Discrete Structures.
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Lecture – 18
Sets and Functions (Contd.)

We are discussing about the set operations, we have read the set union, set intersection, set difference and now we read another operation, which is very important is called the Cartesian product of set.

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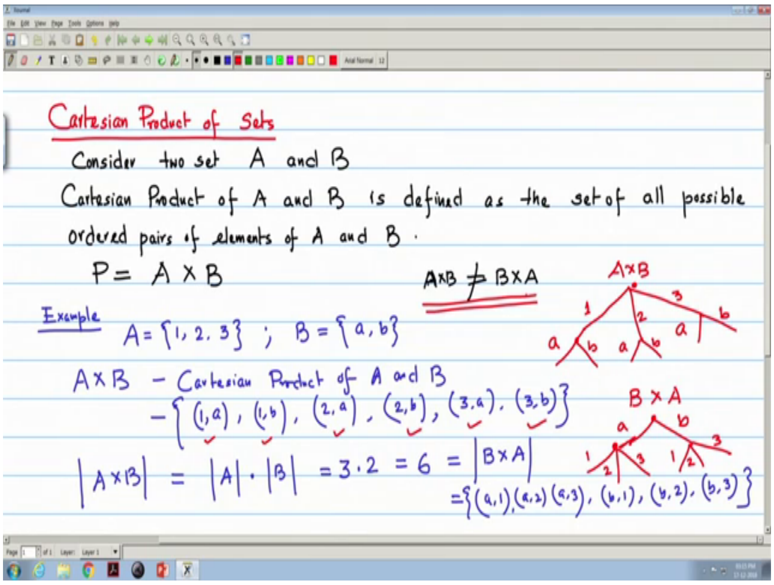

We have defined a set as a unordered set of elements, unordered collection of elements. So, if we consider two sets say A and B; consider two set A and B having some elements. Now, Cartesian product of A and B are the set of all possible ordered pairs of elements of A and B is defined as the set of all possible ordered pairs set is unordered, but Cartesian product is ordered pairs of elements of A and B.

Normally, we denote the product as the A cross B. So, we denote the product say P equal to A cross B, take one example. Let the two sets we define like A is 1, 2, 3 3 elements and B having 2 elements say a and b. So, A cross B is the Cartesian product; is the Cartesian product of A and B and these are set of all possible ordered pairs. So, since A comes first so, it is the ordered pair. So, set of 1 a, 1 b, then 2 a, 2 b. This things the simple pair we take 1 a, 1 b, 2 a, 2 b then 3 a, 3 b and we can if I just give a tree type of structure say give 3 type of structure, say A cross B. So, A has 3 elements, say 1, 2, 3 and B has 2 elements a and b.

So, if we see as if this is my A cross B. So, we get 1 a, 1 b this is 1 a, 1 b, 2 a, 2 b, 2 a, 2 b then 3 a, 3 a, 3 b. So, we get all possible pairs. So, what is the cardinality of this set? So, cardinality of this set A cross B is cardinality of A into cardinality of B. So, in this case cardinality of A is 3 and cardinality of B is 2 so, this is 6. So, this is equal to cardinality of B cross A, what is B cross A? So, if we see that B cross A then B has 2 branches 2 elements. So, 2 branch a and b and a has 3 elements, 1, 2, 3. Since, now the ordering is changed so, B cross A is a 1, if I take B cross A is a 1, a 2, a 3 then b 1, b 2, b 3, b 1, b 2, b 3.

So, clearly the cardinality of A cross B is cardinality of B cross A, but see the A cross B is this set 1 a, 1 b, 2 a, 2 b, 3 a, 3 b and B cross A is the set a 1, a 2, a 3, b 1, b 2, b 3. So, the A cross B is not equal to B cross A, because the two sets are different since, it is considering a order. So, here elements of A are coming first and in the pair of B cross A elements of B are coming first. So, that we can write that A cross B clearly my A cross B that Cartesian product of A cross B is not equal to B cross A, since it is an ordered pair ordering is ordering matters here. So, what is our A cross A or B cross B?

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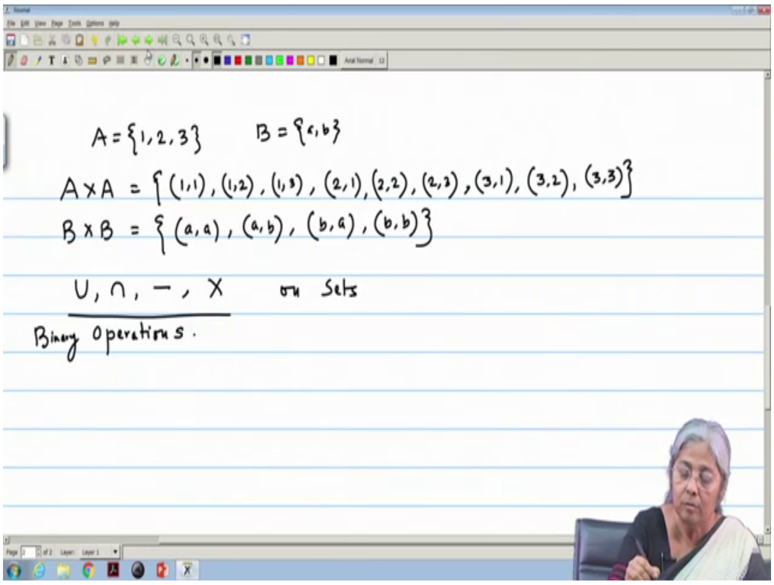

So, the if we consider the Cartesian product of the set itself A cross A, A is the set 1, 2, 3 the same example if we consider. So, similarly we can get this is the pair 1 1, 12, 13 then 2 1, 22, 23, 31, 32 and 33, these are the all possible pairs of is similarly B cross B the Cartesian product of B only this is B is a b. So, this is a a, a b, b a and b b. So, again this is one binary operation like (Refer Time: 10:40) set union set intersection set difference.

So, we have read that 4 binary operations, union intersection, difference and the Cartesian product on sets these are the 4 operations binary operations. Now, we read another very important thing that a different type of representation or some pictorial view of sets, which we can see as a tool, which is very much used or to do some operations on set.

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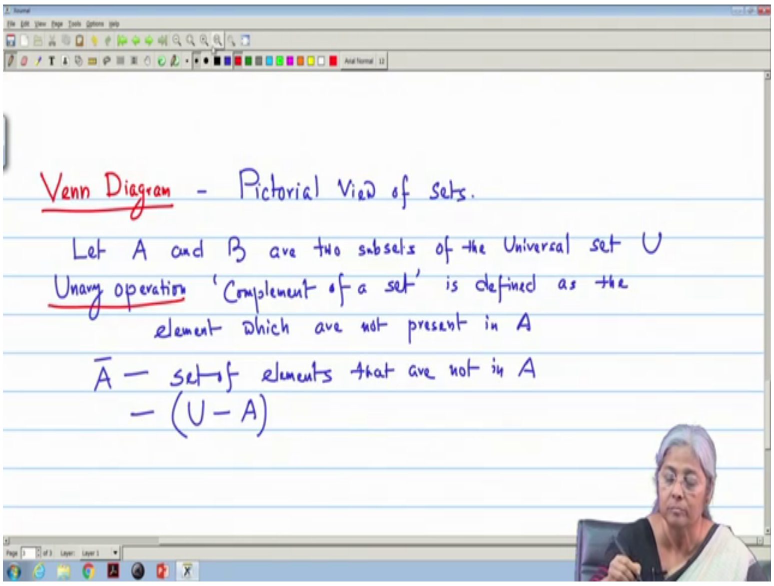

So, it is called the Venn diagram, we use the Venn diagram so, we define Venn diagram. As if this is a pictorial view of sets, it is an end or tool to work on the on sets and the different operation subset ok. So, let AB are two subsets of the universal set U. Now, we have read binary operations, one unary operation, we call the compliment of a set is defined as the number of elements is the elements sorry not number, this is the that elements defined as the elements, which are not present in A if we consider the complement of A; that means, if I normally we denote this thing as a A complement. So, A complement is the set of elements that are not in A is the set of elements that are not in A.

So; that means, if I consider that universal set U and A is the subset of U. So, I can tell that this is U minus A; that means, U minus A contains those elements which are not in A. So, we define this unary operation this is one unary operation that complement. Now, how we represent the all these sets using Venn diagram?

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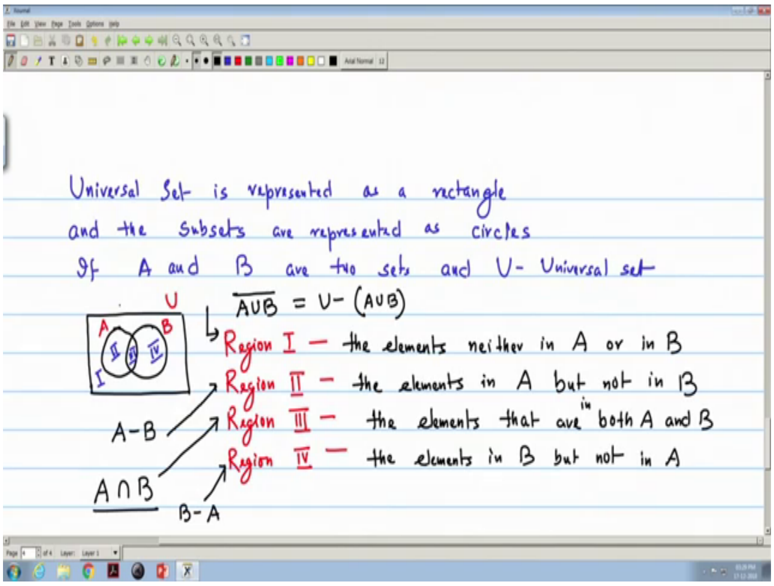

Say we universal set, since it is a pictorial view. So, universal set is represented as a rectangle and the subsets or subsets of universal set or normally, we call the sets are represented as circles as circles. So, if A and B are 2 sets and U is the universal set then we can represent this thing as say this is my universal set U and this is A say this is B. So, if I give the notation, this is U say these circle a first circle that is say A second circle, this is B these are the sets. Now, I give some region as if this is 1 region, 1, 2, 3 and region 4. So, what do we mean by this region? So, region 1 contains the elements I give all the region what they represents. So, region 1 contains the elements that are in the universal set U, but not in A or not in B. So, these are the set of elements. So, region 1 covers the elements neither in a in set A or in B.

Region 2, region 2 is the elements in A only, but not in B in A, but not in B, region 3 that elements that are both; that are in both in A and B that are in both A and B and region 4 like the elements in B, but not in A. So, normally we represent in this way that using the rectangles and using the circles and with these notations that the what are the elements we cover. So, see what do you mean by region 2? Region 2 is both are in the elements that are in both or region 3, the elements that are in both A and B, region 3 in both A and B. So, we know that both A and B; that means, A intersection B.

So, region 2 is actually intersection of A and B. So, elements in A, but not in B region 2, these we can simply write A minus B set difference similarly, region 4 elements in B, but not in A. So, I can tell B minus A, this is the elements neither in A, neither in A or in B. So, this is if I consider this is A union B. So, if I consider this is A union B; that means, they are either in A or B or both. So, neither; that means, it is the complement; that means, this is U minus the A union B. So, the pictorial representation, we how we define the pictorial representation though different regions actually represents the different sets or the different operations on the set like these A union B, A intersection B set difference of A minus B or B minus A or this is the complement of the union of 2 sets. So, normally with this picture we can use in this way. So, if I just see that more on Venn diagram that examples.

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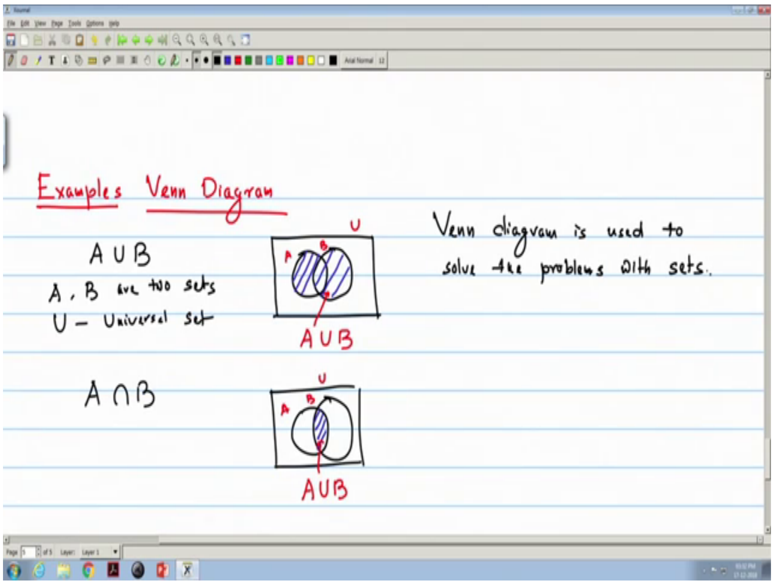

So, if I consider simply A union B; simply A union B as you usual or U is the universal set we are two sets and U is the universal set. So, we can write this is my universal set this A this is B. So, A union B; A union B is this thing. So, this is our A union B all right this is A, this is B and this is my U so, the shaded portion is A union B. Similarly, the A intersection c, A intersection B since, both in A and B. So, this is my this is my intersection B.

Now, this Venn diagram is used to solve the problems dealing with sets, the very easy way we can solve the problems using Venn diagrams. So, Venn diagram is used to solve the problems with sets of that deals with sets and we see 1 example and then it will be it will be some 1 class of problems that normally very easy way we can solve using Venn diagram. So, we take 1 example.

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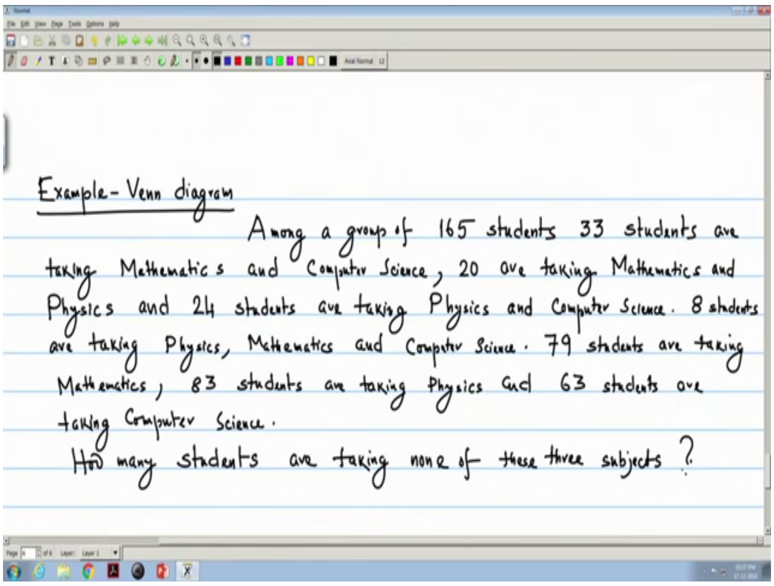

We write the problem say among a group of 165 students, 33 students are taking mathematics and computer science, 20 students are taking mathematics and physics and

24 students are taking physics and computer science. 8 students are taking all 3 subjects physics mathematics and computer science. Then 79 students are taking only mathematics are taking mathematics, if I consider separately 83 students are taking physics and 63 students are taking computers science. The question is that how many students are taking none of the subjects, none of these 3 subjects among these 165 students?

So, this is clearly a problem of set union intersection set difference using mainly these 3 operations. Since already, we have seen that how using Venn diagram, we can represent the set difference union intersection. So, we try to solve using Venn diagram and the that is a very easy way that we can solve this problem.

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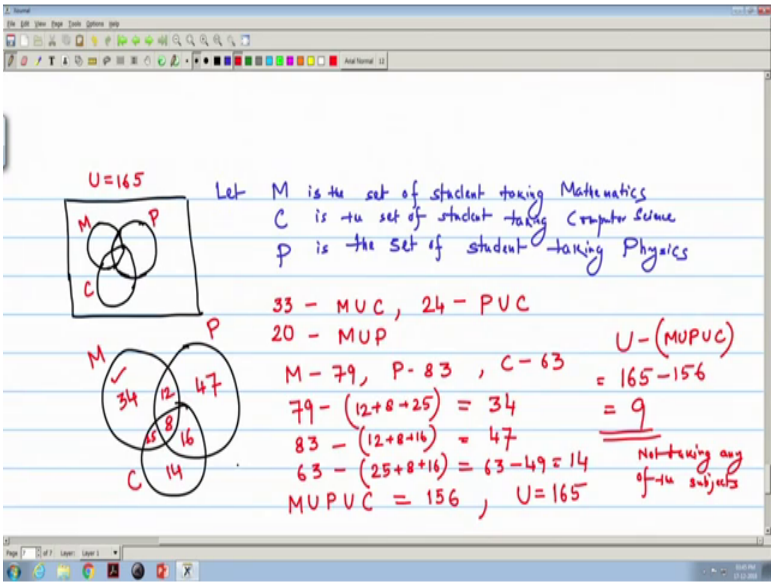

So for this problem, if I draw the Venn diagram first. For this problem, we draw the Venn diagram. So, this is my universal set that U is 165 students. Now here, I have 3 sets, one is my mathematics, one is my physics and physics is 83 students. So, slightly bigger set another is that of my computer science, this is my computer science. So, this is my I represent M the mathematics the set of students, who are taking mathematics, set of students who are taking physics, set of students who are taking computer science by C.

So, let M is the set to students taking mathematics, C is the set of students taking computer science, P is the student set of students taking physics. Now, we have seen that 8 students take the all 3 so, if I just draw this thing. So, this should be 8 so here, it will be a 8. Now, if we consider the 20 students are 33 students are taking; 33 students are taking maths and mathematics and computer science. So, this is my mathematics, computer science and physics.

So, we take that this is my 25 plus 8 together they are taking similarly, my 24 students are taking 25 24 students are taking physics and computer science. So, this will be my 16 and similarly 20, 20 students are taking maths and physics. So, this will be 12 now, I have mathematics students are 79, physics students are 83 and computer science students are 63. So, clearly that I can write these only mathematics students are who are taking only mathematics student here that will be 79 minus this 3, we have to cut; that means, 12 plus 8 plus 25 difference, we take and this will be clearly 34. So, I can write here it is 34. Similarly, if I take only physics student and only computer science student, we can write 12 plus 8 plus 16 is equal to 47 students. So, only physics students are taking 47 and here I can take 25 plus 8 plus 16 is 63 minus 49 is 14 students are taking only computer science.

Now, we get that all union of the set of mathematics physics and computer science students that are together, if I take it will be 156 students and since our universal set; that means, all students will be 165 students. So, the students who are not taking any of the subjects that is clearly that U minus that union of this; that means, 165 minus 156 and this is equal to the only 9 students. So, we get this is my result that 9 students are not taking none of the subjects, not taking any of the subjects. So, this is a very common tool of solving problems using set using the Venn diagram mainly the set operations.