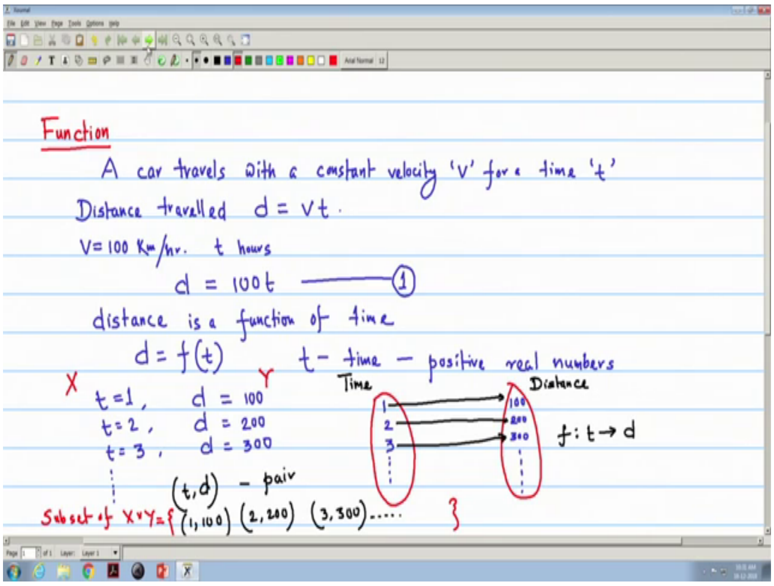
Discrete Structures

Prof. Dipanwita Roychoudhury
Department of Computer Science & Engineering
Indian Institute of Technology, Kharagpur

Lecture - 20
Sets and Functions (Contd.)

Now, we have read the fundamentals of set, we know that what actually we mean by the term set and the different set operations or the different laws that we can apply on set. So, today will read the function.

(Refer Slide Time: 00:39)



First I give the idea of function, say is a very simple example let a car travels with a constant velocity, constant velocity say v for a time t. So, we know the distance travelled by the car say d equal to v t, since it is a constant velocity

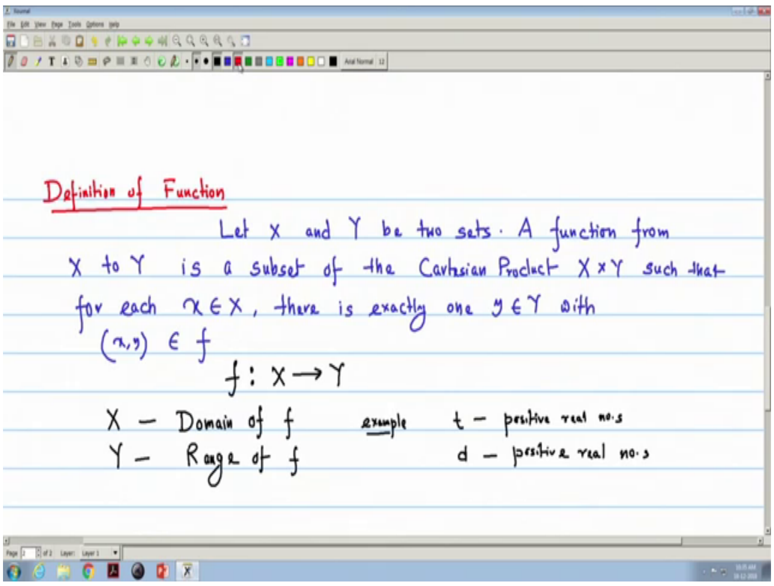
Now, let v is 100 kilo meters per hour and the car travels for t hours, simply we know the distance d equal to 100 t. Now these equation 1 we know that this is distance is a function of time, distance is a function of time. That means, d is a function of t normally we write d is a function of t. What do we actually mean by these? Now say I take different values of t since time t is time. So, t can be some real positive number, positive real numbers, real values it can take so, it can take positive real numbers. So, see initially I am taking only integer value say for t equal to 1. So, my d equal to 100, say for t equal to 2, d equal to 200; t equal to 3 d equal to 300 like that since it is a constant values 100. So, I can get this type of value.

Now, see for each t equal to 1, 2, 3 that for these value d is assigned a value 100, 200, 300 like that. So, we can write now that if I draw in these way as if that this is something called the my time as if I am denoting the time here and this is my distance, this is my distance. So, my t can be 1, 2, 3 or give that values t can take these values 1, 2, 3 this t can take values and for these my distance can be say 100, 200, 300 like that.

So, now if I connect; that means, if I give this thing that it will be a pair, as if it is a pair of t d, this a pair this t d pair and that pair will be 1, 100; to 2, 200; 3, 300 in that way it will go. That means, this is now if I connect as if as if 1 and 100 will be connected, 2 and 200 will be connected, 3 and 300 will be connected in this way it will work. So, this is one example of a function that I can write that this is a f and it gives you that some t to d and see last time we have read the Cartesian product.

Now if I consider that my these, all these time values as if this is something one set called X, X consists of all the time values that the car travels. And, this is the distance travel if the it is a set x then I can tell that this is all the Cartesian product or a subset of the Cartesian product. So, this is a subset of the Cartesian product. So, this is a subset of X cross Y, that X Cartesian product Y, I can write in this way. So, this is the concept of function. So, now formally we can define function, the definition of function.

(Refer Slide Time: 07:15)

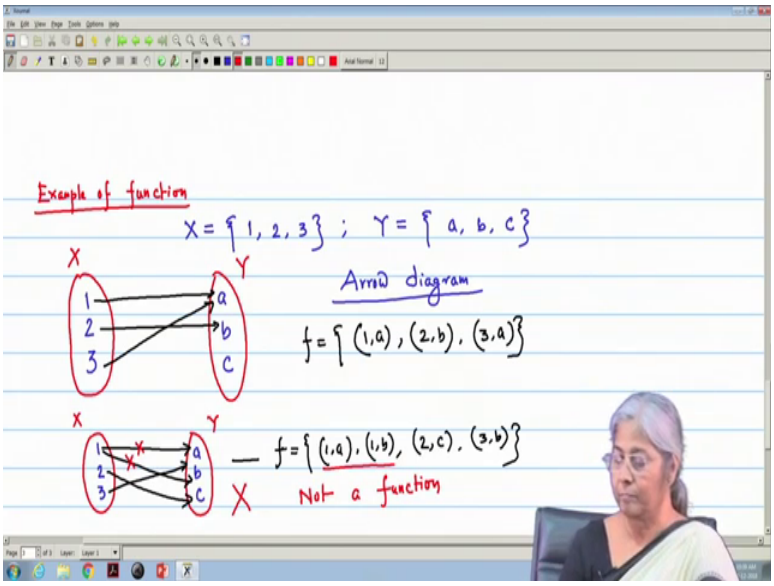


So, let X and Y be two sets, plus just now we have the example we have defined as if X is the set of all real numbers denoting my time, Y is the set of values the distance travelled. Now a function from X to Y, the set X to Y is a subset of the Cartesian product. X Cartesian product y for each x belongs to X there is exactly one y belongs to Y with x y belongs to f.

So, how I define that X and Y be two sets a function from X to Y is a subset of the Cartesian product X cross Y such that for each of the element of the set x, x belongs to X there is exactly one y belongs to Y with x y belongs to f. Normally, we denote that f is we denote like f the function is X to Y this is my function and here X is called the domain of f, domain of the function f, Y is called the range of function f.

Like the previous example that we have taken the t as the positive real numbers. So, when we have considered set x having the values all values of t's then the domain is positive real number, similarly my d is also the positive real number. So, here domain and range all are positive real numbers. So, now we take some example, simple example of function.

(Refer Slide Time: 11:19)

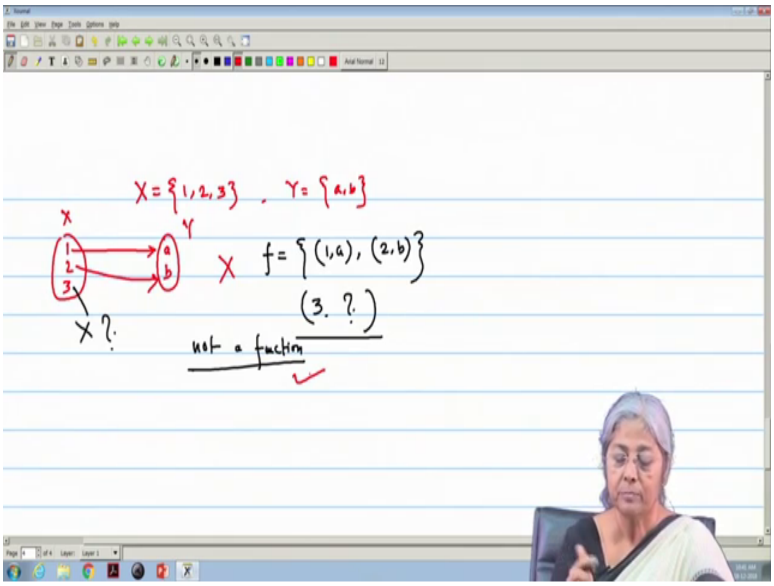


Some simple example of function we see. So, we need two sets let the X is only 1, 2, 3 these are the and Y is I take a, b, c. Now, if I draw the X so this is my set X and this is my set Y, I give only two values 1, X has three elements only and y has also 3 elements a b c. Now normally we represent pictorially by the function as by arrow diagram. So, we use the arrow diagram; that means, we already have seen their function assigns a value of for each value of X to some value of Y and that will be exactly one, exactly one value of Y is assigned.

So, if I give arrow diagram we will draw one arrow from element of X to element of Y, say I give a arrow here then 1 to a, 2 to b again I put 3 to a. So, since the function we have defined as a subset of Cartesian product. So, here the subsets on Cartesian product is nothing, but the ordered pairs. So, it is 1, a; 2, b; again 3, a so this is a function. Now, if I consider, if I consider say a different type of assignment, again this is X, this is Y say I have 1, 2, 3 and here a, b, c. I give some arrow that say 1 is connected to a, again 1 is connected to b, 2 is connected to c, 3 goes to b.

See here this is it is not a function so; that means, these arrow diagram it gives that f equal to 1, a; then 1, b; then 2 c; 3 b. So, this is not a, this is not a function, why? See here 1, a and 1 b; that means, that it violates the definition that or the assignment that 1 is connected to both a and b. So, this is for this reason that it is not a not a function, not an example of a function.

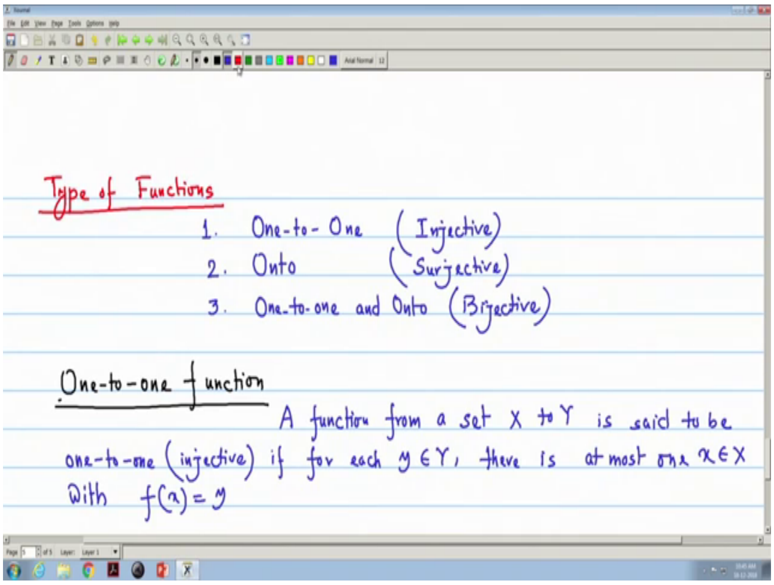
(Refer Slide Time: 16:00)



Now we take another example, say again that same X equal to 1, 2, 3 set and Y equal to a, b. So, if I take X and Y I take 1, 2, 3 and this is a, b then say 1 is connected to a, 2 is connected to b, but 3 is not connected. So, again this is not a function because the definition tell for each X. So, since we do not have here we do not have any, we do not have any connection for 3, we have only the set here f is 1, a and 2, b but we do not have 3. So, this is not a not a function, this is not a function.

Now, we have just now we have seen that this is that function r, last example that this is say a many to one; that means, both 1 and 3 is assigned to a or it can be that only 1, 1 1, a; 2, b; 3, c that type of function can be there. So, for based on their assignments that that one value of X is as how it is assigned to another set. So, we can define that the different type of function.

(Refer Slide Time: 17:54)

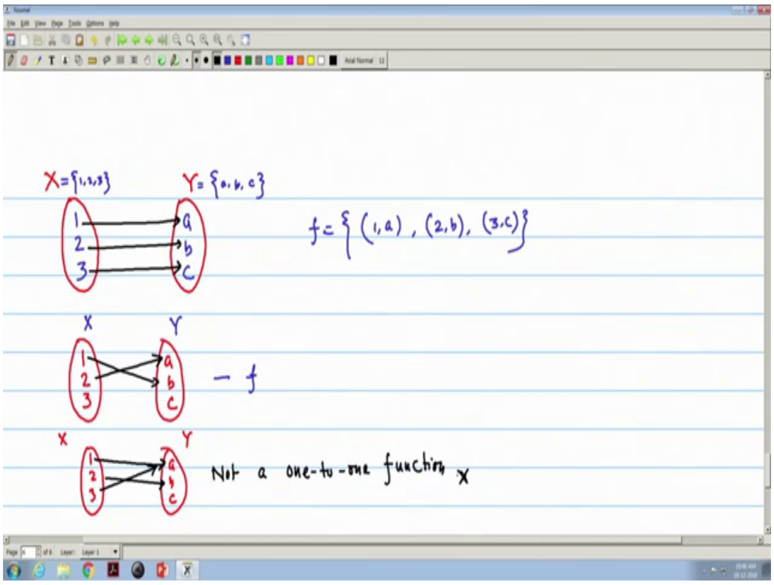


So, normally the type of functions are; we have three type of functions are there [nois1e] one is called the one-to-one function, this is a one-to-one we called it is or injective. We have onto function it is called surjective and one is one-to-one as well as one-to-one and onto and onto and this is called bijective function.

Now, we see that how they are different or how their assignments are from one set to the other set that X to Y how they differ. So, first we see that one-to-one function, you see one to one. So, if was with the definition that a function from X to Y, a function from a set X to Y is said to be one-to-one or what we call the injective if for each y belongs to Y. That means, for each element of Y there is at most one x belongs to X with f x equal to y.

So, here for first thing is that for each y belongs to Y and we have at most one x belongs to X this is my mapping.

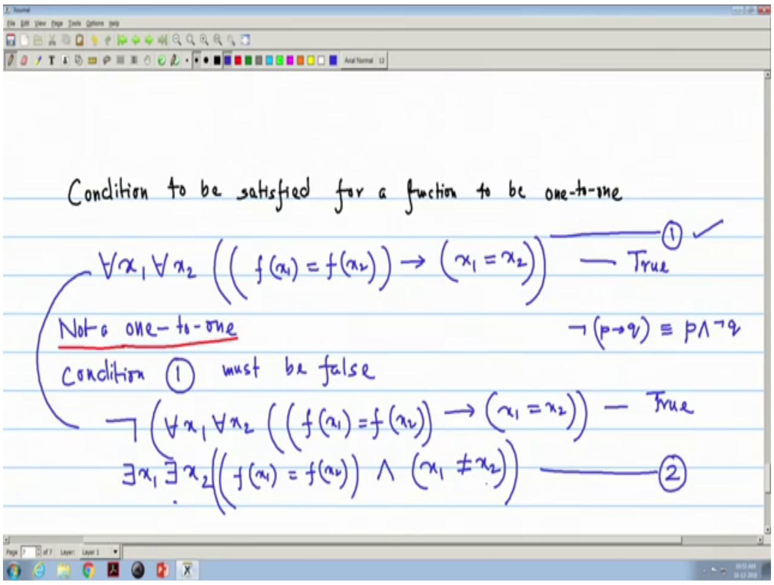
(Refer Slide Time: 21:34)



So, if I take one example that say I take the same example of X to Y 1, 2, 3 here a, b, c as if X is 1, 2, 3; Y is a, b, c and my function is given as here 1, a; 2, b; say 1, a; 2 b; 3, c then if I give the arrow that 1 is assigned to a, 2 b and 3 c. That means, for each element of Y; that means, for each value of a b c at most one value is given ok, at most one.

So, if I take the same example say I have I have 1 2 3 I have a b c then say a is connected to b, 2 is connected to a and say c is not connected then also it is a because for if we see the definition that for each y there is at most one f ok. So, here c is not assigned any value. This is so this is a function ok, c is not connected or assigned, but if I take this type of example. So, it is a function, but it is not a not one to one function, not a one to one function it is not a one to one function.

(Refer Slide Time: 24:39)

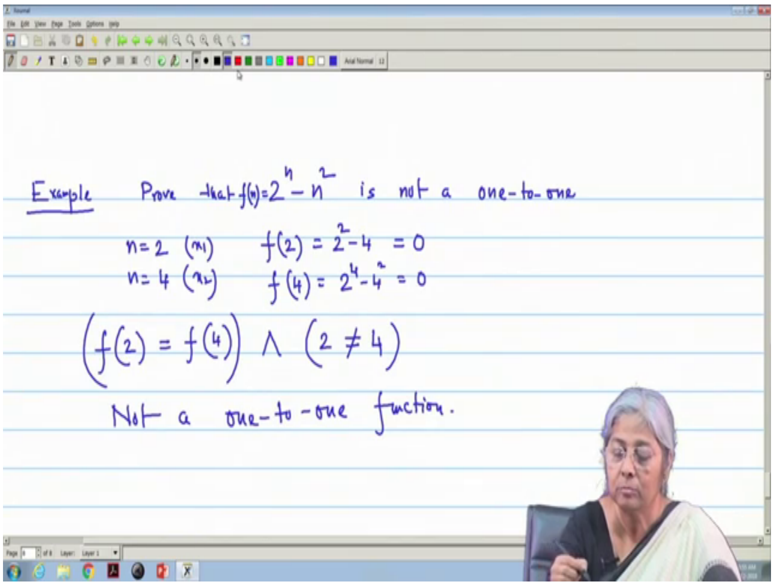


So, what is the condition to be satisfied for a function to be onto? The condition to be satisfied for a function to be onto one-to-one first, one-to-one I write that for all x 1 and for all x 2 if I consider two values then f of x 1 equal to f of x 2 implies x 1 equal to x 2 this must be true. That means, if I get some f of x 1 equal to f of x 2 then x 1 must be x 2 that last previous example we have seen. So, this must be this is the condition to be true.

So, if it is not a one-to-one if it is not a one-to-one then the above condition must be false the, if I give some number that the condition one must be false. So, if I take the negation of that thing, if I take then that negation must be true; negation must be true. If it is not one-to-one, not a one-to-one function, then if I apply the negation rule then I get there exist x 1 and there exist x 2. And, this is negation of f of x 1 and if I remember that negation p to q; that means, negation p or q this is equivalent to p and negation q because negation p or q. So, this is p and negation q. So, directly I can write that, directly I can write that f of x 1 equal to f of x 2 and x 1 negation x 1 x equal to x 2 means x 1 not equal to x 2.

So, this is the condition we have to show or this is the condition that we can if it is true we can show that the function is not one-to-one. So, whenever we if exact and problem is given we have to show that the function is one to one then we have to we have to show equation 1. And if we have to show that function is not one-to-one then we have to show this thing; that means, there exists x 1 x 2 for which f of f of x 1 equal to f of x 2 and x 1 not equal to x 2. So, if we take one example, that one example you see.

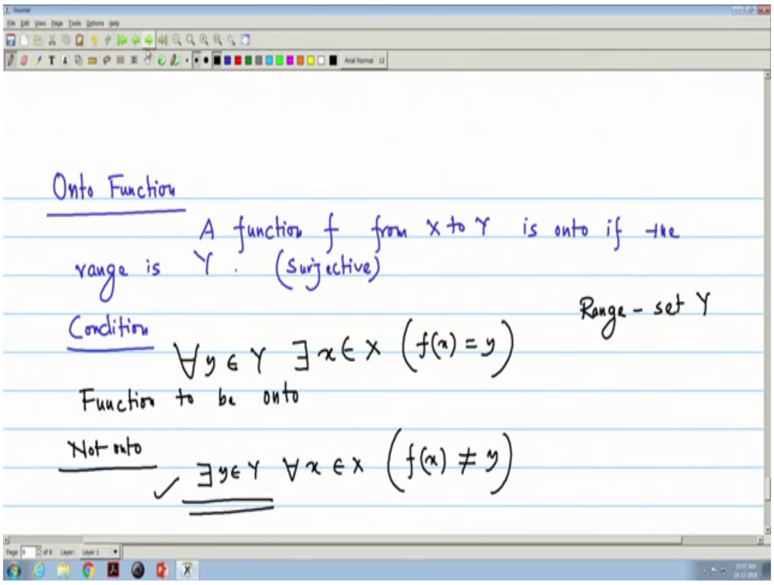
(Refer Slide Time: 29:09)



Say prove that 2 to the power n minus n square. So, if n f of n equal 2 to the power n minus n square is not a one to one. So, not one to one then we have to show that f of x 1, x 2 and x 1 not equal to x 2 there exists some value there exist some x 1 and x 2. So, now if I take that n equal to 2 and n equal to 4 because I have this is x 1, n equal to 4 some I have x 2 say. Now what is f 2? f 2 is 2 square minus 4 is 0; I have f 4 is 2 to the power 4 minus 4 square is 0. So, what we see that f of x 1 equal to f of x 2 and x 1 not equal to x 2; that 2 not equal to 4 so; that means, it is not a not a one to one not a one to one function ok.

Now, we see that our onto function, we define my onto function simply that, a function f from X to Y is onto if the range is Y and it is called the; the onto function is called the surjective, it is called the surjective.

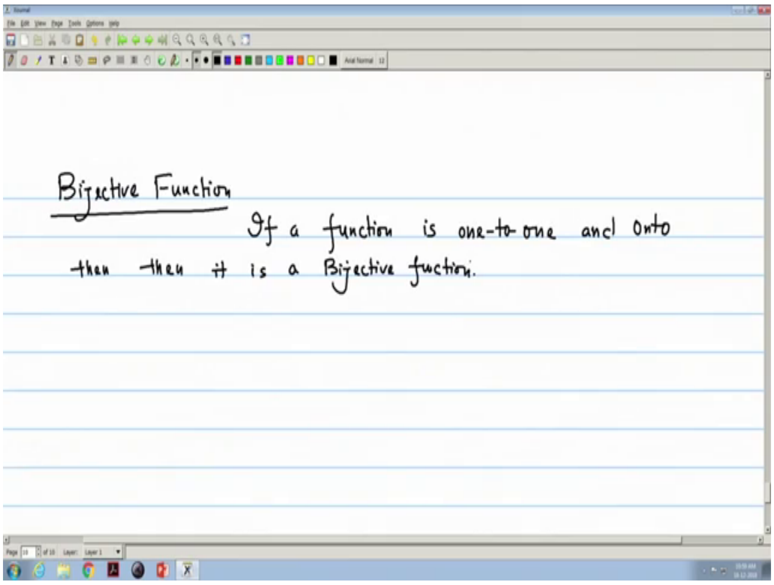
(Refer Slide Time: 31:11)



And what is the condition to be satisfied? Condition is so for all y that y belongs to Y because this is my range there exists x belongs to X, I get that f of x equal to y. And this is the condition to be satisfied that the function to be condition that the function to be onto and if the function is not onto like the previous one we have done that we take the negation and it will be that there exists y then belongs to X for all x belongs to X negation of f x equal to y. So, if x not equal to y.

So, there must be at least one x, at least one y; that means, it is not the range we remember the range is the set of y, range we call the set range is the set y. So, at least one element y exists for which f x not equal to y and then; that means, it is not onto and if I define that our bijective function, our bijective function.

(Refer Slide Time: 33:50)



If a function is one to one as well as onto, one to one and onto and onto then the function is then it is a bijective function. So, we have read the basic concepts of function and the three different type that mainly the injective function, surjective function and the bijective function. And some simple examples that, how or what type of function it is and how to show that if it what are the conditions to be satisfied to show that if it is a bijective or if it is a injective or if it is a surjective. So, bijective the two condition must be added, this the conjunction of the two functions, the two conditions of bijective onto functions and the one to one functions.