Discrete Structures
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Lecture – 07
Predicate Logic

Today, we will read the Predicate Logic. We have read the Propositional Logic and Predicate Logic. We define that it is nothing but an extension of proposition and life project. And mainly, the propositions or the statements that the propositional logic fails to handle predicate logic can handle those things.

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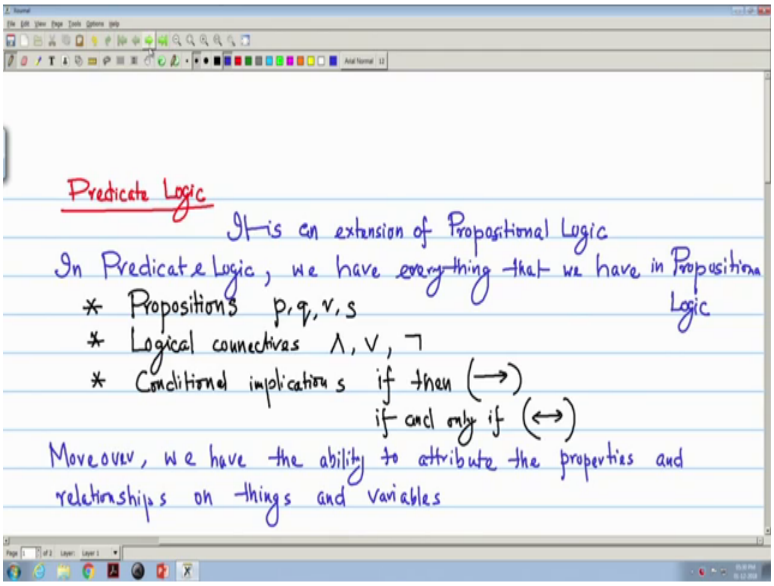

So, we will read Predicate Logic. It is an extension of Propositional Logic. In Predicate Logic, we have everything that we read in Propositional Logic. So, what we have in Propositional Logic? We have propositions like p q p, q, r, s which are nothing but the declarative statements either true or false and we have logical connectives that and or not. We have some conditional implication like if then implies and if and only if that is by conditional.

Now, we have these propositions logical connectives and conditional implications. Moreover, we have the ability to attribute properties and relationships on things or variables. Moreover, over this we have the ability to attribute the properties and relationships on things and variables. Now, these propositional logic that we read have some limitations.

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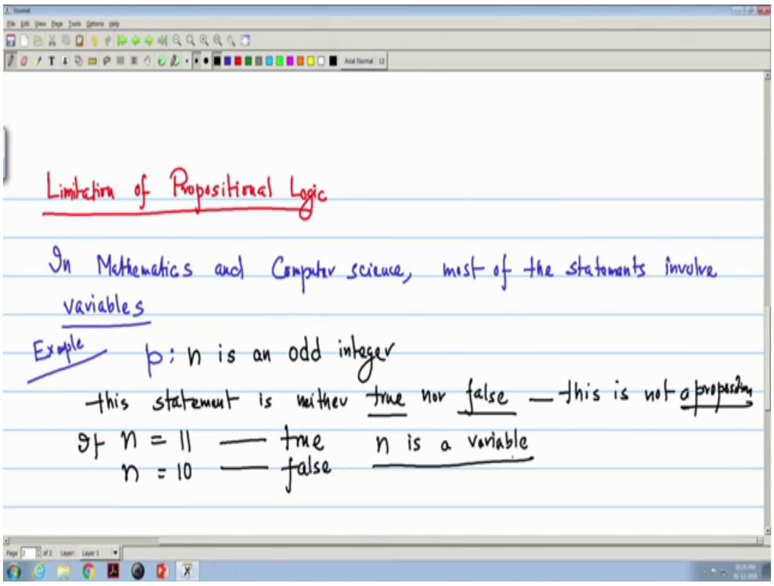

The Limitation of Propositional Logic: see propositional logic discusses about the propositions or the statements which is either true or false; but it is incapable of some statements which are normally used in mathematics or computer science and the predicate logic will handle this type of statements. The propositional logic fails to handle those thing. So, first we see what type of statements that propositional logic are incapable to handle.

We take first an example. See in computer science and mathematics, in mathematics and computer science we use variable. Most of the statements involve variables and the propositional logic that we read or discussed earlier that they cannot handle the statements involving variables. We take one example; little we see one simple statement like p let which tells that n is an odd number; odd integer. Now, this statement is neither true nor false. This statement is neither true nor false. So, this is not a proposition; not a proposition.

So, what? Propositional logic cannot handle this type of statements or this logic involving these statements. Why? Because n is an odd integer. So, if n equal to 11, then it is true; odd integer. But n can be 10 or 110. Then, it is false. So, what is n? So, n is an odd integer; n can take any value. So, we call that n is a; n is a variable; n is a variable.

So, the statements involving variable cannot be handled by propositional logic and for this, we need an extension of the logic of systems.

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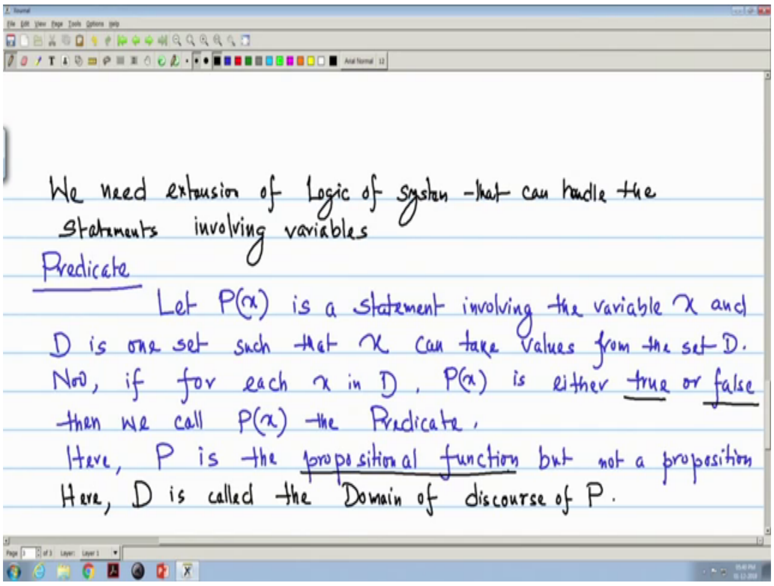

So, we need extension of logic of systems that can handle the statements involving variables.

So, first we define predicate. Let P x is a statement involving the variable x and D is one set such that x can take values from the set D. Now, if for each x in D, P x is either true or false, then we call P x as the Predicate. Here, P is the propositional function, but not a proposition; but not a proposition. But P x P x is a proposition. Since P x can be true or false. P x can be true or false and x takes the value from the set D. Here, a D is called the Domain of discourse. Here, D is called the Domain of P. See P is not a propositional function. P is the propositional function. P is not the predicate. P x is the predicate and x can take the value only from the set D.

So, we define predicate like that.

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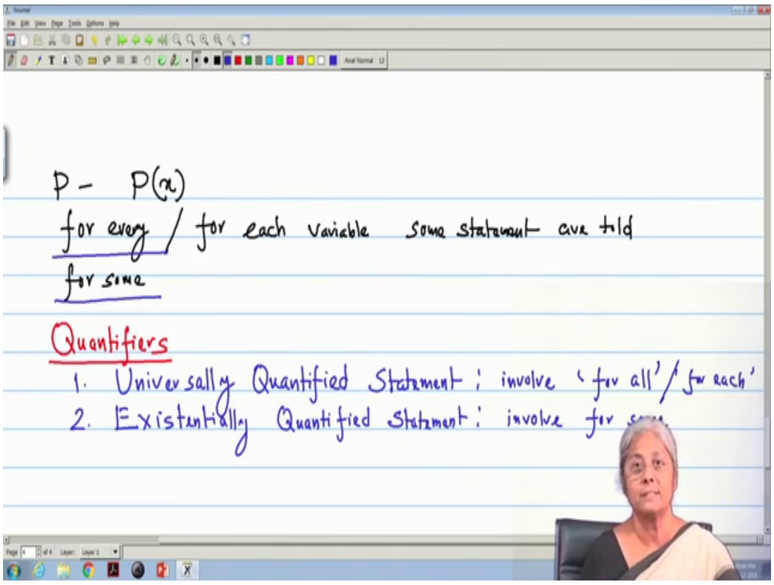

Now, all know what I mentioned that the predicate or the propositional function the P, the propositional function when takes the value x from the set D that actually defines the predicate P x. So, x is here a variable that can take the value from D. Now, in computer science and in mathematics whenever the statements we are dealing or the statements involving the variable.

So, we use the term that variable can be for every variable, for every or for each variable some statement are told or for some for some variable the statement can be told. That means, here these for every variable or for some variable, the statements are normally mentioned. So, how we can handle this type of statements where the variables are either for all variables or for some of the variables or for some of the values of the variables.

Now, based on these 2 type of statements, we define that 2 type of predicate logic and they are normally called the Quantifiers. So, quantifiers are normally of 2 types. One is called the Universally Quantified Statements; that means, those statements they involve “for all” or “for each” every like that or Existentially Quantified Statement which involve for some.

Now, we define the universally quantified statement or existentially quantified statement or the first the universal quantifier or the existential quantifier. First, we see the Universal Quantifier.

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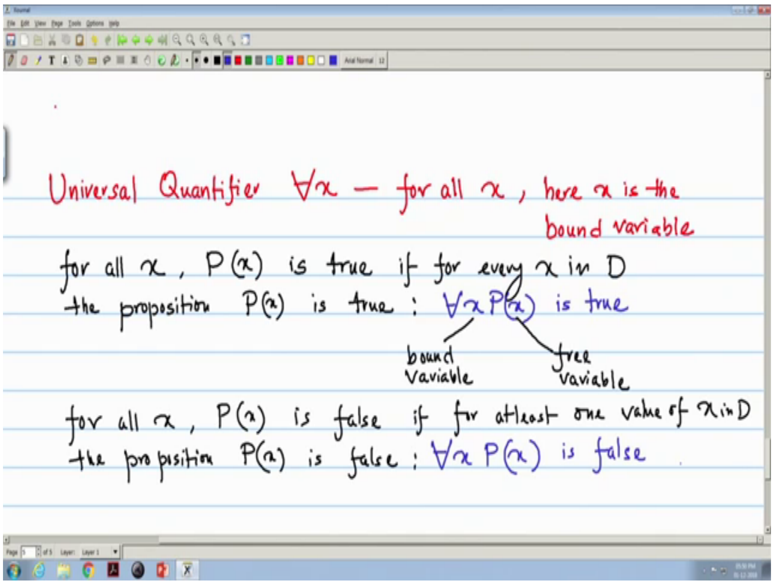

Normally, we denote this thing by for all. So, for all x. This is called for all x, here x is the x is the bound variable; x is the bound variable. How we define?

So, we define the quantifier for all x like for all x P x can be either true or false because the P is the propositional function, but P x is the proposition. So, P x is either true or false. So, how we define that for all x P x is true, if for every x in D, because x can take value from the set D D, the proposition P x is true; for all x if for every x entry the P x is true.

We write this in symbol that which you can write that for all x P x is true. Here, x is these x is the bound variable. We have defined this is bound variable and x in P x is called the free variable. When, P x is false; so, for all x P x is false if for at least 1 value of x in D, the proposition P x is false. How we write? So, we write similarly that for all x P x is false. We take 1 example.

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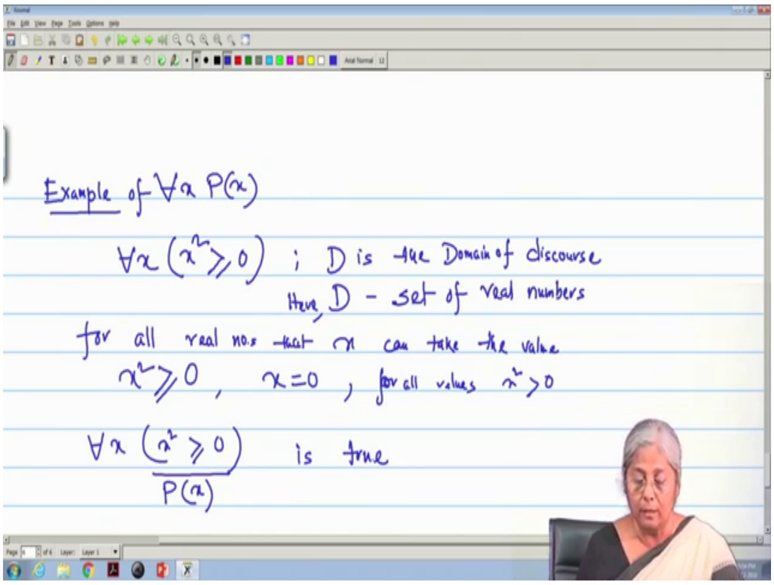

Let one example is that we write that for all x x square greater than equal to 0; you take for all x and D is the Domain of discourse. So, one thing is very important that whenever we will be handling some statements with variables or the predicates, then that we must mention the domain of discourse. That means, the set of elements from where that the variable x can take the value. This is very important, because for these using these values only or depending on the values of D, the statement can be a proposition can be either true or false.

So, if here D is the domain of discourse and D say the D here, the set of real numbers say D is set of real numbers. So, x can take any real numbers. So, for all real numbers; so for all real numbers that x can take the value, we know that x square is greater than equal to 0. It can been never be negative for x equal to 0, it is 0 and for all other values for remaining values; so for all values x square greater than 0. So, the way we define; that means, for all x x square greater than equal to 0. This is our P x, this is true.

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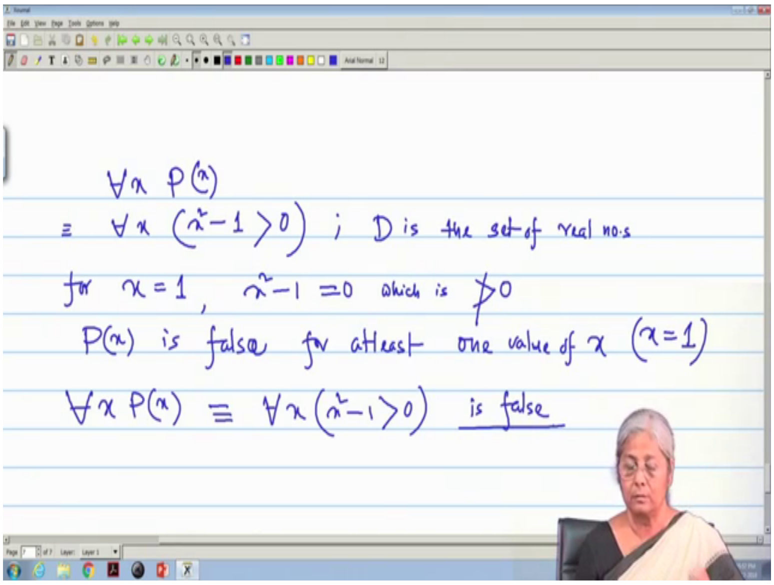

Now, if I slightly change these example like for all x P x which I am telling that for all x, x square greater than or x square minus 1 greater than 1 (Refer Time: 29:49) x square minus 1 greater than 0 and D is the set of real numbers. Now, we know that for x equal to 1 x square minus 1 equal to 0 which is not greater than equal to 0. So that means it is false.

So, P x is P x is false for at least one value of x say x equal to 1; say for x equal to minus 1 also this is true. But if we can identify only one value of x and for that my P x is false, then we will tell that the (Refer Time: 31:29) it is false. So that means, that for all x P x is equal to for all x x square minus 1 greater than 0 is false.

Now, if we consider the Existential Quantifier.

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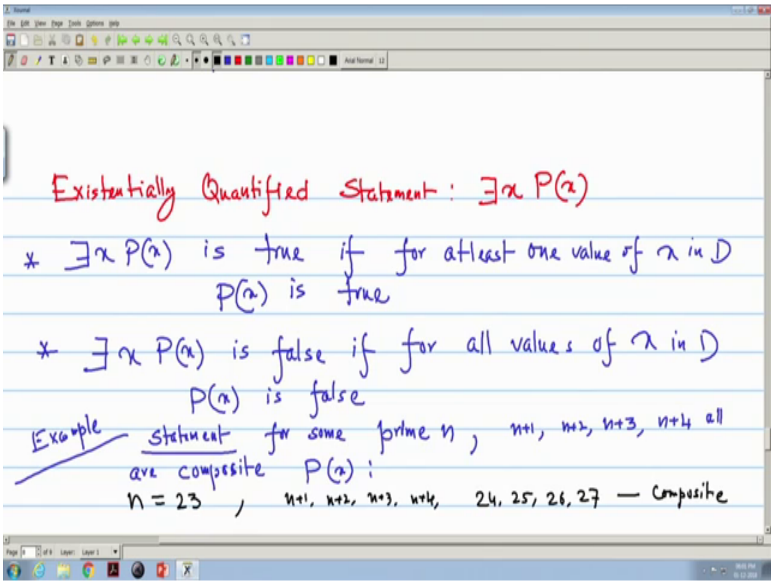

So, we define the Existentially Quantified Statement, normally we denote as there exist x p x. So, here similarly we can tell that there exist x P x is true if for at least one value of x in D P x is true and there exist x P x is false, if for all values of x in D P x is false. Example, we take one example. Say the statement is one statement we make. For some integer n n plus 1, n plus 2, n plus 3, n plus 4 all are it till for some integer or for some prime we take with this change, we change that for some prime integer for some prime n all are composite.

Now, if we take that this is my statement, the P x. This is my statement P x. Now, we have to identify or we have to find out at least one such prime n for which this is true; then it will be true. But if we want to show that this statement is false, then for all prime we have to show that this is false. So, for this example, we can identify we can find out one value n 1 n 1 prime is 23 for which the n plus 1, n plus 2, n plus 3, n plus 4 which are 24, 25, 26, 27 all are composite. So, we get one value of at least one prime, we can find out for which this is true.

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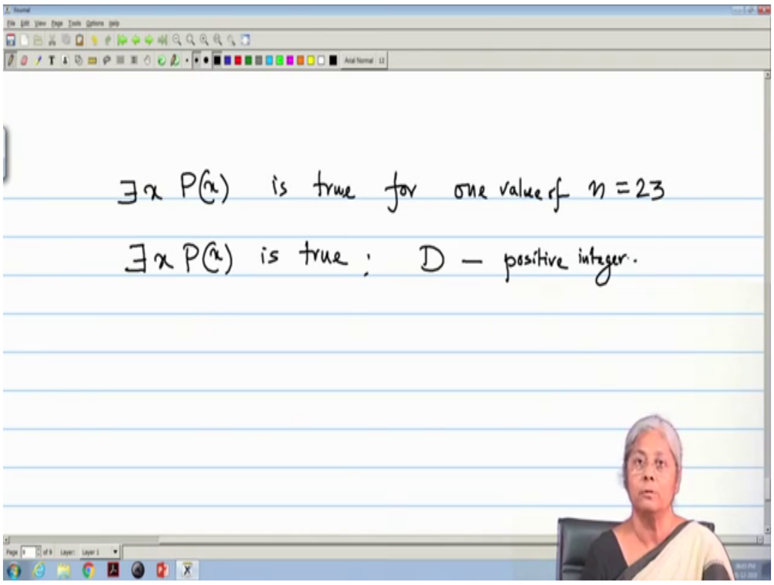

So, there exist x P x is true for one value of n equal to 23. So, these existential quantified statement is true and obviously, here the D we have taken as the positive integer: D is positive integer.

So, we learned that; what is predicate logic; what is the difference from the predicate logic and the propositional logic and the 2 quantifiers that existentially quantifier and the universally quantifier and how they are used to the truth values to find the truth values of the statements. That means, the whether it is the statement is true or false.

So, with this the basics of Predicate Logic we learned.