Discrete Structures
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Lecture – 02
Introduction to Propositional Logic (Contd.)

Last class, we have read the truth tables of the three operations; the negation, conjunction and disjunction. So, today we will start the class with another two operations which are very important and they are they comes under the conditional proposition.

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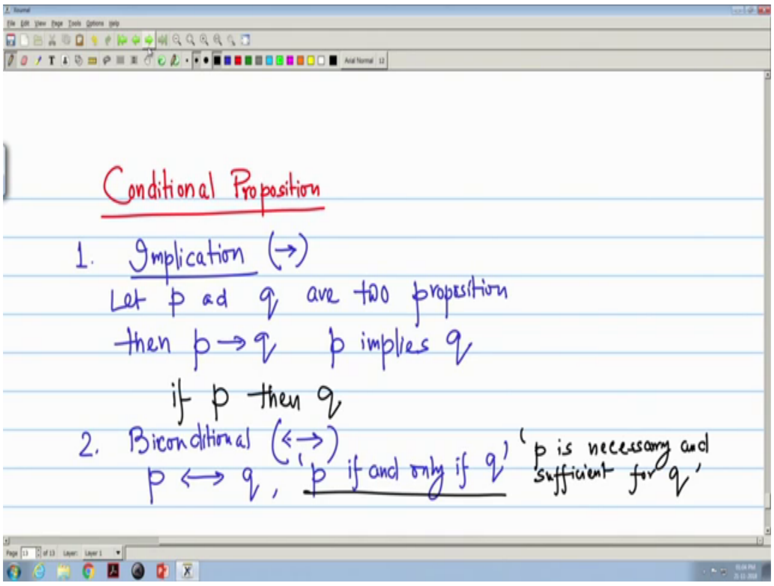

Now, there are two type of conditional proposition, one is implication. How we define? See if we have two propositions p and q let p and q are two proposition then p implies q this is the conditional proposition, we write p implies q. What is the meaning? The meaning is that simple if p then q, meaning is if p then q. So, this is our first type of conditional proposition; that means q depends or the truth value of q depends on p.

The another one is the biconditional. The other conditional proposition is or the operation is biconditional, normally we denote by this. Then if p and q are true proposition then p biconditional q it means that q if an only if p or I can since it is biconditional I can write p if and only if q p and or I can write p is this is same as p is necessary and sufficient for q.

So, these are the two conditional proposition or the two operations that normally it operates on the basic propositions.

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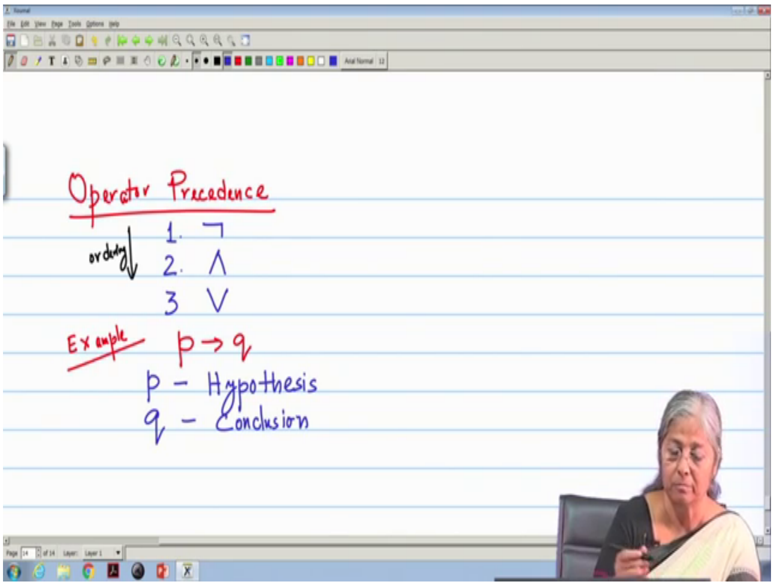

Now, another thing is very important since now we have three basic operations. So, what are the precedence? So, operator precedence; so, when we evaluate or when we will evaluate the truth value of some complex proposition, then the ordering will be the first is negation, second is conjunction third is disjunction. So, this is the ordering precedence operator precedence.

Now, we with since now we have three basic operations and two conditional propositions. So, we see how we can evaluate the compound proposition, ok. We take some example before giving the example another two terms we defined. When whenever we are considering the conditional propositions like p implies q then we tell these p is normally, here p is called the hypothesis and q is called the conclusion q is called the conclusion.

Now what we will see that for when we will evaluate the compound proposition consisting of three basic operations and two conditional operation the implication and the biconditional operator then we will see that for truth values all possible truth values of the proposition that what are the compound proposition takes the value, ok.

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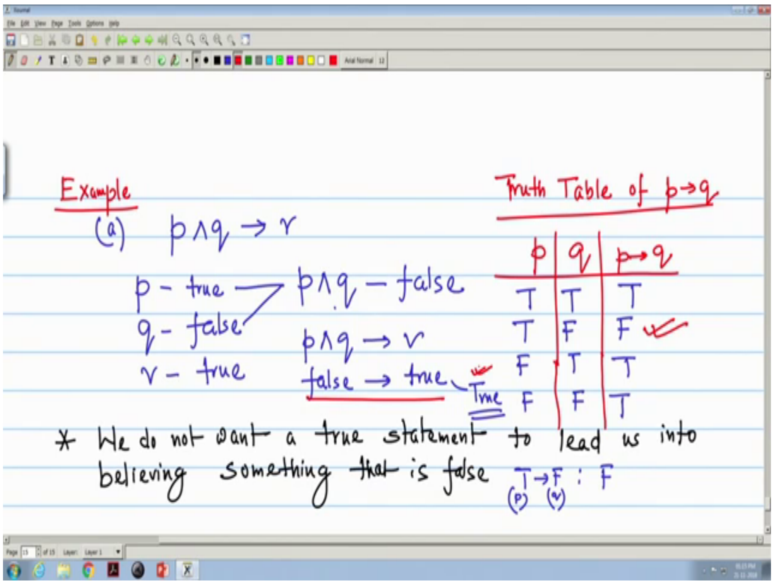

So, we take these example thinks much. One compound proposition we take a p and q implies r. Now, when we try to evaluate, so all possible truth values of p and q we have to take and here r is also there p, q and r. So, I explained first. So, if p let we take the p is true, q is false, r is true. So, if p is true, q is false then what p and q? p and q is false p and q is false because this is true and this is false.

Now, if r is true then p and q implies r; that means, p and q implies r; that means, false implies true. So, we have to know the truth value of implication; that means, if my p and q this becomes false then what will be the resultant proposition? So, what will be the truth table? First we see the, we have to see the truth table of p implies q truth table of p implies q. So, they can take both are true.

Now, if p, q both are true; that means, truth true implies true. So, p implies q is true. Better first I tell the last true condition that if p implies p is F and q is true then F implies true; that means, p implies q is false and true then this becomes true. F implies F, p implies q this is also true. Only T implies F we make it F; that means, accept p is true and q is false all other truth values will give or will result it true value of the proposition.

So, why this is the thing? That means, here the condition is that we do not want to believe that some true value of the proposition implies some false value of the result. So, this is the implication that we do not want this is very important we do not want I write in this way we do not want a true statement to lead us into believing something that is false. So, these gives us that T implies F is F, p is T, q is F and this is F. So, we remember this condition, only this is the condition that if p is true and q is false. So, this is false and this is our truth table of implication.

So, now if we come back to our original example then these p, q this is false and this is false implies true. So, false implies true is this is true, false implies true this is this gives you a true value, this gives us a true value this gives us a true value this is my result. So, how to evaluate this is one example I give that how to evaluate the compound proposition using the three basic operation and the conditional of operator that implication here we have taken implication, ok.

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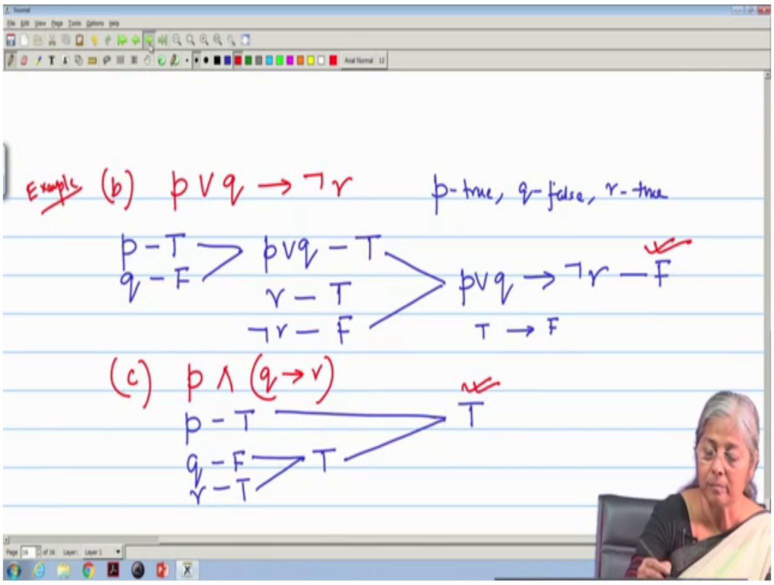

I take another example. I give p or q implies negation r, unless otherwise stated r is always inclusive r or normal not the exclusive one, ok. So, we have values again if we remember I have taken the values p true I have taken p true q false and r true. We try to evaluate with these assignments, ok. So, what is p or q? p true q true q q false p true q false, this gives p or q true p or q true because or anyone. Now, p or q true p or q true and r is true r true, so, negation r is false. So, if we take p or q implies negation r.

Since now we know the truth table of implication. So, p or q is true, so; that means, true implies false which is the only one false condition of the implication, just now we have seen. So, to these will give you the; these will give you the false because that true implies false we do not believe in this philosophy. So, this is the result which is false.

I take another example, I take another example say p and q implies r; p is true, q implies r. Since our q is false, r is true. So, first we see what it gets false implies true this is true. Now true and true, so, this is true. So, this is true result.

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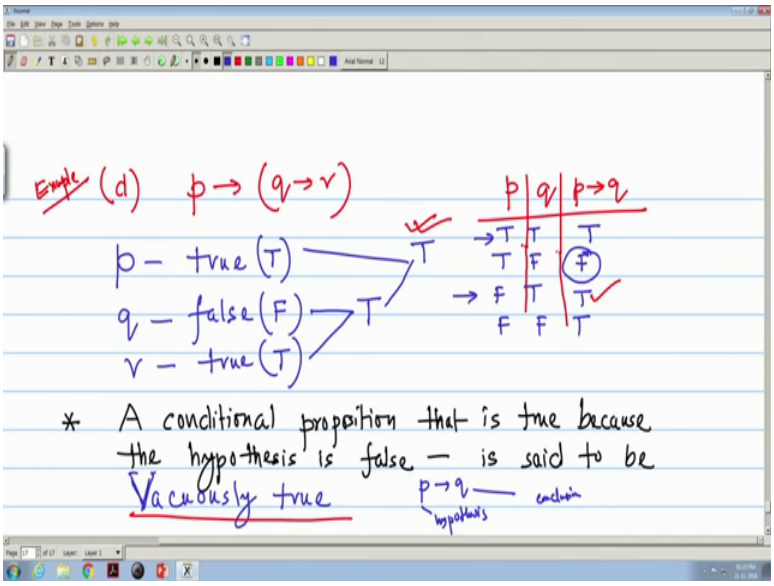

We take another example d I take p implies q implies r, if we remember again quickly I write the truth table of p implies q. If it is all values we have to take both are true, true, true-false only it will give you the false value F T this is T F F this is also T remember. Now, we see that our T is true, now I can write T; then q is false, r is true the example we have taken. So, false implies true, see false implies true, this is true. So, this is true. Now, T implies T; T implies T is this, so, this is true. So, this is my result.

So, in this way we evaluate the truth values of the compound proposition. Another condition we remember this is though when we will be evaluating for different truth values or different assignments that a conditional proposition that is true because the hypothesis is false and these condition is said to be vacuously true. This is said to be is vacuously true. This is our this third situation that p is hypothesis our p is p implies q, p is hypothesis and q is conclusion. Now, if hypothesis is false then the condition is not validated. So, the result is the p implies q is always true. This is sometimes we called by default this is the default situation or it is called the vacuously true, this is vacuously true.

So, what here we see that how actually we can evaluate, how we can evaluate with the all the compound proposition with the three operation and the conditional operation basic operation conditional.

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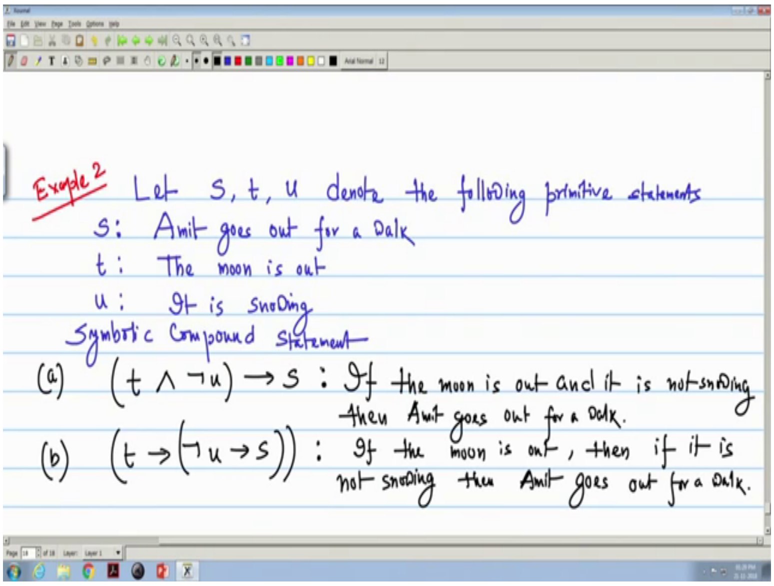

Now, we take another example where that we can tell if the symbolically we can give the proposition then what the meaning of that logic or the meaning of that compound proposition because. These one as well as the reverse one is very important; that means, giving one statement or one sentence written in English how we can denote logically or we can denote with the symbols; that means, the proposition.

So, now we take one example. I write the example that let s, t, u denote the following primitive statements. Now, what are the statements? I write s that Amit goes out for a walk, t denotes the sentence the moon is out, u denote it is snowing. Now, we see the symbolic compound statements using s, t, u what is the meaning of that compound proposition.

So, I write a symbolic compound statements give t and negation u implies s. So, t is the moon is out and negation u is it is not snowing. u is it is snowing. So, it is not snowing and that implies Amit it goes out for a walk. So, I can write that if the moon is out and it is not snowing then Amit goes out for a walk. I take another compound proposition say t implies negation u implies s if I guide this thing t implies negation u implies s. So, I can write negation u means the, it is not snowing. So, if the moon is out then t implies negation u implies this, then if it is not snowing, then Amit goes out for a walk.

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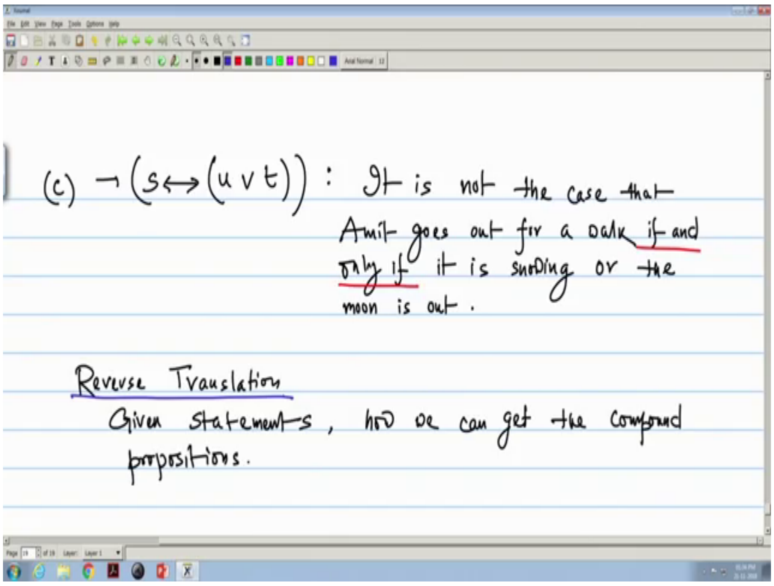

We see the third one, say negation s this one bidirectional condition conditional proposition we see that and u or t. So, since it is negation so, first I write it is not the case. This is one type of that writing the when we translate from symbolical logic to the our statements, since it is negation I write it is not the case and then symbol wise we start writing. So, it is not the case that s means the Amit goes out for a walk; Amit goes out for a walk. Now, what is the physical meaning of biconditional proposition? It is if and only if or necessary and sufficient conditions, for a walk; that means, if and only if u and t; that means, it is snowing or sorry it is or it is snowing or the moon is out or the moon is out.

So, again I repeat for this particular symbolical proposition what we have done? Since it is negation first we have written that it is not the case. Then s is Amit goes for a walk then it is biconditional so, I give if and only if it is u or t u is it is snowing or the moon is out. So, we have we can just translate from symbolic logic to our statements in English. And reverse also many times reverse is also necessary that reverse translation; that means, given statements given statements how we can get the compound proposition, how we can get the compound proposition.

Actually in real life these reverse translation is very much required because whenever we will read the properties of propositions of the different operations then how logically we can evaluate the truth values because finally, we have to find out whether our logic is correct or not, true or false.

So, with this fundamentals of logic are the basics of the logic with the operations the primitive propositions and the operations negation AND and OR and that two conditional propositions, the implication and the biconditional we have read how we can evaluate one composite proposition and we have also learned that given one symbolic notation of proposition how we can write the statements in English. Reverse is also very much necessary that we will read in the next class.