Discrete Structures

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Lecture – 42
Combinatorics (Contd.)

So, we have read the Permutations of a set and what do we mean that our Permutation of a set of n elements. Now this lecture, we will read the combinations of set.

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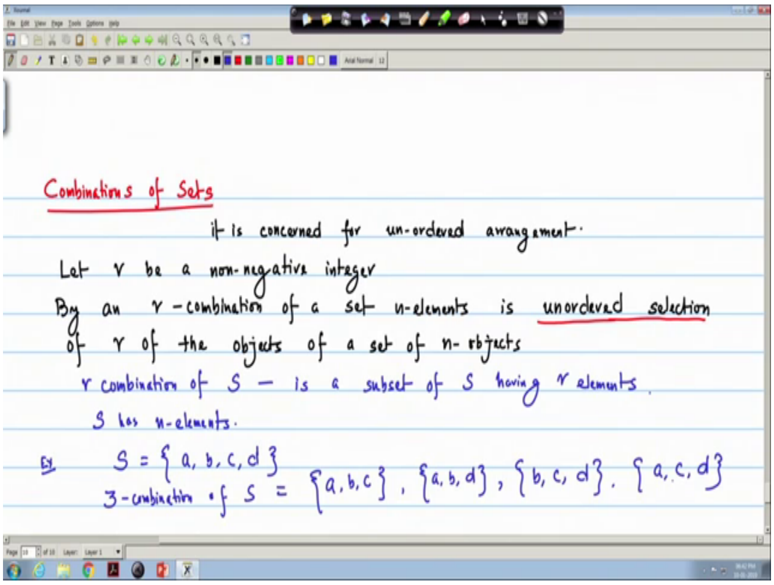

As already I have defined that normally that permutation combination, they are concerned about the counting of ordered or unordered arrangements. So, this is the combination is concerned when we count counting techniques for un-ordered arrangement. So, it is consult for unordered arrangement.

Now, we first define that what do we mean by these combinations of a set. So, let r be a non-negative a non-negative integer. Now, by an r-combination of a set of n elements, what we understand is this is an unordered selection of r of the objects or elements of a set of n elements or n objects; a set of n elements. Now the it differs from permutation is that unordered selection ok.

So, I can write that from the concept of set the definition of set. So, I can write that r combination of since it is unordered. So, I can write the r combination of S having n elements is a subset of S; where, S has out sub or subset of S having r elements and these S has so r combination of S. S has where S has in number of S has n elements or S is an n element set; S has n elements.

So, this is since order is not concerned order is irrelevant. So, I can tell simply this is a r element subset from the set S of n elements. So, if one simple example if I take say S has 4 elements ok. S has 4 elements a, b, c, d; then, what will be the 3 combinations I can take? I can write the 3 combinations; that means, I have to take the subsets having 3 elements from this 4 elements set S.

So, 3 combination of S, these are equal to this I can tell this is a, b, c; I can write a, b, d; then, it can be b, c, d; then, it can be a, c, d. So, these are the 4; these are the 3 combinations. So, this is the difference from the permutation.

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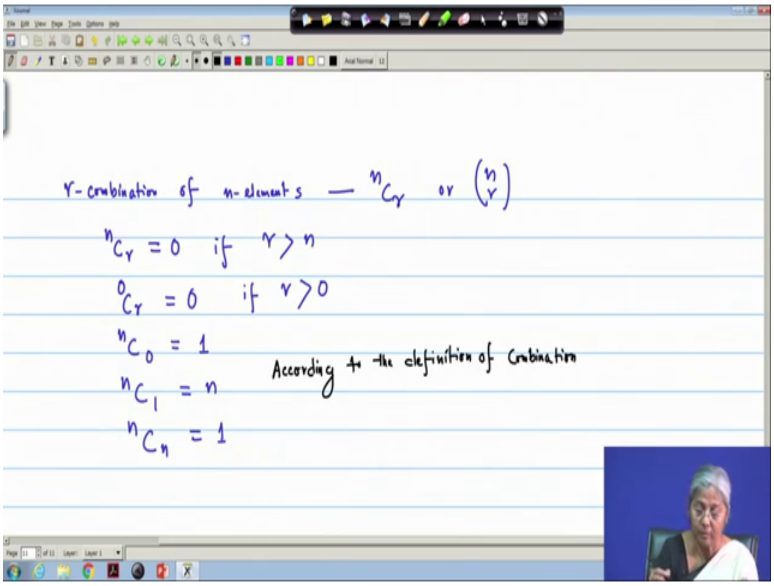

So, normally we denote the combination these r combination; r combination of n elements we denote as n C r or sometimes that n choose r, then sometimes we write in these way also. So, the according to their definition, I can write that n C r equal to 0 if r greater than n because not possible that among that n element set, I can choose a r r combination; subset of subset having r elements and r greater than n. So, it is not possible.

So, this is equal to 0. I do not have any such subsets. Then, I can write 0 C r is actually 0 that if r greater than 0. I can put n C 0 equal to 1, if I think only the way we have defined as if only the empty set I am taking subset; then n C 1 only one element set, since I have n elements.

So, this is n then n C n that is the set only. So, this is again 1 ok. So, this is according to that according to my definition; the way we have defined to the definition of combination or r combination of n elements ok.

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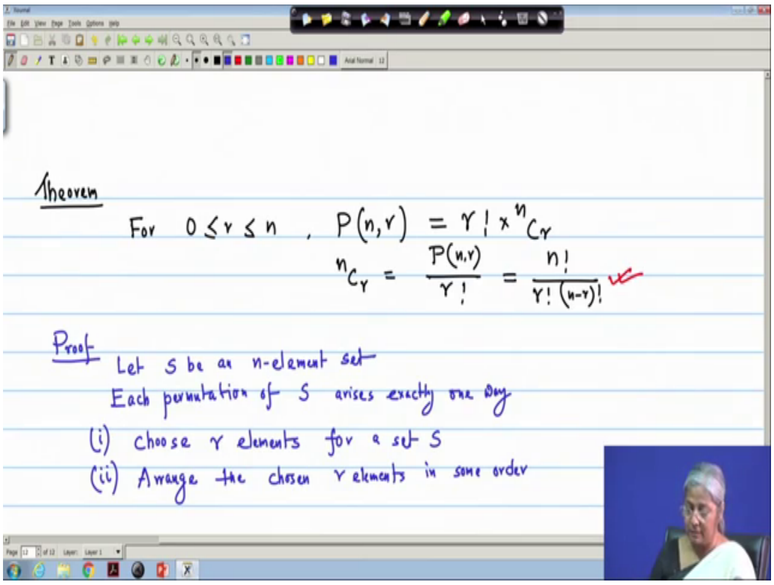

Now, we try to find out the formula and we give it we write a theorem. Write for 0 less than equal to r less than equal to n; then P n, r that in permutation of r; r permutation of n elements P n, r is r factorial into n C r.

So, I can write the n C r is P n, r divided by r factorial. Already we know that P n, r is n factorial divided by n minus r factorial. So, I can write. So, this is my we know thus these are my n r combination of n elements. Now how to proof; how we can give a proof. See now, we know the r permutation of n elements and that is a ordered arrangement or order selection.

So, we start with the ordered selection and see how we can make the ordered selection. So, let S be a n element set. So, now, each permutation of S arises exactly one way; what are this? I choose r elements from a set for a set S and since it is the ordered arrangement, I am considering permutation.

So, I can write arrange the chosen r elements in some order. So, from the definition of permutation, I can write this thing. Now, we see that how we can write. So, though if we see the first one, choose r elements from a set S. So, this is nothing but my definition of r combination of n elements.

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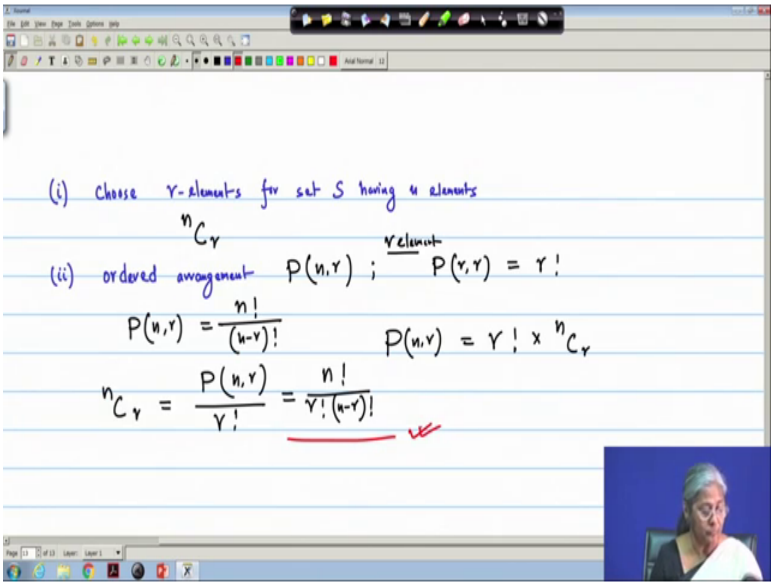

So, the first one that choose r elements from a set S having; choose r elements for set S having n elements is nothing but my n C r. This is nothing but my n chooser and the second one that arranged. So, the arrangement is ordered arrangement in some order. So, that I can write that is my permutation. So, second one I wrote that ordered arrangement.

So, according to the definition, I write that is nothing but my P n, r. Now, here this P n, r is r factorial or sorry a P r, r is r P n, r if it is r r element it is. So, if it is P n, r if it is P r, r; then means from. So, I can write this is my r factorial. Now, we know that P n, r; P n, r we know that already we have got this is n factorial by n minus r factorial. So, I can write that my n C r is P n, r by r factorial.

See you can I have written S this n C r is I can write that this is P n, r by r factorial or actually I can I should write this thing first P n, r is r factorial into n C r because two steps; I have written choose r elements for a set S and array is the chosen r elements array is the chosen r elements means this is P r, r ordered arrangement is P n, r.

So, r for r element for r element it is P r, r and that is r factorial. So, P n, r is r factorial into n C r. So, in C r is P n, r by r factorial and P n, r already we have got n factorial by n minus r factorial. So, this is our expression for the combination; r combination of a set of n elements. So, I got this thing. Now, we see some problems with this combination.

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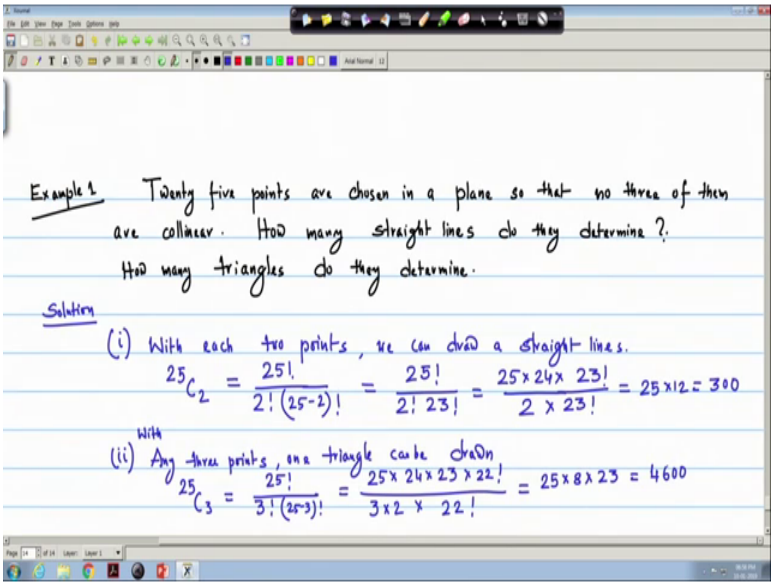

First, we see some simple example. How we can use this formula of n C r ok? Say I have 25 points are chosen in a plane in a plane. So, that no three of the points no three of them among these 25 points are collinear. Then, the counting problem is that how many straight lines we can draw to do they determine with these 25 points. Then, also we can find that how many triangles we can form with these 25 points having this property that no three of them no three points are collinear.

So, first thing is this is a pure counting problems and there is no ordering. So, un-ordered arrangements. So, you can give the solution. The first one that straight lines. So, no three of them are collinear. So, with each two points we can draw a straight line.

Now I have 25 points. So, I have to the problem, I can think that that two combination of 25 elements. So, this is 25 C 2 equal to 25 factorial divided by 2 factorial into 25 minus 2 n minus r and this becomes 25 factorial divided by 2 factorial into 23 factorial this equal to 25 into 24 into 23 factorial divided by 2 factorial is 2 into 1 2 and this is 23 factorial.

So, this becomes 25 into 12 is 300. Now, how many triangles we can give since no three points are collinear? So, with any three points we can draw a triangle. So, I have 25 points and we have to choose three points any three points, the total number of combinations are simply that means, any three points we can write with.

Any three points one triangle can be drawn. So, this is simply 25 C 3. Again, the similar way. So, this becomes 25 into 8 into 23 is 4600. So, we can see that how directly we can apply the formula of the combinations.

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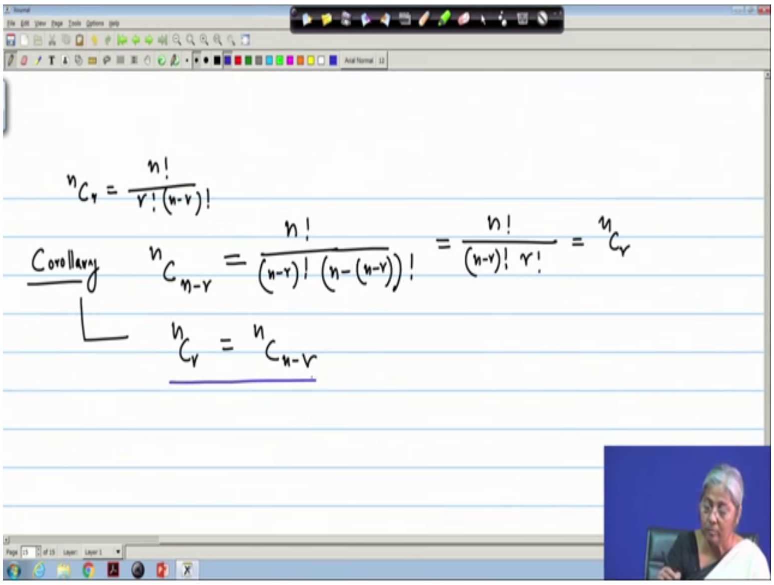

Now, we can the theorem we got that n C r is n factorial by r factorial into n minus r factorial.

So, I can write a corollary that if I write n C n minus r that is I am giving r equal to n minus r. Then, I can write n factorial to n minus r factorial and n minus r is n minus r factorial. This becomes n factorial minus n minus r factorial and this becomes r factorial. So, this is equal to n C r. So, the corollary is that n C r is n C n minus r. So, I can give this thing also.

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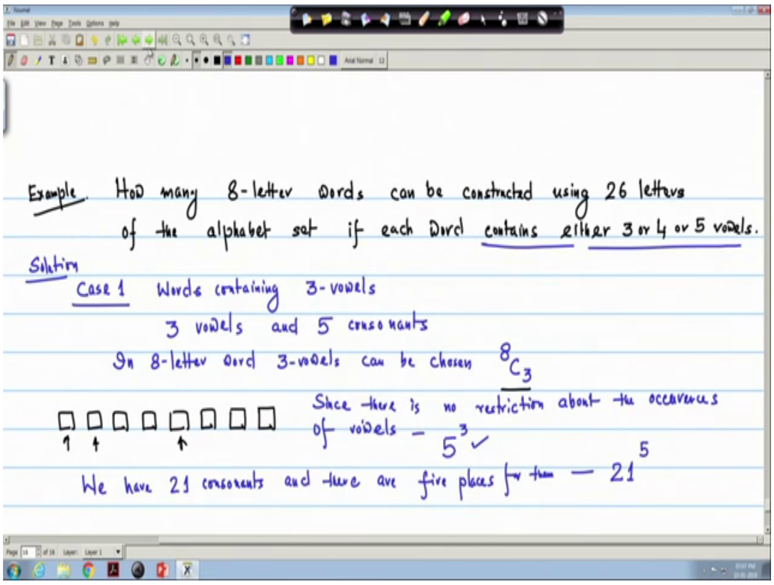

Now, we take another example take some 8 letter say how many 8 letter words can be constructed using 26 letters or alphabets set, if each word contains either 3 or 4 or 5 vowels. So, the problem is that I have to frame a 8 8 letter word and from the 26 alphabets that a to z and the restriction or the properties satisfying the property see we can remember that in the definition of when we started with the definition of permutation and combinations that it the some a ordered or unordered arrangements satisfying some property.

So, here it is the it is contains either the each word contains either 3, 4 or 5 vowels. So, first I consider the three vowels case. So, the case, case 1, I am taking that word contains the words contain containing 3 vowels. Now since there are 3 vowels. So, in this case there will be 3 vowels and 5 consonants. So, in the 8 8 letter word.

So, I can in 8 letter word, 3 vowels can be chosen. How many vowels? 3 vowels can be chosen. Say 8 C 3 ; that means, say if I take in this way say as if this is a 8 letter word 4 5 6 7 8. So, these 3 vowels that I can put in 8; 8 C 3 different ways. So, I can choose here. So, 3 vowels can be the vowel can be here; the it can be here; it can be here.

So, 3 vowels I can put this is my this is the way. Now since there is since there is no restriction that of repetition. Since there is no restriction about that, that how many times I can choose; how many times about the occurrences of vowels. So, there are 5 vowels.

So, I this can be I can keep 5 to the power 3 because I have 3 vowels and there are 5 different vowels.

So, I can put this place. Then the remaining 5 places. So, we have we have 21 constants consonants and there are 5 places for them. Since, 3 places already vowels are there and there are 5 places for them and that we can put I have 21 to the power 5. These many ways I can put the consonants.

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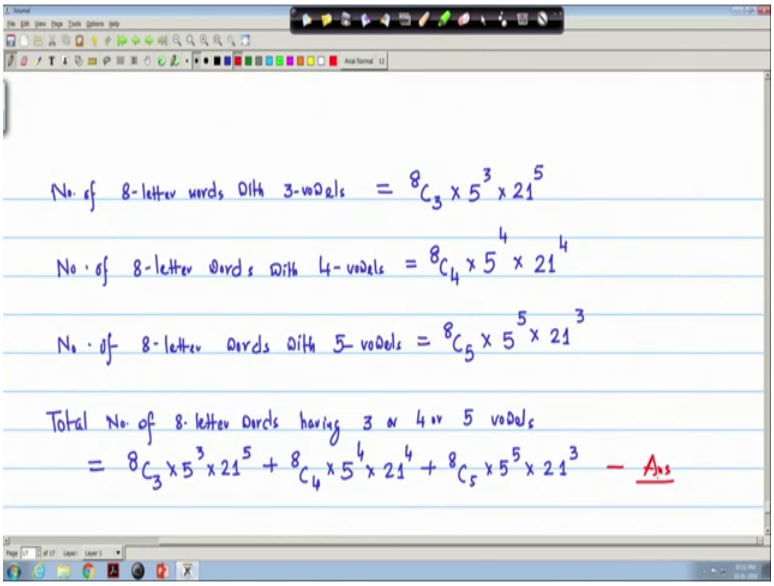

So, the case one that what we are telling that for how many ways that are we can go 3 vowels or the number of words having 3 vowels, I can write. Number of 8 letter words with 3 vowels, this is equal to I got this is the 8 C 3 ways I can put the three vowels. Then, since there are 5 vowels and 3 positions; so, 5 to the power 3. So, I can write that this number equal to 8 C 3.

Then, 5 to the power 3 into 21 consonants to the power 5 places. Now, if we just extend the idea that when number of 8 letter words with 4 vowels. So, I can write. So, if these will be 8 C 4. Since now I can choose 4 vowels from 8 letters 8 C 4 positions there are 5 vowels; 4 places, 5 to the power 4; 21 consonants rest of the rest of the 4 places.

Because I have 8 places; so, this is my rest of the 4 places. Now I have number of 8 letter words with 5 vowels is 8 C 5, the similar way. Then, it is 5 to the power 5 and this becomes 21 to the power 3. So, now, I have that addition principle because my question was that either 3 or 4 or 5. So, the total number of words having 3 or 4 or 5 vowels this is equal to the ok; I have to add because with 3 vowels 8 C 3 into 5 to the power 3 into 21 to the power 5 plus 8 C 4 into 5 to the power 4 into 21 to the power 4 plus 8 C 5 into 5 to the power 5 into 21 to the power 3.

So, this is my answer. So, the, for unordered arrangement for accounting problems, we can use a combination like that. Now sometimes we require this, what will be the total number of combinations.

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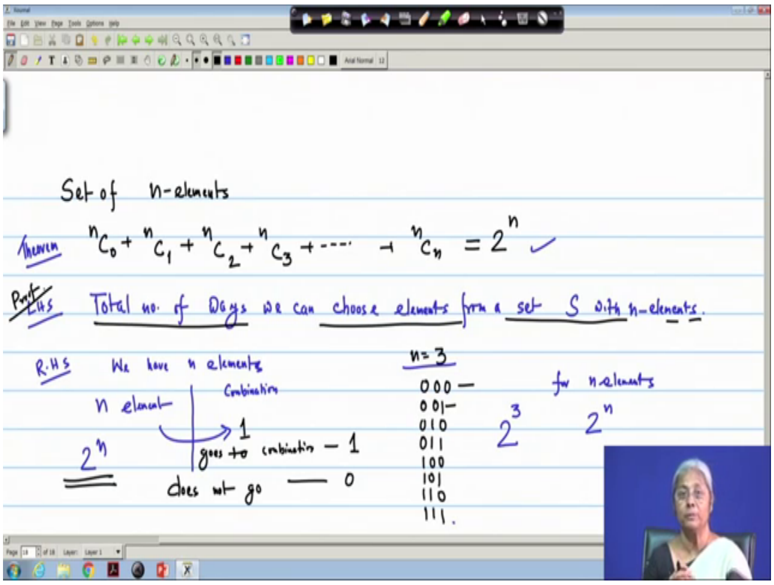

That means if I now consider the a from a set of set of n elements; set of n elements set. Then, the way we number of way we choose the elements say I can start with that as if I start that n C 0 choose n 0 elements, I can choose 1 element. I can choose 2 elements. I can choose 3 elements in that way I can choose up to n elements. That means, the these sum tells about the total number of elements that we can total count that we can choose the elements from a n elements set. I can choose 1; I can choose 2 at a time or 3 and these will be my S 2 to the power n.

Sometimes we write this thing as a theorem also. We can we can write this thing as also a theorem and we can prove this thing. If we if from the definition only actually we can prove this thing that this is this is my the total number of ways or ways we can choose elements from a set is with n elements and this is my say actually this is my left hand side tells that thing I can choose 1 2 3 like that. What my right hand side tip very simple way, I can think if I have we have n elements n elements and as if I am making two partition.

So, whether these n elements and I will be taking some combination or not. Now, that means, if it goes to this combination I am giving 1. If it does not go, then I am putting 0; that means, if it goes to combination 1; if it does not go, I can take it is 0. So that means, if it is in I this is nothing but the all possible ways I can write the thing as 0 0 like this if I have 3 I can tell that it is 0 0 0 or I can write that all possible way if it is only n equal to 3, I can write 000 001 010 011 100 101 110 and 111.

So, these are the way; that means, these I have not chosen any combination. Here only these element goes to combination; and these element goes to combination; and these. So, these are all possible way that we can take the combination; that means, whether and here see we have covered when n equal to 3, we have actually covered all the combinations here and all of we know this is just if we can think in this way. This is a binary representation. So, it is a as if the invariable n bit binary representation.

So, I can write this is 2 to the power 3. So, when this will be some for n elements for n elements I can think we know that these count is 2 to the power n. So, if the RHS we can think that as if these n elements and whether I take the combination or not; that means, if it is goes to that combination partition I am just we are thinking like that; we are defining in this way.

Then, this RHS is nothing but 2 to the power n RHS is nothing but 2 to the power n and the LHS is actually it gives a total number of ways we can choose elements from a set to n elements with n elements and this is definition wise according to the definition the LHS and RHS are same. So, I can put this thing. So, I can treat this thing as a this thing as my proof. Sometimes, we take this as this property as if this as a theorem and then, we use this relation for solving some of the combination problem.