Discrete Structures
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Lecture – 06
Introduction to Propositional Logic (Contd.)

Today, we will read the rules of inferences. What are the rules of inferences? We have read the laws of logic. We have seen that how the validity of one argument is checked by using the laws of logic and the propositional equivalence. Now, we will see that to check the validity of arguments when more than one laws of logics are used, then whether we can combine together and to frame a new rule which is more compact or very easy to use for the validity.

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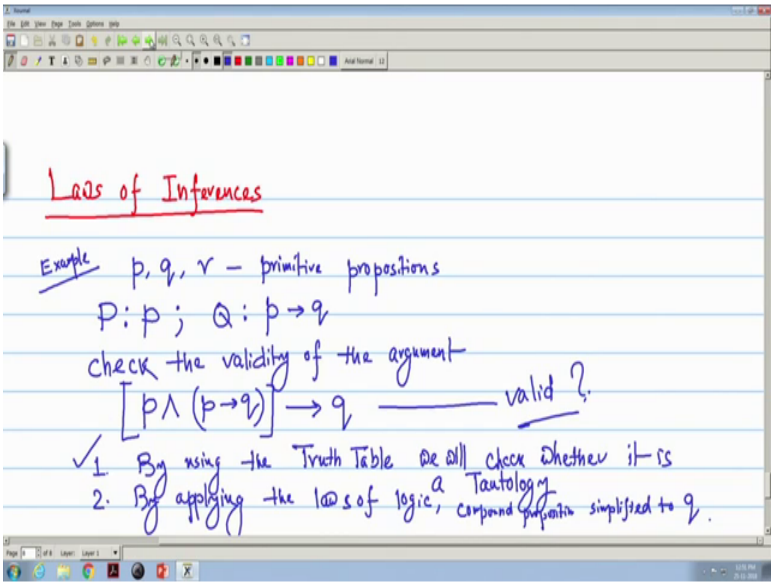

So, we can define that in laws of inference is nothing but the compact form of laws of logics and we can define or this compact form so that it is much easier to apply on the compound propositions to simplify or to validity of the arguments. So, in turn that will be much easier when we will apply the logic to prove some theorems ok.

So, we take first one example. Take one we take one example. Say we have propositions primitive propositions p, q, r; the primitive propositions and we have two compound propositions let capital P is p only and compound proposition Q is p implies q. Then, we want to check the validity. Check the validity of the argument the p and p implies q implies q or not. So, we can do in two ways; one method is by using the truth table.

The truth table, we will check whether it is a tautology or not. Second method we can apply by applying the laws of logic more than one and we can say that whether these implication is true or not or these LHS that the left hand side, I can tell that compound propositions. Compound proposition simplified to q or not; to q or not. So, these are the two ways, we can tell that whether these it is valid or not; whether it is valid or not ok. So, we see by the first method that by truth table.

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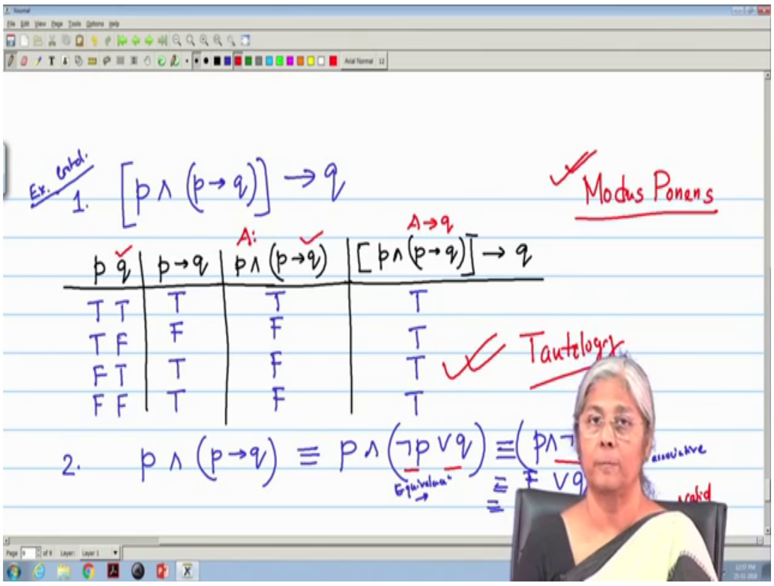

So, our example is continued is p and p implies q implies q, whether it is a tautology or not. We draw truth table here only there are two primitive statements. So, we give p q we required p implies q and p conjunction with p implies q; then, p conjunction p implies q implies small q. We take the truth values of p and q T T, T F, F T and F F. Then, p implies q; p implies q is it is T T so, it is T; other than this is only false these are all true. Now p and p implies q. So, T and T, this is T T and F, so this is F F and T, this is F F and T, this is F.

Now, if I tell this is my say it is something called a compound proposition e; that means, these gives whether A implies q or not. So, I will check A, I will check A and I will check q ok; A and q. So, these implies q so, T T so, it will give me the true value; then, F F true F all are F so, these are all vacuously true. Once that p implies q, p is true; A implies q, A is A is false. Then, it is actually by default that implication is true. So, what we see that this it is a this is a tautology, this is a tautology.

Now, the second method; quickly we can do because it is much easier, then truth table because we have to apply the laws of logic. So, laws of logic we can apply p and p implied q. This is equivalent to p and since, now we know that p implies q is negation p or q. Now we apply the associative rule p and negation p or q; p and negation p is F or q so, which is q only; which is q only. So, here also we see that this is it implies that means, my composition in compound proposition on a left hand side is actually simplified to q so, it is a valid argument.

Now, see here we have applied 3 laws of logic; one is my equivalence that equivalence of implication this is one equivalence of implication. One is the associative so, this is equivalence; equivalence of equivalence of implication. It is associative and this is our absorption F or q is q only. So, these 3 together we can tell that this is the name is that the Modus Ponens. This is the rule of inference is called the it is called the Modus Ponens.

So, instead of applying separately, the equivalence associative law, the absorption together we made one laws of inference. So, now onwards instead of applying separately this thing, we will be using the Modus Ponens. Next we see another example.

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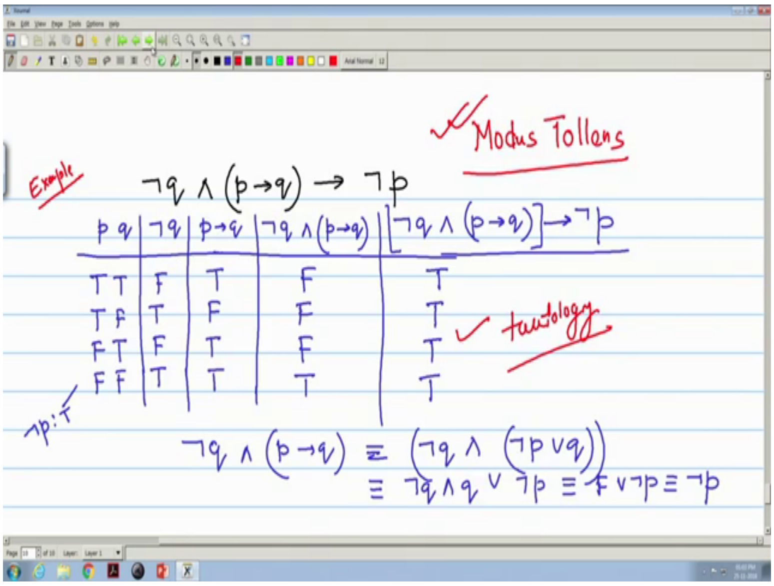

Now, we take the compound position proposition that negation q and p implies q implies negation p as usual p q are primitive statements. Now, we see that truth table, again, if we write p q; then, I need negation q. So, I write that I need p implies q. I write I need negation q and p implies q I write. Then, together the final implication; the negation q and p implies q implies negation p ok. I need a negation p also ok; directly I can write from here. So, we give T F F T and F F.

Now, negation q so, this will be F T F T, p implies q, T T T F T. Negation q and p implies q, so F F F only this one will be T. Now, this implies negation so, negation p is T so, negation is F. So, F F, it is T. Again if so all three F's, directly I can write vacuously true; directly it can write true. This p, so this negation p will be here the negation p is; negation p is true. So, T T and T this will be true. So, again we get that this is a tautology this is a tautology.

So, this rule, we tell that Modus of Modus Tollens and using logic, we can directly prove that thing quickly we do. This is p implies q so, this is equivalent to negation q and negation p or q. The negation q or q, this will be sorry this is the and; this is and so, I can give negation q and q, this can be commutative or negation p which is equivalent to F or negation p which is given to negation p.

So, again that we can combine all these rules and we make is Modus Tollens. So, the combination of the propositional equivalence along with a set of laws of logic, we frame some laws of inference. That means, which will be checking the validity of some implication or the whether the implication is a tautology or not. Now, we enlist these rules. How many such rules are there?

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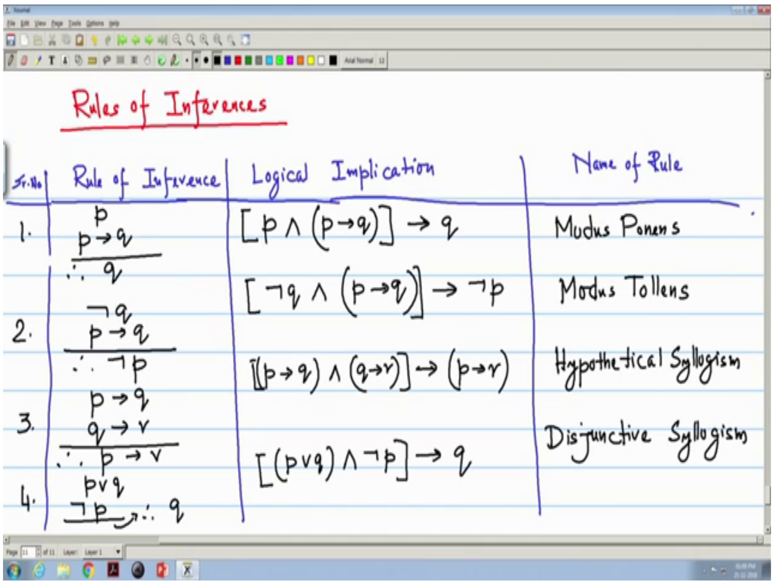

So, we take the rules of rules of inference. The related logical implication that we write here and the name of rule like the last two examples we have seen, the Modus Tollens or Modus Ponens, we have given. So, we give some numbering. The first one, we have seen the p p and p implies q.

This is the convention of writing that therefore, the result is q. The result is q and what was the logical implication? Logical implication is the nothing but the compound proposition that implies the finest result. So, it is the p p and because I have two compound proposition; 1 is the simple one that small p only and p implies q that implies q and name of rule, we have already seen this is called the Modus Ponens.

We have seen the 2nd one; the negation q and p implies q that results negation p. So, logical implication is negation q and p implies q, this results implies negation p, it is called the Modus Tollens. Now, give the 3rd rule as if p implies q, this is one proposition and q implies r another proposition; then, therefore, p implies r. So, it tells p implies q and q implies r implies p implies r and the name of rule is hypothetical syllogism this is hypothetical syllogism.

Now, the 4th rule tells the p or p or q and negation p; p or q and negation p. It gives therefore, the it is q only. So, you can write p or q and p or q and negation p implies q and this is the name is the rule of the this rule is disjunctive syllogism. This is now we have some more rules. Just do that thing.

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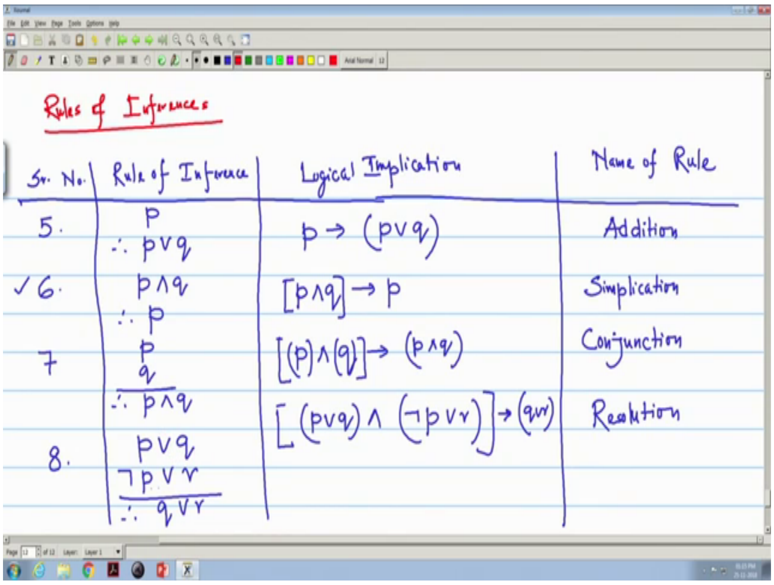

Again, rule of logical implications which are related to the inferences on the name of the rule. So, we have these are very simple that if p; that means, if p is true, then p or q is true. So, logical implication is simply p implies p or q and the name is addition. Since, the disjunction or is called the addition. Now, we know this, we apply the simplification; that means, if p and q is true, then either p or q, I write p it can be q also.

So, I can write p and q implies p. This is simplification see this rule is very important because we have two primitive statements and that implies only one primitive statement. So, by when we will we will do the simplification, these rule is very important; these rule is very important. Then, you have some conjunction that if p and q; therefore, p and q.

So, p and q this is obvious this is actually gives me p and q. So, I should give this thing because, we know the substitution rule that means, small p q can be actually a compound proposition also ok so, this is my conjunction. These rule is very important this is called the resolution so, to simplify the compound proposition, these rule is very important. This is some disjunction of p or q and negation p or r.

So, it gives the result that q or r so, I take the primitive statement p in in disjunction with another primitive statement q and negation p and disjunction with another primitive statement r. Then, they are actually related the other two q and r, they are actually related with the logical connectives the disjunction. So, it the logical implication, I can write p or q and negation p or r this implies q or r and this is called the resolution; this is called the resolution.

So, these are my the 8 rules of inferences. This is the rules of Inferences that we can apply directly when we will try to simplify the compound proposition or we will see the examples later that we will now we will we will try to prove some theorems. Then instead of applying the simple laws of logic that we can apply the these rules of inferences. But one thing whenever we will be applying these rules of inferences, we have to mention that which rule we are using here and that is much easier.

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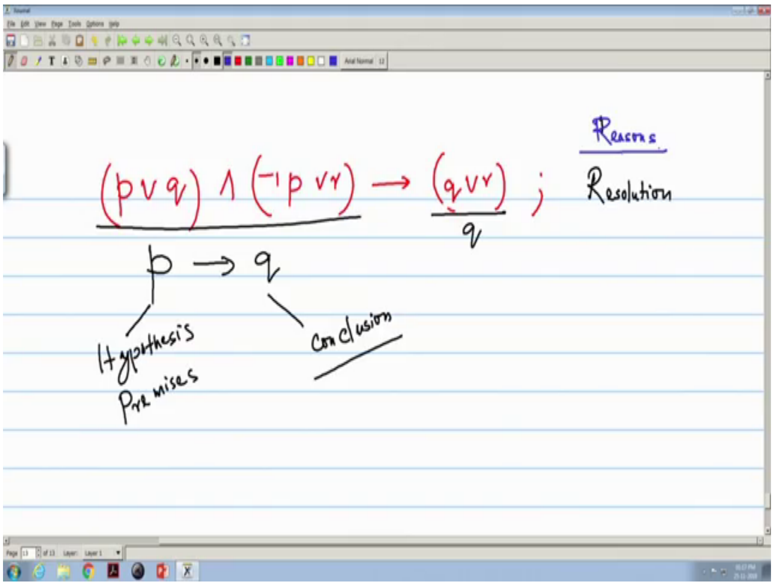

Because now so, we have to remember the name of the rules and then whenever say if we apply the sum p p or q and negation p or q is implies say in if I give that p or q and negation p or r say it is simplifying to sum p or r q; sorry q or r this is q or q or r. Then, I have to mention that it is the application of we are applying the resolution; we are applying resolution.

So, here if we remember that these if I think that this is small p this is my q, then p implies q. So, this is my hypothesis or we can tell premises and this is conclusion. So, some hypothesis are given and from these hypothesis, what we can conclude whether we get the conclusion and by using what rule of inferences that we have to mention as a reasons for simplification. Like we have to write that this is these are the reasons for simplifications.

And we will do the examples next day. Then, we can write properly and we will be see we will see that how we can apply these rules of inferences and what are the formal way of writing these hypothesis conclusion and then, the simplification applying these rules of inferences.