Discrete Structures
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Lecture – 35
Recurrence Relations (Contd.)

So, we are discussing about the solution of Recurrence Relation, how what are the different techniques to solve recurrence relations. In the last 2 lectures we have discussed, how to solve the linear homogeneous recurrence relation with constant coefficients when roots are equal or roots may not be equal.

And with the second order equation or second order equation or second order recurrence relation, we have read that how with examples that how we can solve that thing. Now today we will see now how we can solve linear, but non-homogeneous recurrence relation with constant coefficients.

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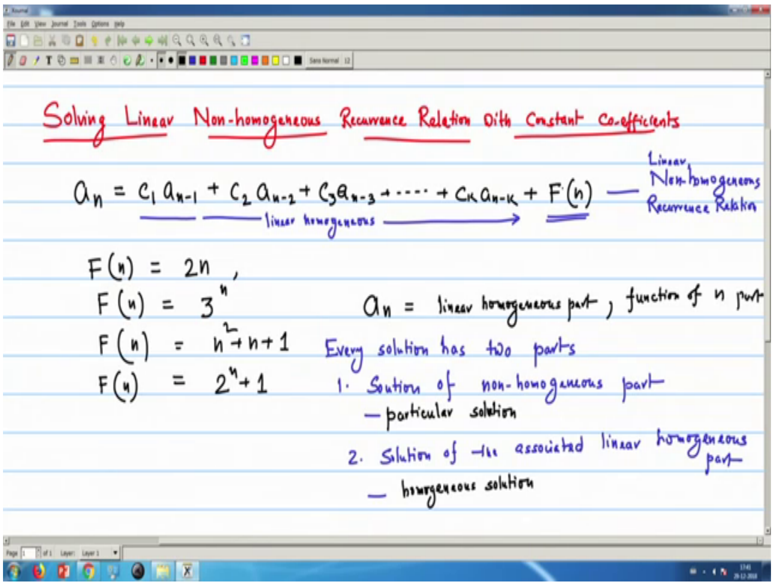

So, we will be reading the solving linear non-homogeneous recurrence relation with constant coefficients. So, first we see that: what are the forms of non-homogeneous reconciliation. If I remember that my linear homogeneous recurrence relation was a is C 1 a n minus 1 plus C 2 a n minus 2, C 3 a n minus 3. If it is of order k, then C k a n minus k this is my linear homogeneous recurrence relation. Now if I add a function of n here then this becomes a, this becomes a non-homogeneous recurrence relation linear since the power is up each term is 1.

But we have then what are the; what is the solving techniques of this linear non homogeneous reconciliation? Now see up to this part it is same this is my linear homogeneous part this linear homogeneous recurrence relation with constant coefficient, only here this is the non homogeneous part that is a function of n has come. Now this function of n this can be of the form different form function of n can be 2 n, function of n can be of say 3 to the power n or function of n can be n square plus n plus 1 or it can be any other that which contents of n say 2 to the power n plus 1.

So, this type of this function can be there. So, how we can solve this thing? Now see here that linear non homogeneous recurrence relations then I have two parts. If I consider that a n I have a it I have a linear homogeneous part, I have a linear homogeneous part from the above equation we see and we have a function of n part function of n part. Now so, every solution is of the form or I can write first every solution has two parts; one is the solution of a particular solution. That means, two parts, one is the solution of non homogeneous part, we call this is the particular solution we call these as a and second is that solution of the associated linear homogeneous part linear homogeneous part. Normally we call this as a homogeneous solution, we call it is as the homogeneous solution.

So, as if the recurrence relation or the equation has two part; one is the linear homogeneous recurrence relation with constant coefficient plus this F n that is as if this is a particular function it has two part. And when will get the solution, solution is also two parts that solution of non-homogeneous part; that means, for F n this is called normally the particular solution and the solution of the associated linear homogeneous part, we call the homogeneous solution. So, we can write that we have the recurrence relation.

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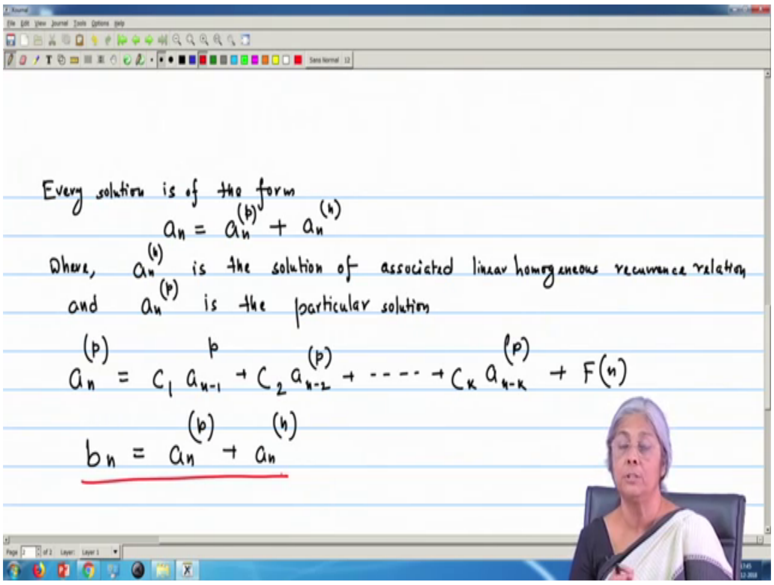

So, every solution every solution is of the form, that a n equal to a n equal to a n particular n p plus in h there are two parts. So, where a n h is the solution of associated linear homogeneous recurrence relation and a n p is the particular solution. So, actually a n p, I can write a n p equal to the C 1 a n minus 1 p plus C 2 a n minus 2 p, C k a n minus k p plus F n. So, you can write that I if my solution b n I can write a n p plus a n h. So, now, with some examples we see that or how we can find out the particular solution and the homogeneous solution of a non homogeneous recurrence relation linear non homogeneous recurrence relation.

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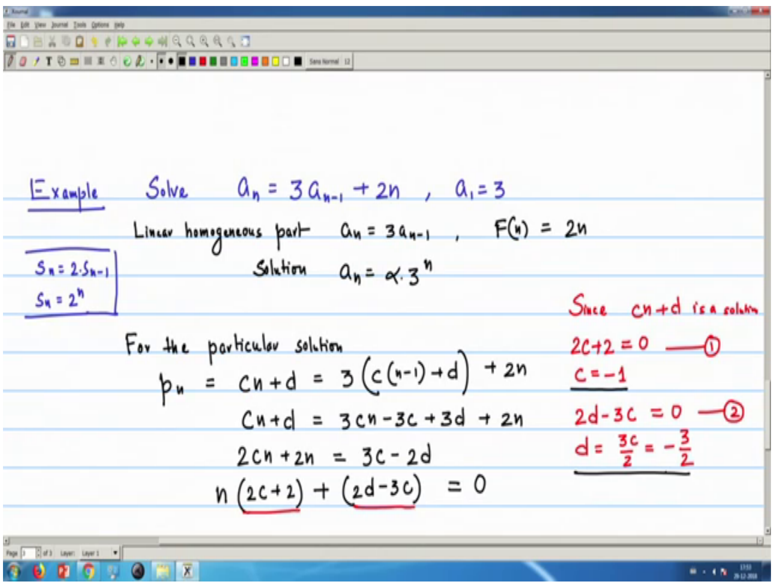

So, I take one took one example it first we take a simple example. We take an example that solve a n equal to 3 a n minus 1 plus 2 n and the initial condition is a 1 equal to 3. So, the homogeneous linear homogeneous part is, I can write that a n equal to I can write the linear homogeneous part as a n equal to 3 a n minus 1 and the F n the function of n is as a 2 n ok.

Now already we have seen that the solution of these a n equal to 3 a to the power n minus 1; if I remember that our number of subsets opposite if we remember quickly that S n equal to 2 S n minus 1. When we have we have framed the recurrence relation of a number of subsets of a set having n elements, we got that this is my recurrence relation and the solution by applying the technique of iteration, we got the solution is S n equal to 2 to the power n that we can we can remember.

So, it is of the same form; S n equal to 2 S n minus 1 here n equal to 3 a to the power n minus 1. So, the solution I can write, the solution it is of the solution is of the form a n equal to alpha 3 to the power n. Since here there is only one linear part was there only linear homogenous part, but this is a hybrid type here it is an linear homogeneous part as well as one function of n is there. So, I take one coefficient alpha, but the solution is has is of the form 3 to the power n, that I can get from my that simple recurrence relation that already we have read we absorbed also iteration. So, this is of the form this.

Now I have F n equal to 2 n so, that I can write the particular equation. So, this is linear homogeneous part and the particular solution for the particular solution I can write the p n equal to C n plus d it is of the form because it is 2 n simply. So, I can write that C n plus d, I can write 3 because my equation is 3 n minus 1 plus 2 n.

So, it is 3 C n minus 1 plus d plus 2 n. If we remember that the way we have done a n minus 1 p, a n minus 2 p, but here I have only one first order equation. So, I have only one term. So, I have given this term C n minus 1 plus d plus 2 n because I have considered it is the solution is of form C n plus d since my function F n is 2 n. So, if I write then C n plus d, C n plus d equal to 3 C n minus 3 C plus 3 d plus 2 n.

So, if I write 2 C n 2 C n plus 2 n equal to 3 C minus 2 d. So, I can write this equation as n into 2 C plus 2 if I take here plus 2 d minus 3 C equal to 0 since C n plus d is a solution. So, what I get here? C n plus d is a solution. So, as if this is C and this is my d. So, I can write that since C n plus d is a solution. So, this it will be a solution then to C plus 2 equal to 0 because this is equal to 0. So, 2 C plus 2 equal to 0; that means, C equal to minus 1; C equal to minus 1 and 2 d minus 3 C equal to 0; so, d equal to 3 C by 2 equal to minus 3 by 2. So, I get the coefficients I get the 2 coefficients C and d, C equal to minus 1 and d equal to minus 3 by 2. So, what is my particular solution?

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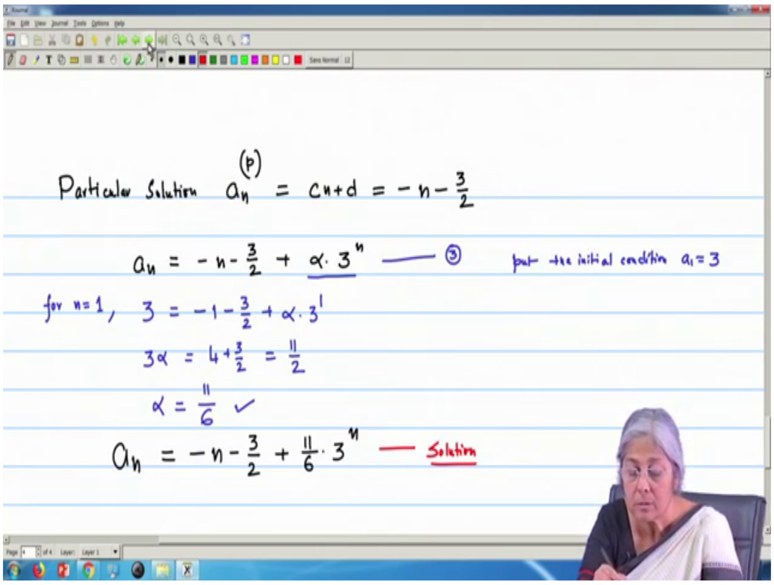

So, my particular solution that a n p equal to is C n plus d C n plus d. So, equal to minus n minus 3 by 2. So now, we have to find out that my solution total solution that a n I can get a n is minus n minus 3 by 2 plus alpha 3 to the power n, because alpha 3 to the power n was my solution for homogeneous linear homogeneous part. So now, we can put in this equation, we can put the put the initial condition a 1 equal to a 1 equal to 3 that was the condition. So, I get if that means, n equal to 1. So, for n equal to 1, I have 3 equal to minus 1 minus 3 by 2 plus alpha 3 to the power 1. So, 3 alpha equal to 4 plus 3 by 2 is 11 by 2 1 minus 3 by 2.

So, I have alpha equal to 11 by 6. If I put a 1 equal to 3, this is 3 this is minus 1 minus 3 by 2 plus alpha to the power 3 to the power 1. So, alpha 3 to the power 1 is 4 plus 3 by 2, I think it is 11 by 6. So, now, I can write my solution the final solution is minus n minus 3 by 2 plus 11 by 6, 3 to the power n. So, this is the solution this is the solution of that given linear non homogeneous recurrence relation. You now we see another example where, the function of n is of different type and we see that how we can solve that thing.

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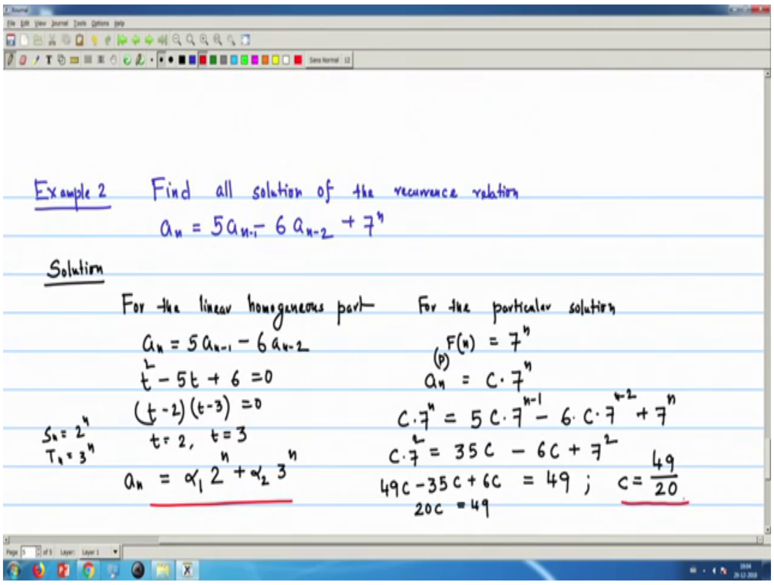

So, we take another example, and I write that find all solutions of the recurrence relation is 5 a n minus 6 a n minus 2 plus 7 to the power n 5 n minus a n equal to ok. So, we see the solution the way we have done; first we see the linear homogeneous part for the linear homogeneous part. So, we know that if we put the form of V n equal to t to the power n that type of solution then we get, since it is a second order linear homogeneous that homogeneous part; that means, this is a n equal to 5 a n minus 1 minus 6 a n minus 2.

So, I get t squared minus 5 t plus 6 equal to 0, and this becomes t minus 2, t minus 3 is equal to 0. So, t equal to 2 t equal to 3. So, my solution becomes that a n equal to alpha 1 2 to the power n plus alpha 2 3 to the power n what earlier we have done that b is n b S n plus dt n. Here is n equal to 2 to the power n if we remember that earlier we have done that, S n equal to 2 to the power n and T n equal to 3 to the power n, I have given earlier that solution is that bs n td n that alpha 1 to the power n plus alpha 2 3 to the power n.

So, this is my linear homogeneous part. Now, we see the you see the of particular solution see here that over for the for the particular solution. So, my function is F n equal to 7 to the power n. So, I get my particular form a n equal to C into 7 to the power n some constant, this is of the solution is of this form. So, if I write that C 7 to the power n, np equal to that 5 a n minus 1. So, there I get C 7 to the power n minus 1 minus 6 C 7 to the power n minus 2 plus my F n 7 to the power n. So, if I divide both sides by 7 to the power n minus 2, I get C 7 square is 35 C minus 6 C plus 7 square. So, this becomes 49 C minus 35 C plus 6 C equal to 49 that gives C equal to 49 divided by this is becomes 20 C equal to 49. So, C equal to 49 by 20 so, this is my particular solution.

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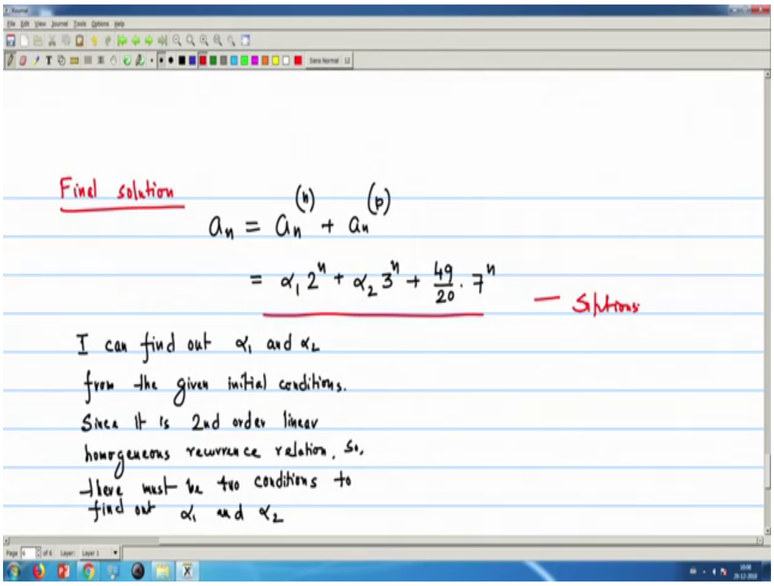

So, my final solution becomes. So, my final solution it has 2 part that a n equal to a n h plus a n p and that becomes that alpha 1, 3 to the power n plus alpha 2 alpha alpha 1 2 to the power n. So, they give alpha 1 2 to the power n alpha 2 2 to the power n plus particular solution is 49 by 20. So, C is 49 by 20 7 to the power n. So, this becomes 49 by 20 into 7 to the power n.

Now if I get some conditions like then I can find out I can find out alpha 1 I alpha 2, you can find out alpha 1 and alpha 2 from a given from the given initial conditions. Since it is a second order linear homogeneous equation so, I need at least 2 conditions to find out alpha 1 and alpha 2. So, there must be 2 conditions to find out alpha 1 and alpha 2 and solving those equations, I can get the.

So, once we get the ones we get the values of alpha 1 and alpha 2, we get all the solutions of we get all the solutions of though given recurrence relation. So, we have learned that how to solve the linear non homogeneous recurrence relation. Mainly non homogeneous recurrence relation has 2 part the homogeneous part. Homogeneous genius part will be solving the similar way we have handled the linear homogeneous recurrence relation and only for the function of n part that we will get the particular solution and the technique we have just now illustrated. So, with this we have we finished the lecture of the solving recurrence relations.