Discrete Structures
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Lecture - 03
Introduction to Propositional Logic (Contd.)

We have learned the Proposition and the operations on it the conjunction, disjunction, negation and the two conditional proposition the implication and the biconditional. Today, we will read or when can we tell the two propositions are same; that means, the relation between two propositions. Now, the simplest relation between two proposition are the equivalence, that means, given two propositions whether they are same or whether they are different.

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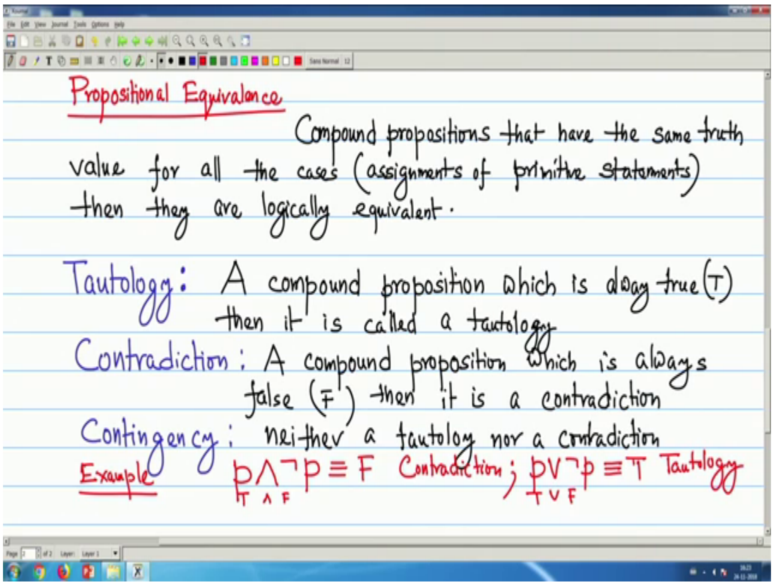

So, we first define the propositional equivalence. So, compound proposition that have the same truth value for all the cases; that means, for all assignments of primitive proposition. If the compound propositions have the same value for all the cases, I can tell the assignments of primitive statements, then they are logically equivalent.

As I mentioned that this is the simplest relation between two proposition or two statements. So, before we learn the equivalence between the different type of compound propositions having the logical connectives of and or negation or the conditional propositions. We define three basic terms one is called the tautology. A compound proposition which is always true, then it is called a tautology.

Now, if your compound proposition is always false, then it is called a contradiction. Proposition which is always false, then it is a contradiction. And the proposition which is neither a tautology nor a contradiction, that means, neither it gets a value true always or false always, then it is called the contingency, neither a tautology nor a contradiction.

So, the simple example of tautology very simple example. So, I have a proposition P. Then if I consider P and it is the law I take the logical connectives the and or the conjunction with negation P. So, P and negation P this is always false, because at a time the definition of proposition is either it is true or it is false. So, either if P is true, then negation P is false. So, true and false that will give me true and false. We give the that will give you the false statement, that means, P and negation P this is equivalent to a false statement. This is we denote as a F. So, this is a always this will be a contradiction; this will be a contradiction.

Similarly, if I consider the or now if I take the P or negation P, then it is always true, because either P is true or negation since it is a P is true and if it is negation P is false. So, if one is true, then always it is true. So, it is equivalent to a true so, this is a tautology. Now, we formally define the equivalence, the propositional equivalence, we will define.

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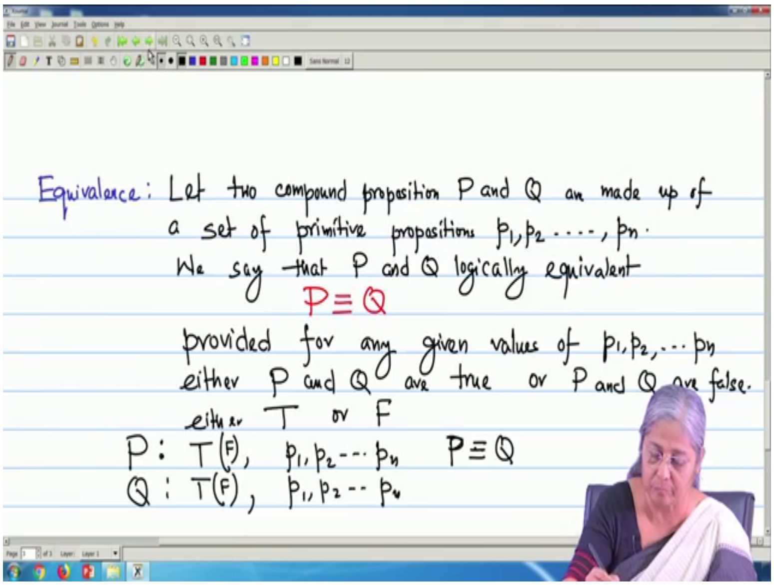

Suppose that we have two propositions P and Q, two compound propositions. And they are made of the same set of primitive propositions p 1 to p n. Let two compound propositions; two compound proposition P and Q are made of a set of primitive proposition p 1, p 2 to p n. We say that P and Q are logically equivalent. They are logically equivalent according to definition that if any values of p 1 and p 2, that means, a set of values of p 1 to p 2 n that P and Q have the same values either true or false.

Normally, we define or we denote the equivalence relation as we denote this as P is equivalent to Q. Provided for any given values of p 1 to p n, either P and Q are true, or P and Q are false, that means, the for the same set of primitive propositions p 1, p 2, p n that P Q have the same values, either true, it is either true or false, then they are called the logical equivalence. Now, why it is important? Because, we are considering that we are now we want to see the relation between true statements and we want to check.

Now, the simplest thing if we can derive the one it is a given two compound propositions P and Q. If we derived P and we get that for some values of p 1 to p 2, P get the values true, that means, if P P is true for a given p 1 to p n values. Similarly, if we see that Q is also true for the same values of P p 1 to p n the primitive proposition, then we can take that P is equivalent to Q.

Now, instead of this true that P can also be P can be false also, then this time Q must be false. Then we can tell that they are actually the same statements that means P and Q are the same statements or we call that logically equivalent. Now, we will use this equivalence relation to check that any other set of compound statements whether they are true or what are the what actually the relation between them ok.

Now, the first relation we will see that it is some rule, it is called the De Morgan’s rule, that actually gives a relation between the two compound proposition using the basic operations or the logical connectives the conjunction, disjunction and negation.

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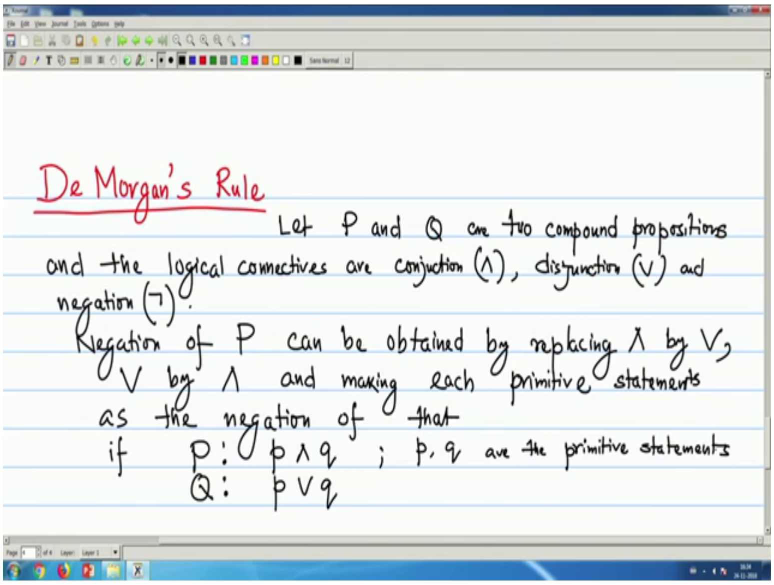

So, first we see the our De Morgan's rule. So, what the rule tells, say suppose we have two compound propositions P AND Q. Let P AND Q are two compound propositions. And the connectives are the logical connectives between the primitive propositions. So, logical connectives, we can write are the conjunction, disjunction and negation.

Now, the De Morgan's rules tell that the negation of compound proposition P say can be obtained by replacing, AND by OR, the OR by AND, and making each primitive statements as the negation of that. Then what I can take say if P is p and q, and Q is p or q. Here P Q, capital P Q are the compound propositions. And small p q are the primitive statements or primitive propositions, basic propositions or the primitive statements.

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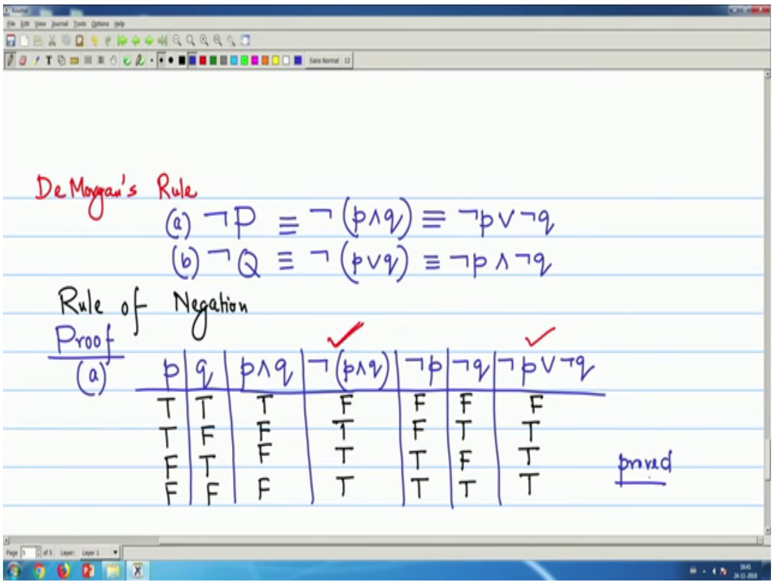

Then De Morgan rule tells, so what it tells that the conjunction is replaced by disjunction, disjunction is replaced by conjunction and the each primitive statement is replaced by it is negation. So, if I take the negation P, if I take that my negation P that is equivalent to negation p and q, since my P is the composite proposition that is the conjunction of p and q. So, this is equivalent to the P will be replaced by a negation p this AND becomes the OR, Q becomes the negation q.

So, what will be my negation Q? Negation Q is equivalent to the p OR q and that will be equivalent to negation p AND negation q. So, De Morgan's rule tells that the rule of negation it is called the this is called the rule of negation. And when we will be deriving or to make simplify the some compound propositions, then always that it is very important to apply the De Morgan's rule, so we will get the different form using the primitive statements and it is negation.

Now, this is a rule. So, how we can prove that thing? We prove first we see the a. Now, we can see the equivalence we have defined that they are equivalent if they take the same value for all assignments. Since, here our primitives statements are only two p and q. So, we will take all possible assignments of p and q and that we can do by using truth table.

So, we see the truth table. Now, we have the compound proposition P is p AND q. So, first we compute p AND q then we have to find out the negation p AND q. So, this is our left hand side. Now, we find the negation p, negation q and then negation p OR negation

q. And we see that what we get. p, q have four values it can be T T, T F, F T, and F F. So, p AND q - this will be true; this will be false; this will be true; this will be false. The negation - so it will becomes false; sorry, this is also since it is and this is also false; so this becomes true; this becomes true; this becomes true.

Now, what is negation p? P is T T T F. So, negation p is F F T T. Negation q is, q is T F T F. So, negation q is F T F T. The negation p OR negation q – F OR F, it is F; F OR T, it is T; T OR F, it is T; T T, it is T. Now, if we see that our this is our left hand side that negation p AND q. So, this is F T T T. And this is our right hand side that negation p OR negation q, F T T T. So, what we see that for all possible assignments of p q, when they are taken or both are true, or both are false, or one true, one false, then we get actually that they are taking the same true value, either when they are false, both are false, or both are true. So, it is actually that the De Morgan's rule or the a part is proved. So, they it is proved the De Morgan's rule it is proved.

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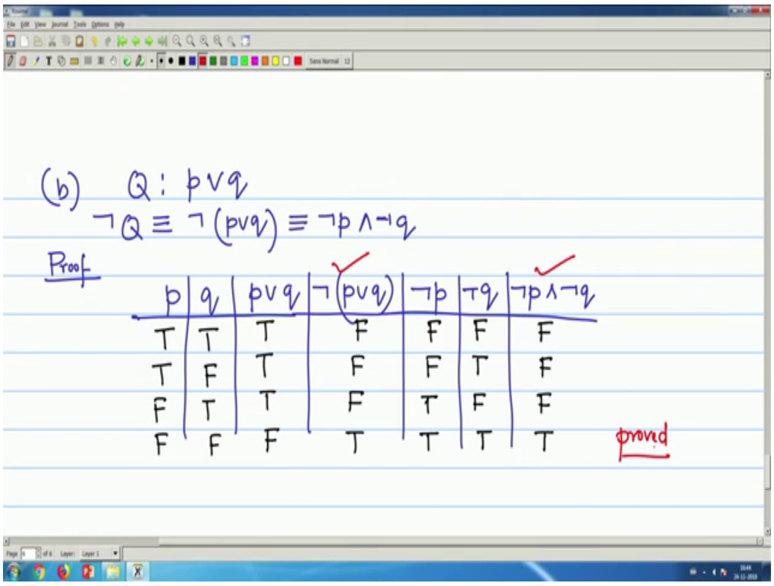

Now, quickly we can see the b part; b is our compound proposition Q. Q is our p OR q. So, what De Morgan's rule tell that negation Q negation Q is equivalent to negation p or q. And using De Morgan's if we compute the negation it will be negation p and negation q. Now, if we prove, then again we can draw the truth table p q. Then this time it will be p OR q, then negation p OR q, then this negation p negation q and negation p and negation q. We take the both are true, true, false or both are false. If one is true, then p OR q is true, it is true; only when both are false then they are false.

So, when I am taking negation, it will be false, false, false, only this case it is true. Now, similarly negation p is false, false and this is in this case it is true, false, this is true, this is false, this is true. So, this is negation p and q so, it is false, this is also false, only this is true. So, again we see this is my LHS; this is my RHS. And they are taking the same truth value for the same set of primitive statements that is small p and small q. So, the De Morgan's rule is proved here ok so, it is proved.

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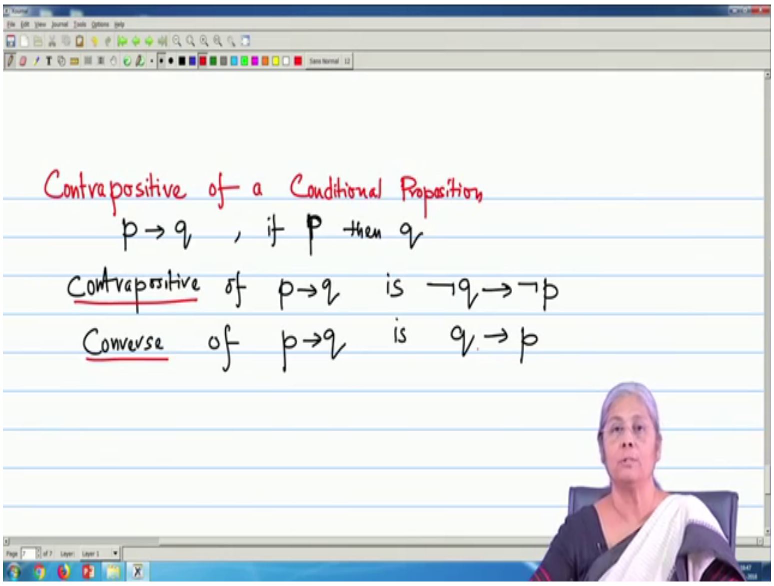

Now, we see another form of or alternate form of logical or conditional propositions. This is we call the contrapositive of a; contrapositive of a conditional propositions this is also very important, when we will be evaluating some compound propositions. We know that conditional proposition is that p implies q; that means, if p then q; if p then q. Now, contrapositive is that negation q implies negation p contrapositive of p implies q is negation q implies negation p.

And converse of p implies q is q implies p. So, the difference between the contrapositive and the converse is that for contrapositive this is a negation q implies negation p and converse is simply the q implies p. This is also have often we use this thing for evaluating the compound proposition not to simplify the compound proposition. And next we will see that how we can use this thing for the equivalence checking.