Discrete Structures
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Lecture - 05
Introduction to Propositional Logic (Contd.)

We have read the laws of logic the propositional equivalence. And today will shall begin the formal study that how we can use the implication to valid that one or to check the validity of the propositions. Or in other way, we can tell that given an argument whether that argument is valid or not by using the implication whether we can tell that thing.

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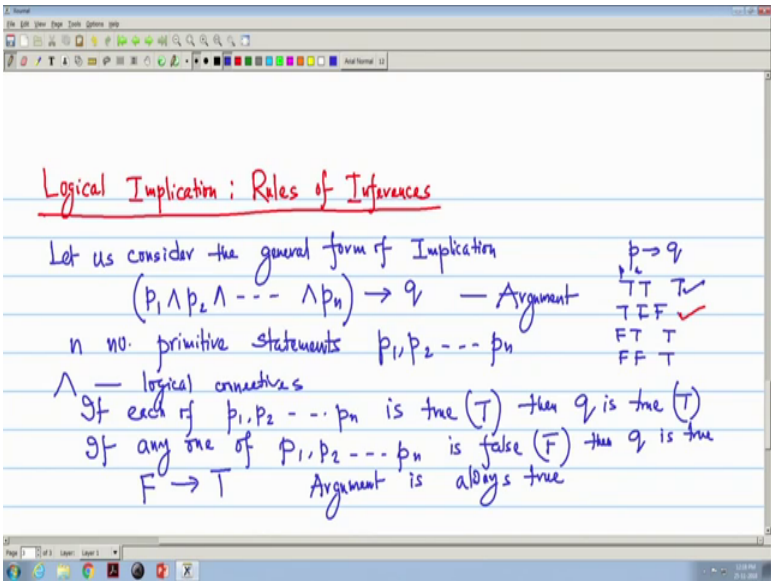

So, it is the logical implication, which formally we can tell that finally some we will be giving some rules of inferences. So, let us consider one implication the general form of implication. Let our p 1, p 2 to p n, these are primitive statements and the conjunction of these n primitives this implies q.

We call that this is an argument, this is an argument. Now, whether this argument is valid or not. Now, one thing is that why we will be reading this thing, what is the importance of the validity of the arguments? Because our main objective is that to give some mathematical proofs. So, when this logic will be applying to prove theorems, so that time will be seen that given one argument or given some compound propositions whether it is valid or not.

So, if we explain this implication or the argument, what we see that we have n number of n is a positive integer, n number of primitive statements primitive statements say p 1 to p n. Now, since they are logically connected with the connectives and that is conjunction, they are connected with the connectives and is the logical connectives.

So, if each of the p is that means, each of the primitive statements p 1 to p n they are true, then the left hand side will be true, since it is a conjunction, and then q will be true. So, if each of p 1, p 2, p n is true, then q is true, q is true. So, this is valid. When it will be invalid or even if we consider the so it is we remember that the implication truth table that p implies q that if it is these are p, q, then result is true. It is TF, then it is false; if it is FT, then it is true; if it is FF, then it is true. So, only this is the case where it will be giving a false result, it will be giving a false result ok.

Now we see that when it will be giving a false result that means, if any one of p 1 to p n is false, since they are connected with the conjunction n, so then also q is true. Why, because the left hand side becomes F. And f implies T, if q is true, then this is actually by default it will be true that is it is vacuously true, by default it will be true. So, we have to check the validity of the argument; that means, if we see that what is the conditions are on what cases that the this becomes always the argument is always true. Lastly, we have read that if the result is always true, or if we see the compound proposition is always true for all the assignments of its primitive statements, then it is actually a tautology.

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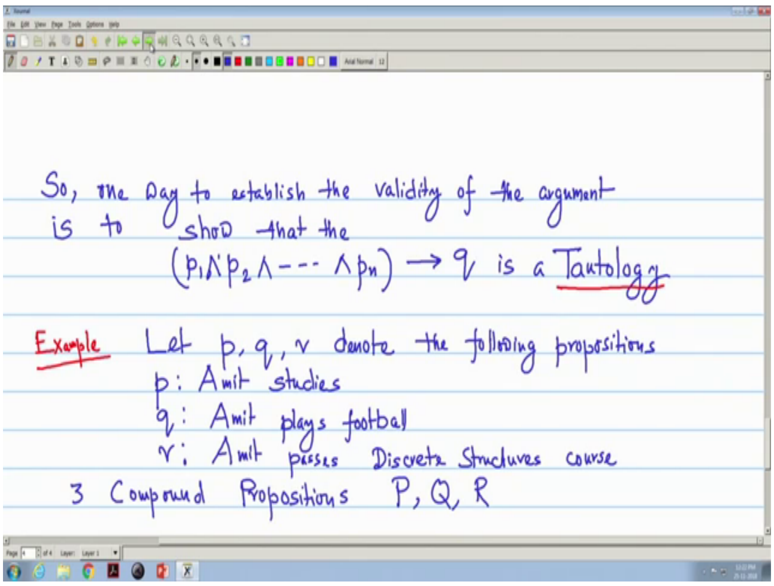

So, we can so one way of establishing the validity of the argument is to show that the implication that is general form that p 1, p 2, p n to q is a tautology. And we can tell that then it is a valid statement or this compound proposition is valid ok. So, we take one example. Let p, q, r denote the following propositions, what are the propositions, see p, I give that Amit studies; q Amit plays football; r Amit passes discrete structure course. Now, we see the take 3 compound propositions, we take 3 compound propositions, let be denoted by capital P, capital Q and capital R.

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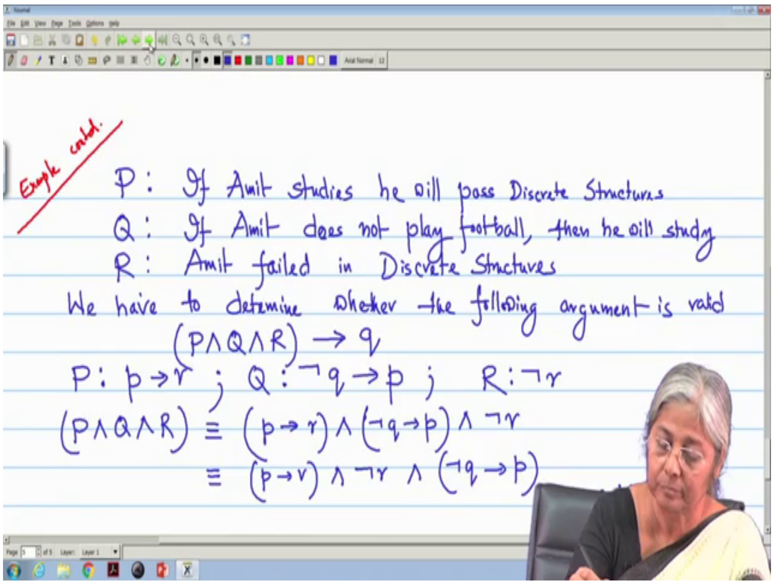

Now, may write those the statements of the compound proposition. See, P is if Amit studies, he will pass in discrete structures. Q, if Amit does not play football, then he will study. R, Amit failed in discrete structures course. Now, we determine we have to determine whether the following argument is valid, that means, the argument we write the our conventional form that P and Q and R, this implies small q ok. Now, small q is these are the compound propositions. And when they are connected with the logical connectives conjunction which implies q, and q is the Amit plays football. So, we have to check this thing.

Now, last lecture, we have learned the substitution law. So, we can apply this substitution laws here, that means, before that we write the compound propositions in terms of our primitive statements. So, what is P? P is if Amit studies, he will pass discrete structures Amit studies is small p. So, small p implies r, because he will pass discrete structures is quickly we see the previous thing r is Amit passes discreet structure course, Amit passes discreet structure course. So, this is q. The compound proposition Q is if Amit does not play; that means, negation q, then he will study it implies he will study means it is p, because Amit studies is small p.

Then what is R? R is Amit failed in discreet structure. So, Amit passes that was the primitive statement small r, so Amit fail, so it will be negation r. So, now, we replace the capital P, Q and R. So, if we start with the compound propositions connected by the conjunction. So, it is equivalent to p we write small p implies r then AND q is negation q implies p r is simply negation r. Now, we can apply our laws of logic here to simplify to check the validity of the given argument. So, we apply the commutative thing. So, we can write p implies r, I can take negation r first and then negation q implies p. So, we will continue to next page.

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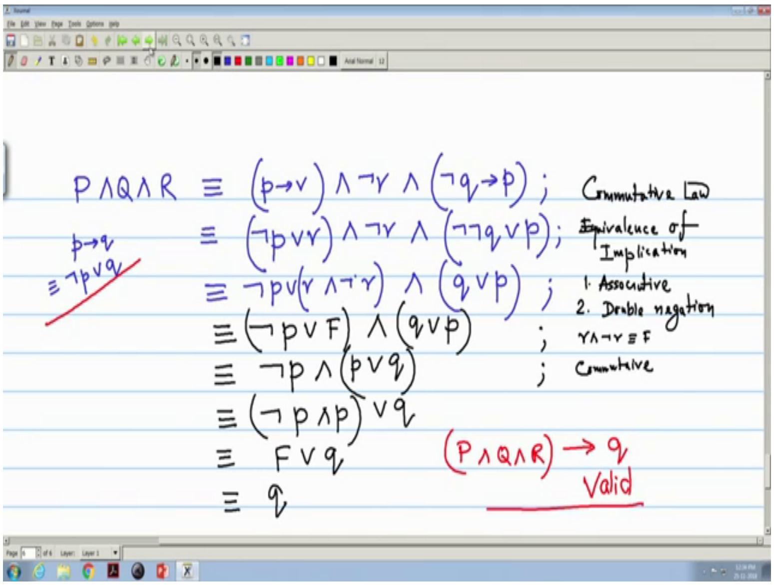

This is equivalent to p implies r and negation r and negation q implies p. So, we have write our Commutative law ok. Now, what is p implies r? Last day we have seen that it is with the propositional logic this is becomes the p implies p implies q, p implies q is equivalent to negation p or q. So, I can replace this is p implies r. So, this is negation p or r AND negation r. And similarly I can apply here also the implication that negation of negation q, so this becomes because negation of negation q or p. So, it is equivalence of implication.

Now, I can associative law we can apply. So, negation p or r and negation r we can apply r and negation r double negation. So, this becomes q or p. So, here we have applied two rules; one is our associative, one is associative; second is double negation. Now r n negation r this is always F. So, negation p or F, now this becomes q or p. So, now, this is equivalent to since it is OR, so this becomes negation p. And here also I can apply q or p is same as p or q ok.

So, we can similarly we can apply that here r and negation r is equivalent to or F. And here some commutative. So, this becomes again associative negation p and p or q, or we have just now we have use that negation p and p is F, F or q. So, this is equivalent to q, so our that P, Q, R. P, Q, R connected with conjunction implies small q that is it is it is valid.

Now, we see that to check the validity, we have used the propositional equivalence as well as the laws of logic. Now, today we will see that whether we can still simplify it or a group of laws of logic can be combined to apply the or to check the validity of the argument that means mainly the implication, because our argument validity of argument we have defined whether some implication is a tautology or not or it is valid or not. So, here we have seen using the laws of logic. Now, if we remember the definition of tautology that our final result or the truth value of the compound proposition that should be always true.

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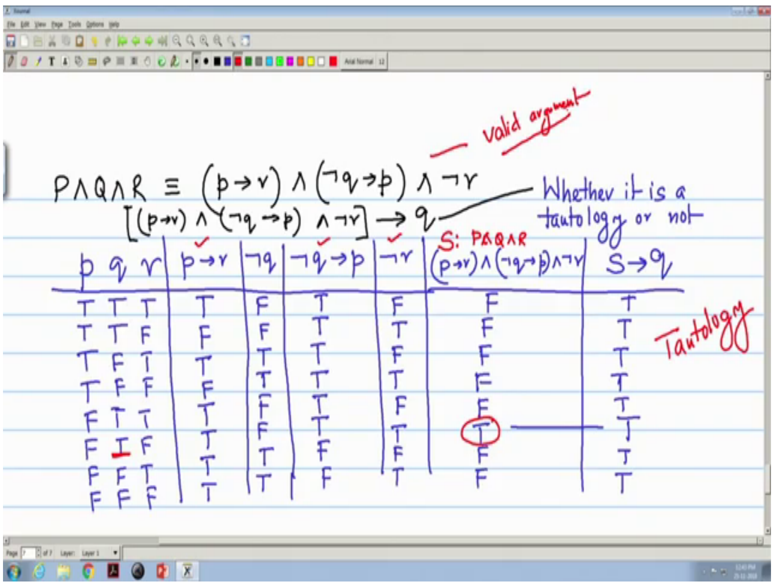

So, the same thing we see that our compound proposition was that P conjunction Q conjunction R, it was equivalent to p implies r and negation q implies p and negation r. Now, we see that whether it is a tautology or not whether, whether it is a tautology or not. So, we do the truth table and see the thing. We make the truth table. So, we have three primitive statements p, q, r ok. If it is a tautology, then we have to write in a different way that we can write that p implies r and negation q implies p and negation r whether this implies q, this we have to check.

So, I need p implies r, then I need negation q, so that we can compute negation q implies p, then I have negation r, then I can calculate that p implies r negation q implies p negation r. And then if I mark this thing as a S say I apply this S is p, q, r. So, this is S, then whether S implies q. So, I take all assignments; I take. So, I take all assignments of p, q, r. We know that p implies r; that means, this T implies T is true T implies F only this is false. Again this is true again T implies F it is false then all it is F, so all are vacuously true. So, if p is F, I can directly write these are all true.

Now, I have negation q. So, negation q is F T, then again F T. Negation q implies negation p. So, negation q this is F, whenever these two are F, it is vacuously true now TT. So, they are true. Again this is vacuously true. And then again these two are false. What is negation r? Negation r is F T F T F T F T. So, if I take the, and of p implies r negation q implies p and negation r, because this will give me that S, it will give me the S. So, S is TTF. So, this becomes F. Again, this is F 1 F means it will be F again this is tt F it is F F, T, T, F, T, T F, it is F. T, T, T, this is only true T, F, F, F and T, F, T, again it is F.

So, S implies Q. Now, see this is S, and except except this one, all cases it will be vacuously true. So, except these where S implies q, q is here this T and this is also T. So, this becomes true. So, I can write this becomes this becomes true, and or remaining cases since my S is all F. So, that S to q, that means, this becomes always vacuously true, these are all vacuously true. So, remaining all cases, it will be, it will be true.

. So, what we see that S implies q, this is a tautology because it is taking the truth value always true. So, earlier that with the same example using the laws of logic we have validated the compound proposition, and now with this truth table method what we have seen that S implies q is a tautology. So, the argument is valid. So, this is a valid argument, this is a valid argument.

Now to check this valid argument, we will try to frame the laws of inferences, so that our validity of the proposition of argument or to check the compound propositions, the conditions or the properties will be much easier. So, we will now see that how this laws of logic can be used to combine a set of logic to one laws of inference.