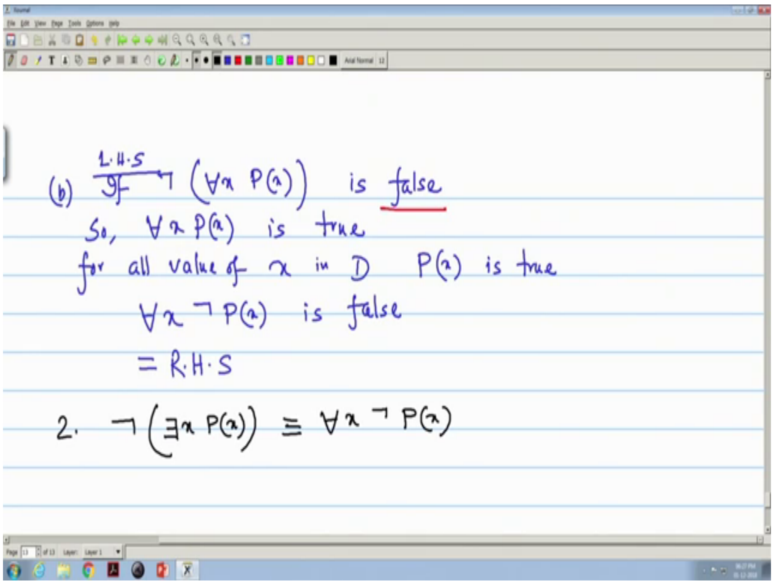

Now, another part is that we have to consider if it is false now we see that if though we see the b part that if negation for all x P x is false. So, if this is false so; obviously, similar way we can tell that for all x P x is true. Now, for all x true P x true; that means, that for all values of x in D since it is a universally quantified statement, so, for all values of x in D P x is true.

Now, for existentially quantified statement if it is a true then what will then for all values of x this can be false; that means, that for all values of x P x is true. So, for the all values of x negation P x is false which is nothing, but our RHS and this is the left hand side we started for the other case when it is false. So, I can just write this is this is our proof for when we have considered that it is the universally quantified statement is false, this is false and here we can tell that whenever the our universally quantified statement is true. So, both the cases we can prove and this is our demand answer.

So, similarly we can prove the existentially quantified statement that is there exist x on negation of there exist x P x is equivalent to for all x negation P x similar way we can prove. So, we see the De Morgan's law, De Morgan's law we can apply to find the negation of the universally quantified statement or the existentially quantified statement.

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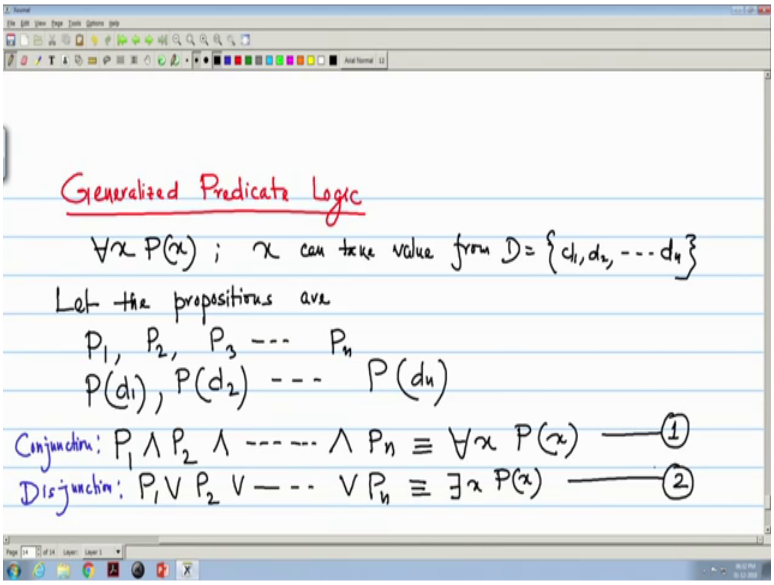

Now, how we can generalize this thing we can write the generalized statements. So, the concepts the basic concepts that we have applied to prove the De Morgan's law that we see the generalized De Morgan's law or generalized predicate logic. So, let us consider a universal quantifier statement for all x P x and x can take value from the domain of discourse D; that means, which are D is the value of d 1, d 2 up to d n. And let the statements the propositions are P 1, P 2, P 3, P n which are nothing, but the P of d 1 when x takes the value d 1, d 2 like P d n; that means, when x takes value d 1 from D P d 1 which is P 1 which can be either true or false that is a proposition and similarly for P n.

Now, if we take the conjunctions of all propositions; that means, we take the conjunction, so, P 1 and P 2 up to P n. So, what are the physical meaning of this conjunction? This physical meaning of this conjunction this is equivalent to for all x P x because for all x means for x takes each value from D that is d 1 to d 10 d n and for that the statements becomes or the proposition becomes P 1 to P n. So, if I take the conjunction of P 1, P 2, P n then this is for all x P x.

So, similarly if I take the disjunction if I take the disjunction, so, this is my the conjunction if I take the disjunction. So, this will be P 1 or P 2 or P n and we know we can write that this will be there exists x P x when that any value one value of x; that means, one of P i’s are true. So, we can now we see what we can tell from here. We give as if these are the conjunctions of all the propositions or the disjunctions of all the propositions or little. Firstly, we see how they are whether they are true or false. So, we immediately we do not write the universal or existential statements we see that.

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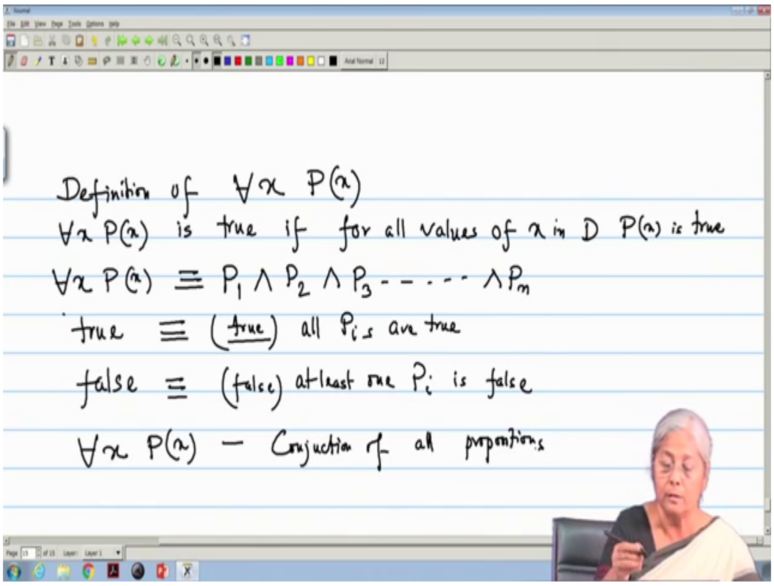

Now, from the definition of for all x what we know that for all x P x we write that for all x P x is true if for all values of x in D P x is true. So, for all values of x in D, for all values of x the propositions become. So, I can write then for all values of x P x that I can write that is equivalent to P 1 conjunction, P 2 conjunction, P 3 conjunction P m. So, when it will be true since it is a conjunction. So, if all are true then only this is true; that means, LHS will be true if this conjunction is true; this part is true, if everyone is true.

Now, all P i values of all P i’s are true all P i’s are you can write all P i’s are true then only I can tell that this is are true and since it is conjunction. So, it will be false when any one or at least one P i is false. So, this is my generalized form of predicate logic which is nothing, but so, what we can conclude that for all x P x is nothing, but the conjunction of all propositions.

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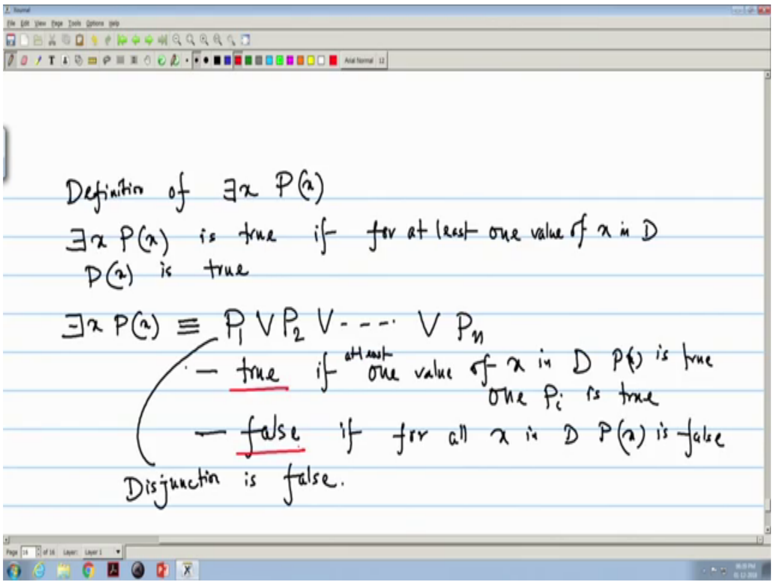

Now, if we see the there exist x; so, definition of there exist x P x when it will be true there exist x P x is true if for one value, at least one value of x in D P x is true and false when for all values of x P x is false and false. So, I can write is P 1 or P 2 or P n, since we know that it will be true for this will be it is true if one value of, at least one value of x in D P x is P i is true; that means, P x is true; that means, one P i is true. And, when it will be false when it will be false if for all x in D P x is false; that means, all P i's are false; that means, this P 1 or P 2 or P n this disjunction is false.

So, we can generalize our existentially quantified statement or the universally quantified statement by only using our conjunction connectives or the disjunction connectives of the propositions or the predicates. So, once we know these generalized De Morgan's law, so, any number of predicates that we can relate or the with the existential quantified statements or the universally quantified statements any number of quantified statements that we can relate with the simple the negation rule and the number of conjunctions and the number of disjunctions of this thing.