Discrete Structures
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Lecture – 36
Counting Techniques and Pigeonhole Principle

Today, we will read a Pigeonhole Principle which is an elementary, but very important commentarial principle. We can use to solve a variety of problems and the problems using arrangement.

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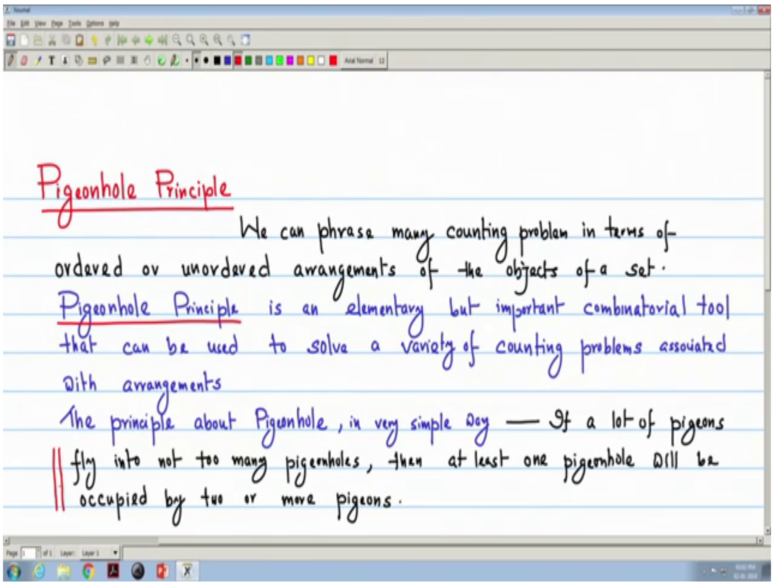

So, the study of arrangement is of object is an important part of discrete mathematics. So, we write Pigeonhole Principle. The study of arrangements of objects is an important part of discrete mathematics. We can face many arrangement or many counting problems in terms of arrangement, I am for ordered or unordered arrangements and there we can use this pigeonhole principle to solve the problems. So, this is very elementary, but very important counting techniques or the principle that we use for counting or in combinatorial problems.

So, I can write the introduction that we can; we can phrase many counting problems in terms of ordered or unordered arrangements. And pigeonhole principle is an elementary; I can tell this is a combinatorial tool that can be used to solve a variety of counting problems associated with arrangements.

So, very simple way if I tell this principle that, the term is pigeonhole. So, as if a lot of pigeons fly into not too many pigeonholes, then each pigeonhole must contains more two or more than two pigeons. So, very simple form I can tell this term, that from where this pigeonhole terms are coming.

So, simple way if I use this term that the principle about pigeonhole, in very simple word if I tell. I can tell that, if a lot of pigeons fly into not too many pigeonholes, then at least one pigeonhole will be occupied by two or more pigeons. And this simple rule or principle, we can apply and we can solve a interesting problems or we can conclude surprising way, that conclusions are sometimes it very surprising though simple.

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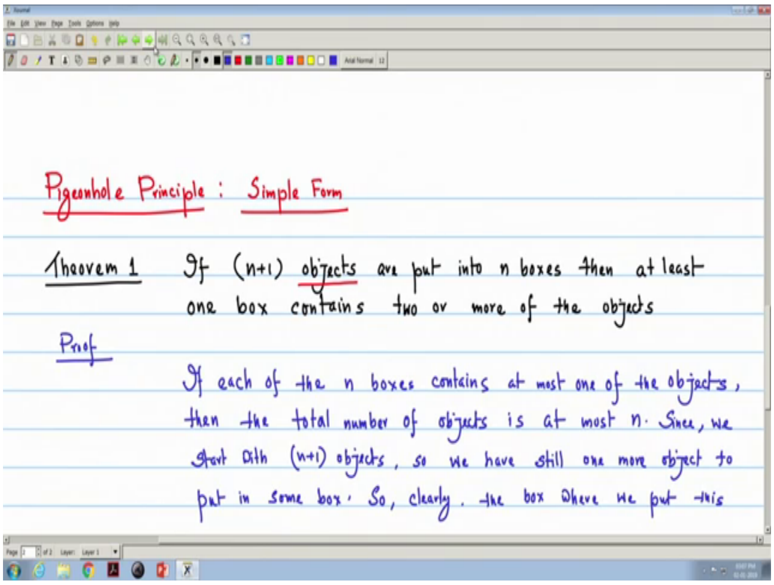

So, first we in a simple form or more mathematically, we try to state the pigeonhole principle. A simple form we tell, you can write the statement as a theorem. So, if n plus 1 objects are put into n boxes, then at least one box contains two or more of the objects.

So, the earlier statement I made, there here these n objects are similar to the pigeons and the box are the pigeonholes. So, how we can prove? So, statement is n plus 1 objects are put into n boxes, then at least one box contains two or more of the objects. So, if we prove, the proof is very simple. So, if each of the n box contains at most one of the objects, then the total number of objects is at most n. Because there are n boxes we have considered.

Now, since we start with n plus 1 objects. So, we have still one more object to put in some box. So, clearly the box where we put this n plus 1th object or the last object or the last one that contains two objects and it is proved.

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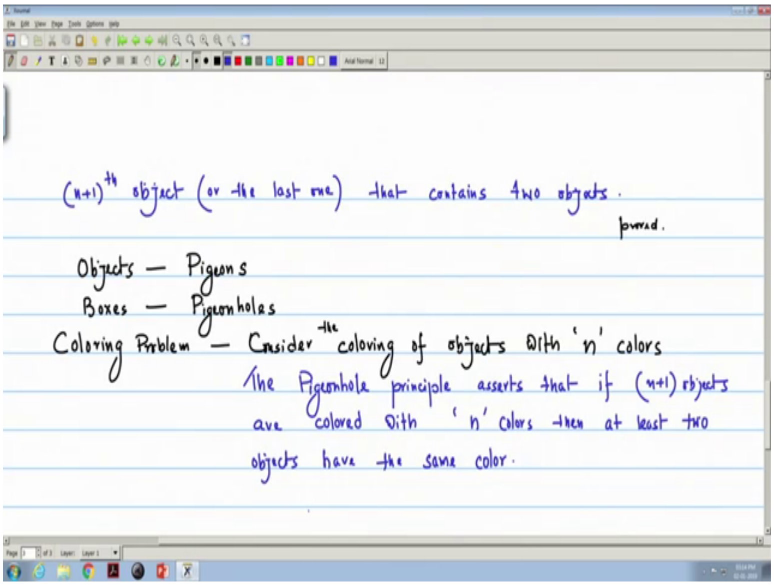

So, here we have; we have considered that each of the n boxes; if each of the n boxes contains at most one of the objects. Otherwise, that if we do not consider this that at each box contains at least one, then automatically that one of the n box; one of the n boxes must contain two or more than two objects.

Now, here something we must see that, this pigeonhole principle it does not guarantee in finding the one of the box, which contains two or more than one objects or it does not guarantee to identify the box or how the boxes are arranged like that? It only states that, the arrangements are such that one of the boxes; at least one of the boxes contains toward more than two objects. And, but with using these simple statements, we can solve a number of problems of different classes of problems.

Now, the problems or the problems we have stated or the principle, we told sometimes we called pigeonhole; sometimes it is also popular as shoe box problem, that what the objects as I already mentioned the objects are pigeons and the boxes are pigeonholes.

Now, this problem can be of say coloring problem ok. Say, so instead of; so, here actually we can write that these objects, I can tell these objects are same as that of pigeons and boxes are similar to pigeonholes. Now, instead of putting objects into boxes, I can consider as a coloring problem, then I can consider these as a coloring problem say, coloring of balls or coloring of objects, then I can think that coloring problem that we have to consider color the coloring of n balls; oh bit rewrite, coloring of objects with n colors; n number of colors.

And, we can tell that, the pigeonhole principle asserts that if n plus 1 objects are colored with 'n' colors, then at least two objects have the same color. So, here the pigeons and the pigeonholes are different, as if the pigeonholes are the colors and the pigeons are the objects.

So, when we try to solve the problem using the pigeonhole principle, that first we have to identify that which are the or which objects we are considering as the pigeons and which objects are considered as the pigeonholes. Since, our pigeonhole principle tells about the relation between the pigeon and the pigeonholes. So, we have to identify this thing. Now as I mentioned that a number of problems or a variety of problems, that can be solved using pigeonhole.

So, in this lecture we try to classify the problems that we use, that one class of problems that can be used by or that can be solved by using pigeonhole. First we consider a very simple problem that normally we solve using pigeon. And these classes of problems we are telling as if these are some application or different application classes. So, this is one application 1 we are taking.

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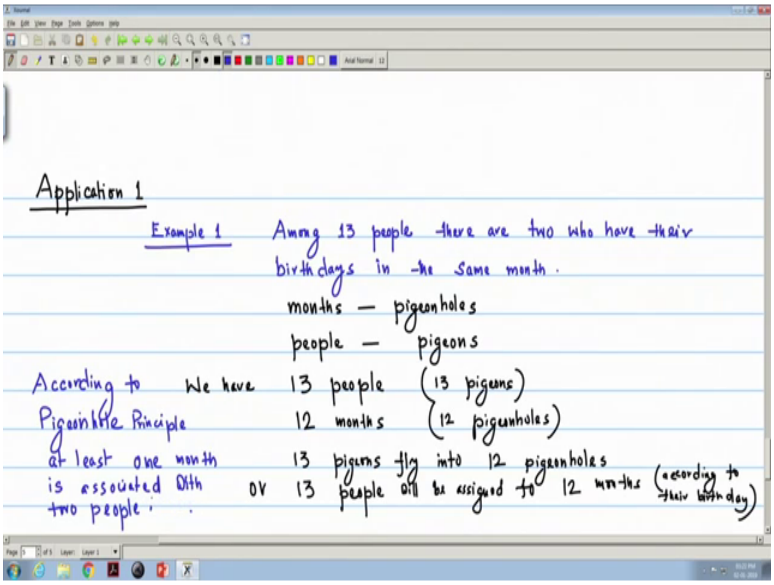

Say, we are giving some one example we are taking, say example 1. Say, among 13 people, there are two who have the birthdays in the same month. So, first we identify who are the pigeons and who are the pigeonholes. As, if here we have in year we have 12 months. So, here as if the months are the pigeonholes and the people are the pigeons.

So, I have I have 13 people; so we have 13 people that means, is it 13 pigeons and that we have to put into in 12 months. And, we have 12 months that is 12 pigeonholes. So, 13 pigeons we have to put 12 pigeonholes or I can write, this is similar to that 13 pigeons fly to; fly into 12 pigeonholes, which is same as that of or 13 people will be assigned to 12 months, according to their birthday.

So, according to pigeonhole principle that one month must contain two people. So, I can write according to if we apply pigeonhole principle. So, according to pigeonhole principle at least one month, because one pigeonhole contains two people, so one month is associated with two people who have the is associated with two people who have their or different way I write at least is one month whose, but they are in the same month.

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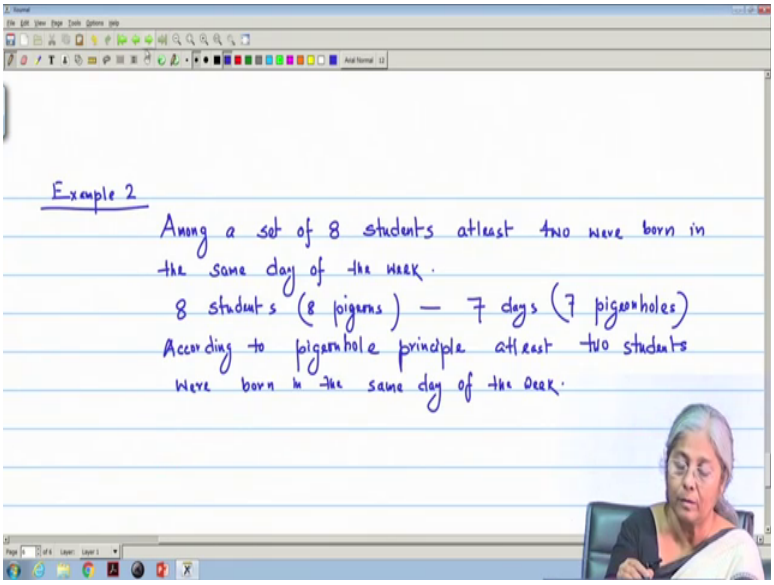

Now, the similar problem I can take with under application 1, I can take another example say, among the set of 8 students at least two were born in the same day of the week. Similarly, here the pigeons are the students and the pigeonholes are the; so, 8 students I can think that 8 pigeons and assigned to 7 days in the week; so 7 days, which is 7 pigeonholes.

And, so, according to pigeonhole principle, at least two students, because if I consider at most one student in one day they are born in one day of the week, then the eighth one must be in one of the days where already we have considered that some other student has was born. So, two students were born in the same day.

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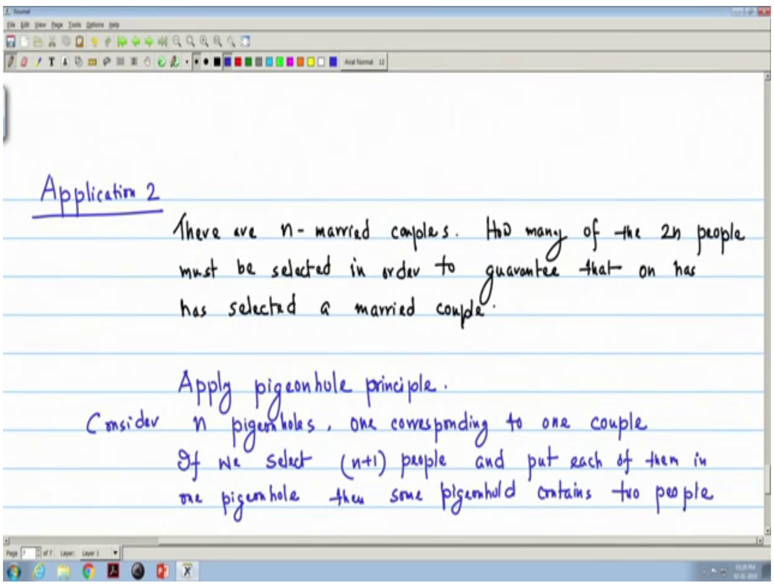

This is another similar but slightly different; so, I am writing as a application 2 say the problem is there are n - married couples. How many of the 2n people must be selected in order to guarantee that one has selected a married couple.

So, we can directly apply pigeonhole principle. So, apply pigeonhole principle and as if we consider n people or n pigeonholes or boxes (Refer Time: 34:11) write n pigeon holes, considered n pigeonholes, and one corresponding to one people or to each of the n couples to one couple. Now if we select n plus 1 people and put each of them in one hole, then some pigeonhole contains two two people.

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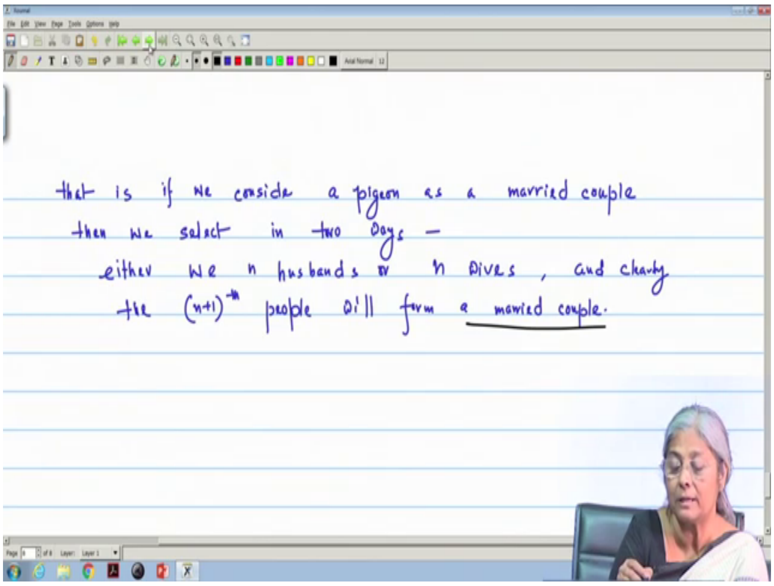

That is if we consider, a pigeon as a couple; a married couple, then we select in two ways either we take n husbands or n wives, and the and clearly the n plus 1th people will form the form a couple; will form a married couple.

So, this problem directly gives this is accounting that, because my question was that how many? Now, question was that how many of the 2n people must be selected and the answer is that n plus 1, if we select n plus 1 people then we can get a married couple. So, this is one, we have seen that application 1 and 2 that is two classes of problem very simple type of problem, that we can solve using pigeonhole principle.

There are other principles related to the pigeonhole, that are similar to that the statements that we have met. And, that we can write as a that in a different form that we can tell; one is, I can write that if n objects are put into n boxes and no box is empty, when each box contains at least one object; contains exactly one object.

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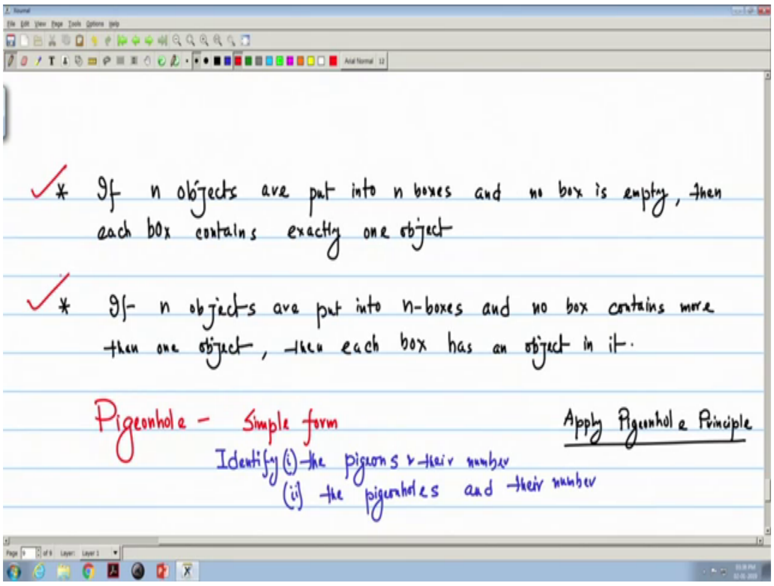

Same thing; same statement, I can write that, if n objects are put into n-boxes and no box contains more than more than one object, then each box has an object in it. So, normally these are the three statements or the or three statements that are used for the pigeonhole in simple form; these are the pigeonhole in simple form, that we use to solve the some arrangement problems or the counting problems.

So, here the main principle is that, we have to; we must identify given a problem, we must identify the pigeons the objects, which are same as that of pigeons and their number and their numbers this is one and the pigeonholes; pigeonholes and their number. And, then some analogy between the pigeon and the objects and the pigeon holes and the objects and then apply pigeonhole principles, and then we apply pigeonhole principle; in any one of the form. One we have given as a theorem one and these are two other different ways we can state the pigeonhole principle.

So, here these lecture very simple problems; one class of simple problems as an application one and application two, that we have read that, how to solve using pigeonhole. In the next lecture, we will see some mathematical or numerical problems, how we can solve using pigeonhole principle.