Discrete Structures

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Lecture - 19
Sets and Functions (Contd.)

So, last lecture we have read the some basic properties of set and mainly the operations on set. Today you will see some very useful properties on sets using the binary and the unary operations on the sets and mainly these are the properties that are very useful to solve the problems using sets. So, first we list those properties and we can prove later the all the all these properties. So, first thing is that some useful properties of sets.

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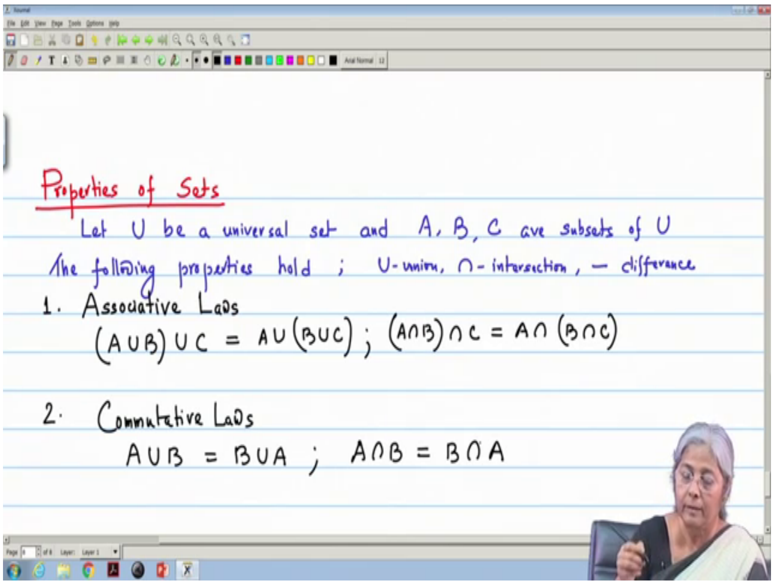

So, we consider three sets and a universal set U. So, let U be an a universal set and A, B, C are three subsets of U. So, the following properties hold on A, B, C and U and all the operations that we read on set and any of these property actually we can write as a theorem since, we can prove these properties also. So, the first property give the serial number 1 and this is something called the, we call these properties as the laws since we can prove this property. So, this is first is associative laws of set.

So, we have operations on, here we have we consider these three sets and the operations are union, intersection and set difference, we use this thing. So, associative laws we can write if we take union of two A union B union C this is same as that of A if I take A union

B union C and for union operation whatever law is valid, that is true for intersection also. So, if we write if we just replace union by intersection we will get A intersection B intersection C that is same as A intersection B intersection C.

Now, we see the commutative laws, if I consider only two sets here. So, A union B is B union A and similarly I can tell A intersection B is B intersection A.

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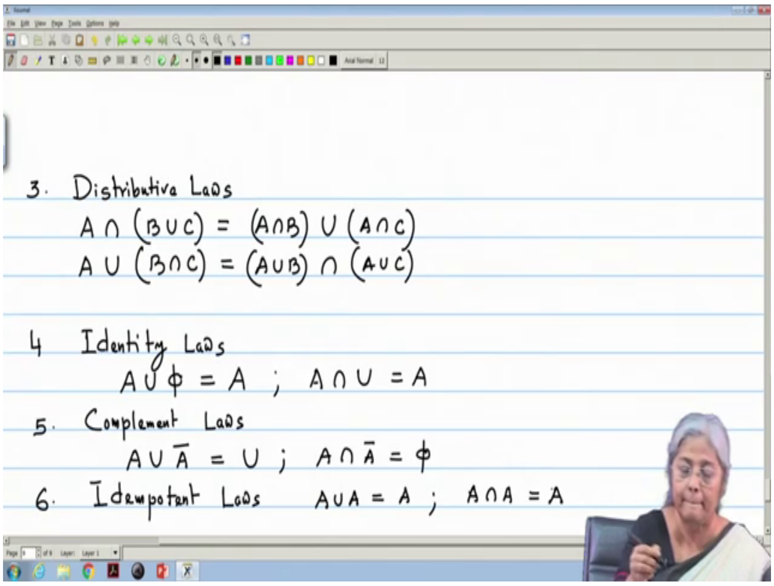

So, if we continue the third rule there is the important distributive laws on sets, we write that A intersection B union C is A intersection B union A intersection C. If we exchange the operations that intersection and union that is same A union B intersection A union C, later we can prove this distributive laws.

Now, identity laws so, if I take the union of a set A with my null set then we will be getting A only because null set means these having no elements and union is that that elements which are either in a or in the null set. So, it is actually thus we can tell this is addition. Similarly if I take A intersection with my universal set then again this is the set itself so this is as if the identity. Now I have the complement laws. So, A union A complement is the universe only, because if I take the subsets of A and A complement; that means, all the elements of universal we are (Refer Time: 07:46). Similarly if I take A and intersection A bar then this is actually a null set, because these are A and A complement are the disjoint sets having no common elements they are disjoint.

Then idempotent law; so, if I take A union A which is A itself, similarly if I take A intersection A this is A itself.

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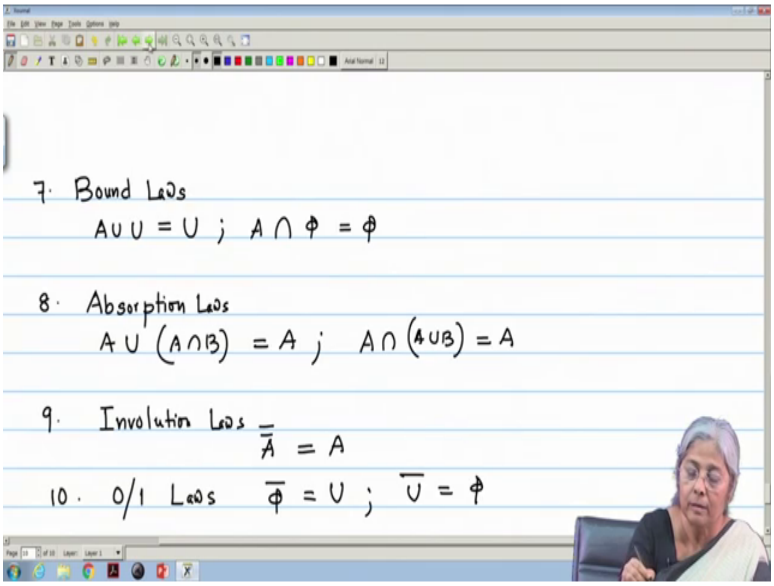

7th law I can tell it gives some thing on the bound. So, this is the bound laws either lower bound or the upper bound. So, A union with universal set is the upper bound that the universe only. Similarly if I take the A intersection with the null set which is the lower bound is the null set. Now, I have absorption this is very much important when we try to simplify some set operations.

So, absorption is A union A intersection B is A only, similarly if I exchange intersection and union I will be getting A only. Now, involution if we take the double negation of to a complement of set; that means, A complement of complement that is the set itself then our something called the 0 1 laws. That means, if I take the compliment of my null set that is nothing, but my universal set and if I take the compliment of universal set which is my empty set or the null set.

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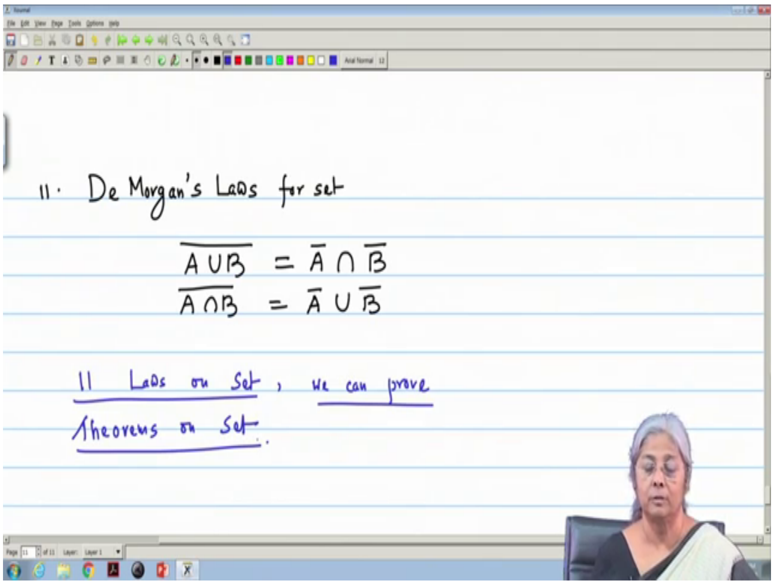

I have another laws left which is the our well known De Morgan's law on set De Morgan's laws for set and the operations are union and intersection and the complement. So, we can write that if I take A union B complement of that I will get the A complement union is replaced by intersection and B complement. Similarly if it is A intersection B complement this will be A complement union B complement and we can prove.

So, these 11 laws; the these 11 laws on set though we are telling these are the property actually any one of this law we can prove this law on set that we can prove logically. So, they are sometimes we call these are also the theorems on set. Now, we see that how to prove some property, first we consider on say commutative law; the very simple one first is consider.

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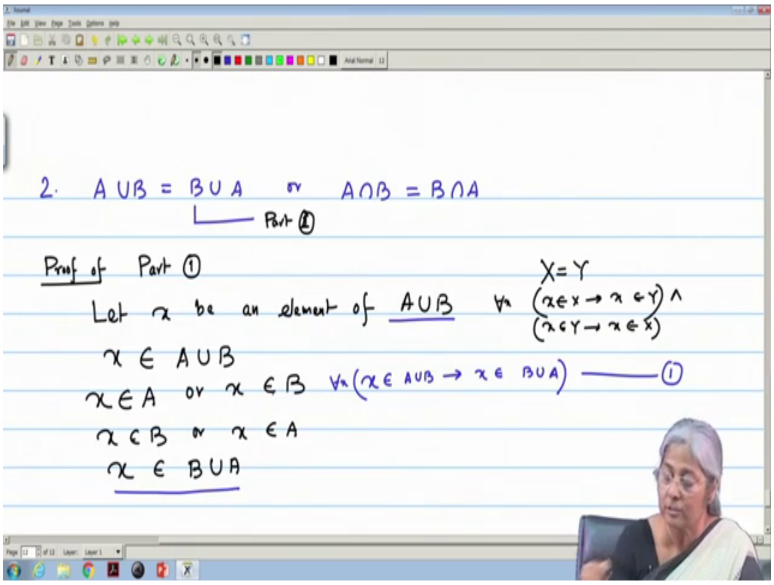

So, our this is my law 2 that we told that A union B is B union A or A intersection B is the same as that of B intersection A. So, this is one set say first we the first part we take this one and we prove equal this is part A; we call this is my part A and we prove part 1 and we prove that thing. So, proof of part 1 and similarly we can prove part 2. How we can prove? So, see these are A union B, if this is one set similarly B union A is a set. So, let x be an element of A union B. So, how we can show that two sets are same? If we have to show that X equal to Y then we have two that if x belongs to X implies x belongs to Y and x belongs to Y implies x belongs to X for all x for, all x we have to prove this thing.

So, first we say let x be an element of A union B. So, since x belongs to A union B since it is a union; that means, x is either x belongs to A or x belongs to B because either A or B or both. So, I can write that x belongs to B or x belongs to A; that means, x belongs to same as that of x belongs to B union A. That means, if I consider that if x B and element of A union B I have considered then I have shown that x belongs to B union A also so; that means, x belongs to A union B implies x belongs to B union A. Now we have to show this is for all x this is for all x. So, this is one we have shown now we have to show that if I consider x belongs to.

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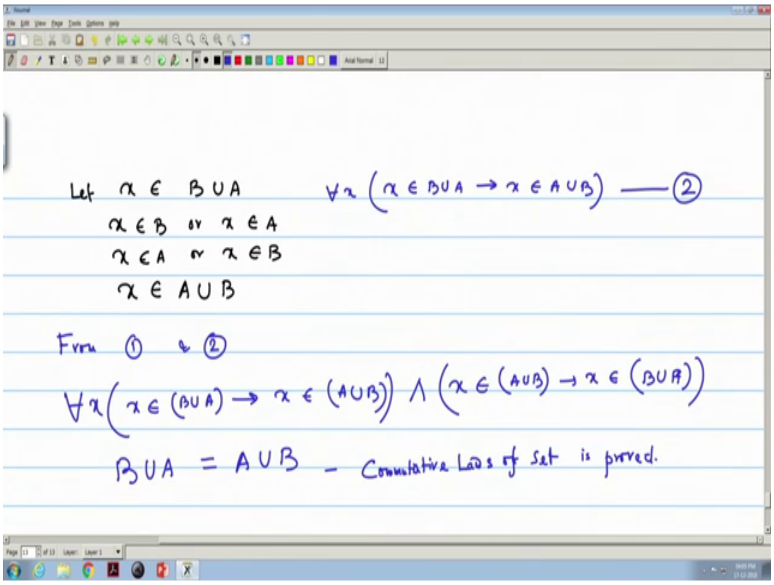

Now, the for the reverse one that let x belongs to B union A; that means, again I can tell that x belongs to B or x belongs to A. So, I can again I can write x belongs to A or x belongs to B. So, I can write x belongs to A or B. So, what I can write that for all x for all x that x belongs to B union A implies x belongs to A union B. So, this is my 2.

So, for all x we can show the first one and from 1 and 2. So, you can tell from 1 and 2 that for all x that x belongs to B union A implies x belongs to this is two and x belongs to A union B implies x belongs to B union A. So, I can tell that B union A equal to A union B. similarly I can take I can prove that the intersection also. So, my commutatively or commutative law of set laws on set is proved. So, this is this is trivia we proved the distributive law ok.

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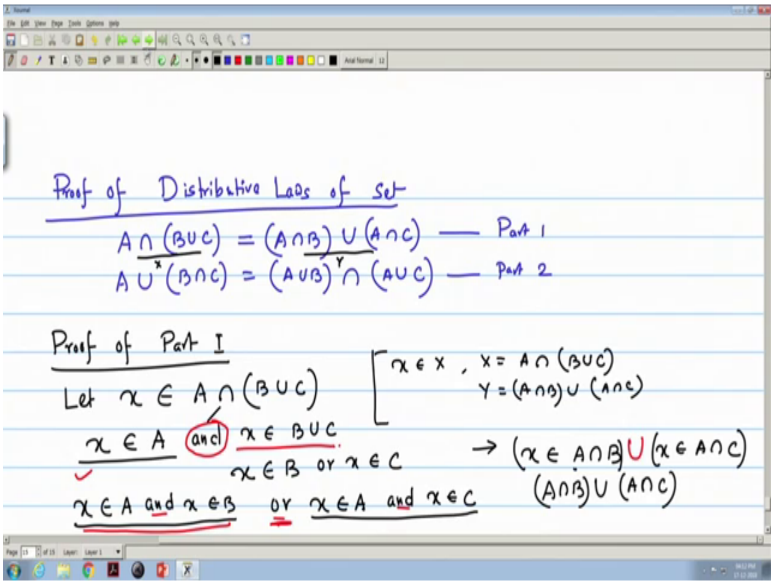

We proof of distributive laws of set ok. Now, we write one part one of the distributive laws that A union or x A intersection B union C equal to A intersection B union A intersection C or I can write A union B intersection C is A union B intersection A union C again similarly I can tell this is my part 1 and this is my part 2. So, it prove the part 1. So, take the proof of part 1, again the same logic say I can tell my LHS this is my x this is my x this is my y. So, X equal to Y we have to the logic is that for all x we have to show that x belongs to X implies x belongs to Y and x belongs to Y implies x belongs to X.

So, let x belongs to capital X means A intersection B union C I have assumed that my; that means, I can tell that what I have assumed x belongs to X here X is A intersection B union C and Y is that A union B A intersection B union A intersection C ok. So, what does it mean physically? That if x belongs to A intersection from the definition of intersection and union since it is the intersection; that means. So, x belongs to A and because it is a intersection so this is this is and x belongs to B union C ok. So, x belongs to A this is constant and so it is there I take this is always there.

Now from this x belongs to B union C what we can write? Now these it means that x belongs to B or x belongs to C. So, now, if we consider; that means, I have this and that means if I consider now I can write that x belongs to A and x belongs to B or since x belongs to A and so x belongs to A or this is and x belongs to C.

So, this is one part this is one part I can write now this is and again this is this is and this is and this is or. So, I can write that x belongs to A and x belongs to B the first I can write this is equal to x belongs to A intersection B since it is and x belongs to A intersection C and this or this or I can replace by union, this should be a union. So, now, what I get x belongs to A union B A intersection B or; that means, union x belongs to C. So, how I can write this thing how if I now just right x belongs to A union B; that means, a intersection B union A intersection C. So, I started that x belongs to we have assumed let x belongs to A intersection B union C and which implies that x belongs to which implies that x belongs to A union intersection B union x intersection C.

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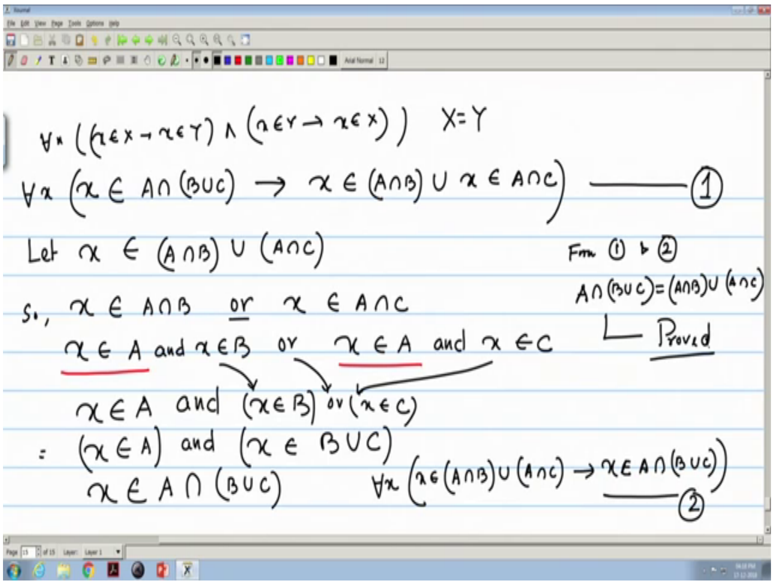

So, the first thing what we have assumed that; that means, that x belongs to A intersection B union C implies x belongs to A intersection B union x belongs to A intersection C; that means, these and this is for all x for all x. So, this is we get for all x can tell for all x I get this is one implication 1. Now if I consider let x belongs to A intersection B union A intersection C, since from the definition of union then I would write; that means. So, the x belongs to a intersection B or x belongs to A intersection C. So, x belongs to A intersection B means x belongs to A and x belongs to B and this or again I can write or x belongs to A and x belongs to C. So, again here we see that x belongs to a in both the cases that x must be A.

So, I write this x belongs to A both the cases it is there and if I consider in this way and this is here this x belongs to B or x belongs to C. So, same as that I can write x belongs to A and together I can write this x belongs to B since it is odd so it is union C. So, x belongs to B union C and x belongs to A. So, this is same as x belongs to A and means union and this intersection and is intersection and this is B union C. So, what we have assumed? We assume that x belongs to this set and we infer that they or it implies that this. So, I can write now that. So, we can get that for all x for all x x belongs to A union B union A intersection C implies x belongs to A B union C this is my 2.

So, since for all x, x implies x belongs to the A union B union C remember that x belongs to capital x implies X belongs to capital Y and x belongs to capital Y belong implies x belongs to capital X for all x and then this is nothing, but my X equal to Y. So, here from 1 and 2, from 1 and 2 we can infer that my capital X is A intersection B union C and capital Y is A intersection B union A intersection C. So, it is proved. So, my part one of my distributive laws of set is proved and similarly we can prove the part two also. So, in this way any one of the 11 laws that we have enlisted that we can prove.

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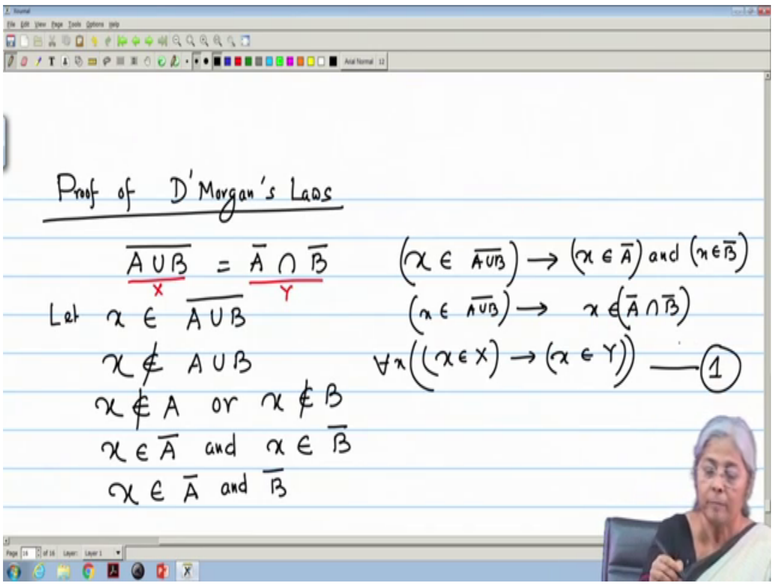

Quickly we can see the proof of our De Morgan's law proof of one De Morgan's law. If we take the De Morgan's law one part that this is this is a law set. So, again I can similar way I can take that let x belongs to this is my capital X say this is my X and this is my Y. So, the similar way I can tell that this is A union B complement. So, if x belongs to A union B; that means, x does not belongs to A union B because if it exists in the complement part if I remember the Venn diagram also. So, this is these x must not be in that a union b. So, if I write; that means, x does not belongs to A since it is union. So, or x does not belongs to B; that means, x must not be in A or in B or anywhere in A or B.

So; that means, x belongs to A complement, since we are in the universal set U similarly I can take x belongs to B complement. So, what it means? That means, x belongs to A complement and here it will be and B complement A complement and B complement because x x must not be in A x must not be in B; that means, x must be in A complement and must be B complement. So, we can tell that when we have taken that x belongs to A union B complement that implies x must be A bar and x come must be in the B bar complement; that means, x belongs to A union B implies x belongs to A bar intersection B bar or I can other way I can x belongs to capital X because this is my capital X that implies belongs to capital Y and this is for all x we have done.

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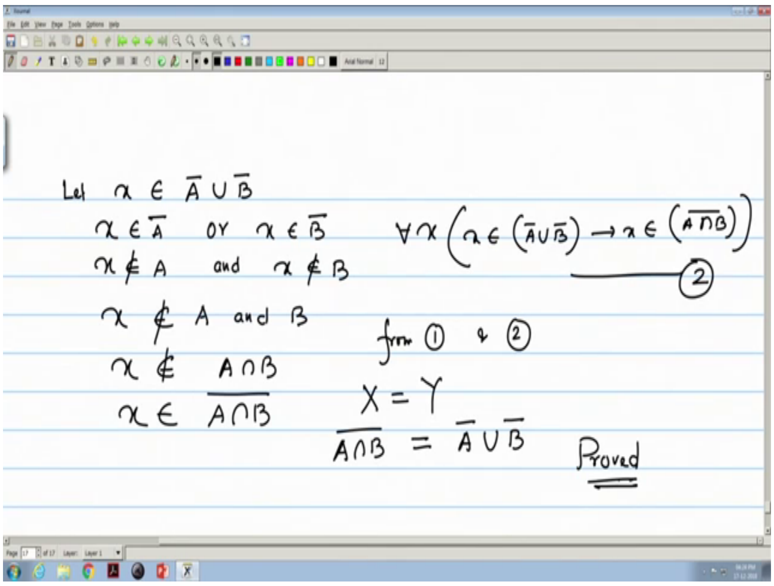

So, this is one similarly the same way if I consider. Let x belongs to a complement intersection B complement since we have taken the union. So, this means x belongs to A bar or x belongs to B bar then x does not belongs to A and x does not belongs to B. So, x belongs to either a bar or B bar; that means, x does not belong to A and x does not belongs to B; that means, x does not belong to A and B; that means, x does not belong to

A intersection B. So, x belongs to A intersection B complement. So, x for all x what we get? That x belongs to A bar B bar implies x belongs to A intersection B complement.

So, this is my two. So, again from one and two we can infer that X equal to X equal to Y; that means, X is my A union A intersection B union inverse is the and that is A bar complement B bar and this is the one part of De Morgan's is proved. So, in this way that any one laws that we can prove using the elements of set whether they are they exist in some set or whether they do not exists in the complement in this way mainly from the concept of the set the complement of a set and the three operations that we have read on the set mainly the intersections, the union and the complement of the set.

So, we have read the fundamentals of set the operations the basic definitions and now we can proceed for that functions and relations using this set where, these basic concepts of sets are very much required. And, as we have started with the discussion that this is the very basic concept the set is the very basic concept of any mathematical streams mathematics computer science or any discipline we required this concept.