Discrete Structures
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Lecture - 16
Sets and Functions

Today, we will reach the Sets and Functions.

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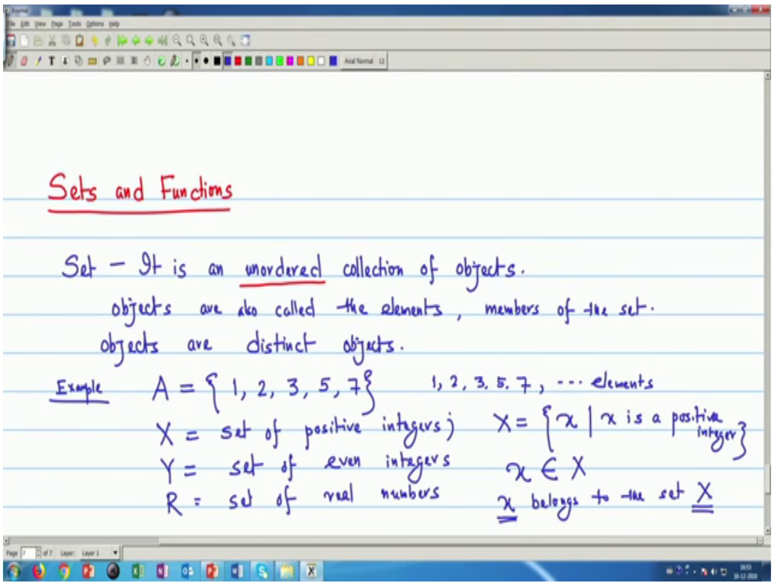

This is if very basic concepts in mathematics and almost in all streams of engineering and science, we require the knowledge, the concepts of this two. So, first we read the set.

We define set, very simple way if we define the set is an, it is an unordered collection of objects. These objects are normally sometimes we call the elements or members, called the elements or sometimes we call they are the members of the set. Another property of set is that unordered collection that are normally the elements we are considered these are or these objects are, objects are distinct objects. And as already mentioned in the definition or unordered; that means, in which order it is coming that is set is in the definition of set we do not concerned or set is not defined. So, that is why in the definition itself we mentioned that is unordered collection of objects.

Like we tell the set of even integers or even before that I can write normally we write the notation is say A is a defined as the set say it is 1, 2, 3, 5, 7. So, this is the notation normally we use to define a set. So, here these 1, 2, 3, 5, 7, these are the elements or objects of the set, these are the elements of the set and like I can tell that X is set of positive integers or Y is set of even integers. Similarly, I can define that R is set of real numbers and if these if I denote that or I can these thing also I can write say set of positive integers I can write X is x such that x is a positive integer.

So, here both the definition are same. Here, if I write x is set of positive integers same as I also can write I also this thing also I can write that x is small x, x is a positive integer; that means, all x which is a positive or x which is a positive integer that must be one element of this set. Normally, we denote that the x belongs to X, this is the notation we use that x belongs to the set X.

So, normally this is the notation we use the small letter to denote the element of a set and the capital letter which denotes the set itself.

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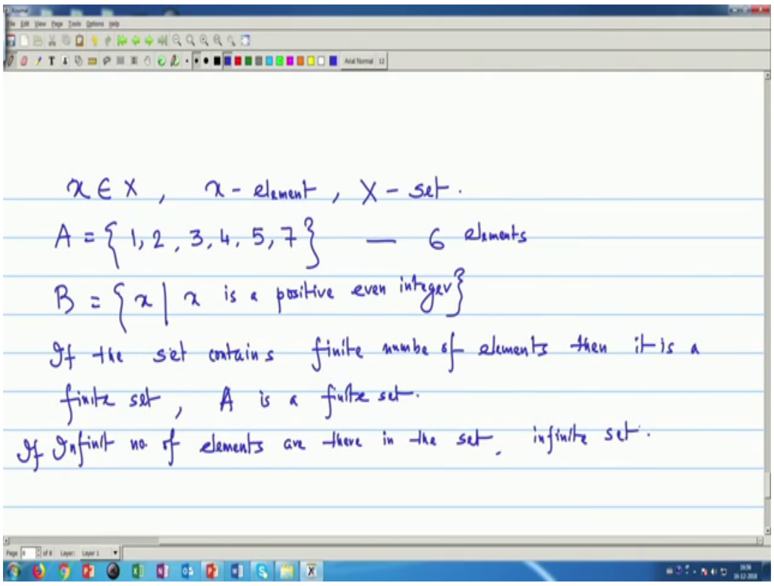

So, normally we write that x belongs to X, here x is the element and capital X is the set. Now, just now I give example that one set A which has only say 5 elements 1, 2, 3, 4, 5, 7, say 6 elements ok. There are 6 elements, on there can be 60 elements and I can take that x such that x is a positive even integer. So, here the number of elements how many elements are there only 6. So, if the set contains finite number of elements, then it is a finite set.

So, in this example A is a finite set. Since it contains only six elements, but set B, B we have defined B contains all the positive even integers and how many positive even integer we have, this infinite number of positive even integer 2, 4, 6, 8 just we 100, 100, 200, 4 there can be infinite. So, there is something called infinite set; that means, if infinite number of elements are there in the set. It is a infinite set. So, the in how many elements are there in the set that actually gives a the what type of set it is. So, number of elements is a important thing in the set and we define that is as the cardinality of the set.

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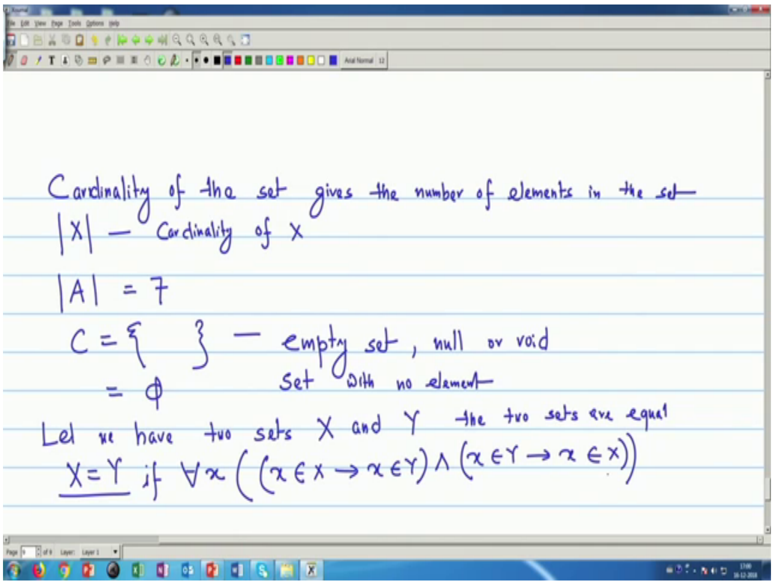

Normally, we denote so the cardinality of the set gives the number of elements in the set.

We define or we denote that as the cardinalities is in this way. So, this gives a cardinality of the set; that means, the number of elements, is the cardinality of X. So, previous example this is equal to 7 for finite set A. Now, there can be some set that where no element is there; that means, I can have a I have a set where there is no element. So, normally this is called a empty set or null set or void; we just null or void. So, this is defined as set having a set with no element. It is another notation we use this is the null set now when can we set or when can you say that two sets are equal say, let we have we have two sets say X and Y.

So, we can tell X equal to Y, so, you can tell X equal to Y say if for all x x belongs to X implies x belongs to Y and x belongs to Y implies x belongs to X. Then, two sets are equal. So, what the meaning; that means, all elements of set X and all elements of set Y are same, then the two sets are equal and that in propositional logic using propositional logic, we can write X equal to Y if for all x, x belongs to X implies x belongs to Y and x belongs to Y is x belongs to X.

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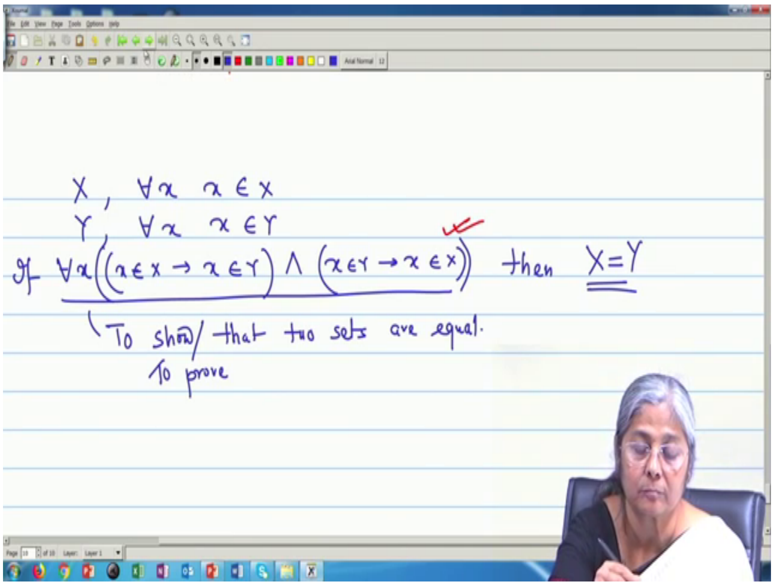

That we know that X is a set for x set x for all we can write that for all x x belongs to X; that means, small x are the all elements of X.

Similarly, if we consider y, then for all x x belongs to Y, then the proposition that if we get that for all x x belongs to X implies x belongs to Y; that means, for all elements of X, again they are the elements of Y and similarly all elements of Y implies that x this implies they are all elements of X, then if this is true, then I can tell then two sets are equal. So, this is the way to show that two sets are equal when we have to prove to show or to prove, then we have to show this proposition and logic to be true.

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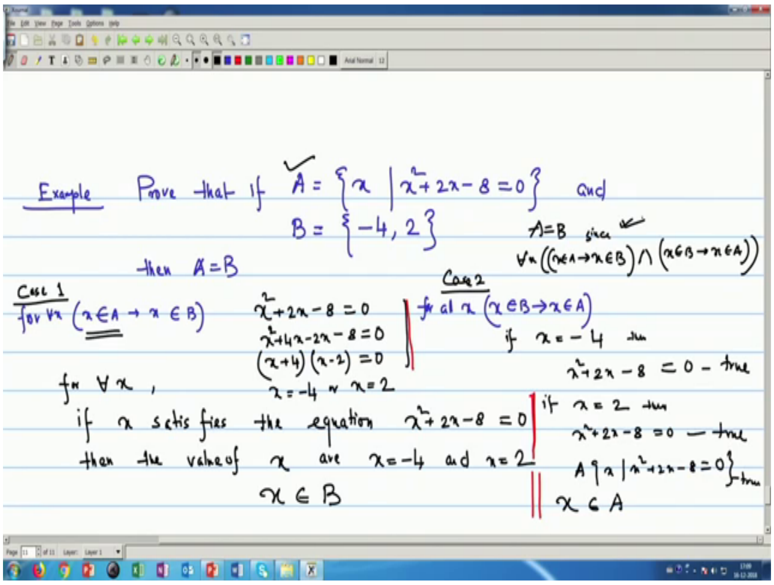

Now, we take one example ok, we take one example say prove that if A equal to x such that x square plus 2 x minus 8 equal to 0 and B equal to say I have two elements minus 4 and 2 that there equal, then the two sets are equal. So, first x belongs to A implies I have to show that for all x what we have to show for all x x belongs to A implies x belongs to B. So, what type of set A is? A is x such that x square plus 2 x minus 8 equal to 0. So, if I put what x square plus 2 x minus 8 8 equal to 0. So, what value of x, this equation is true. So, if I solve x square plus 4 x minus 2 x minus 8. So, this is x plus 4 x minus 2 equal to 0. So, x equal to minus 4 or x equal to 2.

So, x belongs to A; that means, for minus 4 or 2 that if x belongs to A; that means, x equal to minus 4 or 2. So, this implies that if x belongs to A; that means, for all x that satisfy. So, for this case this is actually our case 1 that for all x, the x if x satisfies the equation x square plus 2 x minus 8 equal to 0, then the value of x are x equal to minus 4 and x equal to 2. What do you see that B is the set of elements minus 4 and 2. So, x belongs to. So, x belongs to b. So, our now what is our case 2 or case 2 is for all x, for all x x belongs to b implies x belongs to A.

Now, we see x belongs to B, x is minus 4. So, if x equal to minus 4, then x square plus 2 x minus 8 equal to 0, it is true. If x equal to 2, then x square minus plus 2 x minus 8 equal to 0 true; that means, for x equal to minus 4 and 2, A x that x square plus 2 x minus 8 equal to 0 this is true; that means it is x belongs to A. So, what we see that that both the cases that for if x belongs to A, then it implies x belongs to B and x belongs to B, it implies x belongs to A. So, we prove that A equal to B since we have proved that for all x x belongs to A implies x belongs to B and x belongs to B implies x belongs to A. So, this is the way we should show that if two sets are equal.

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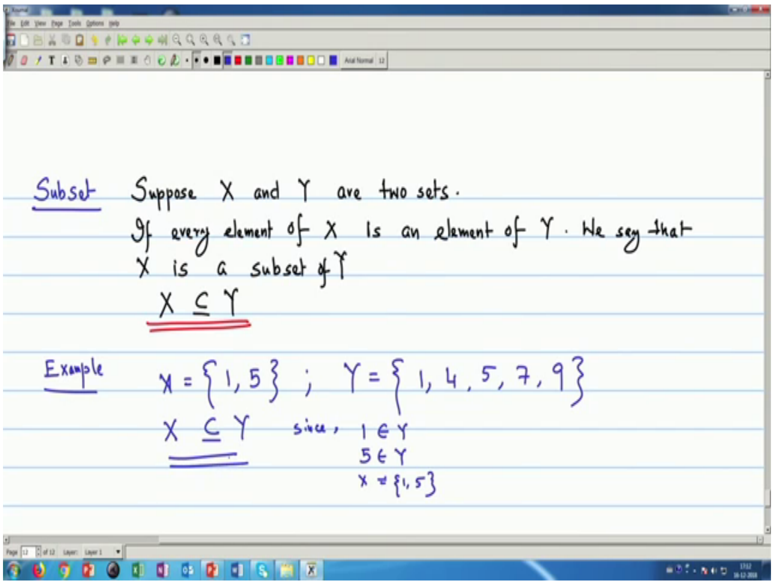

Now, we define the subset, subset opposite, define subset. Suppose X and Y, suppose X and Y are two sets. If every element of x is an element of Y, we say that X is a subset of Y and we denote that X is a subset of Y. So, this is my notation of subset. So, we take example is very simple example we can take, say my X is 1 and 5 and Y is the set say 1, 4, 5, 7, 9 like that. So, what we see that every element 1 is 1 belongs to Y, 5 belongs to y. So, I can take that X is a subset of Y. Since 1 belongs to Y, 5 is also belongs to Y and my X is 1 and 5 only.

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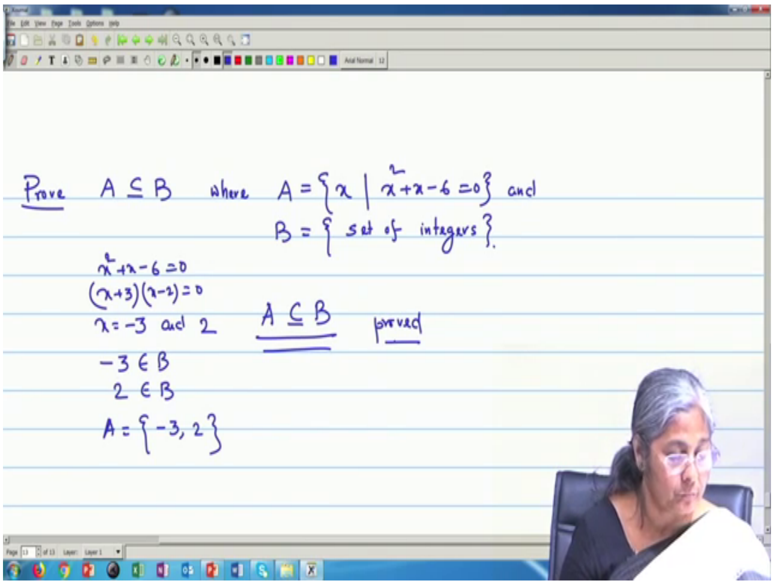

So, this is my subset notation. You see the example one another example. We see that prove, A is a subset of B where A equal to x such that x square plus x minus 6 equal to 0 and B is the set of integers, B is set of integers. So, A is all such x that satisfies these equation and we know that x square plus x minus 6 equal to 0; that means, for x values are x plus 3 into x minus 2 equal to 0. So, x equal to minus 3 and 2 and B, the set of all integers, so minus 3 is also belongs to B, 2 a belongs to B, A is actually the set of minus 3, 2 because that satisfies this. So, my A is the subset of B, proved A is the subset of B.

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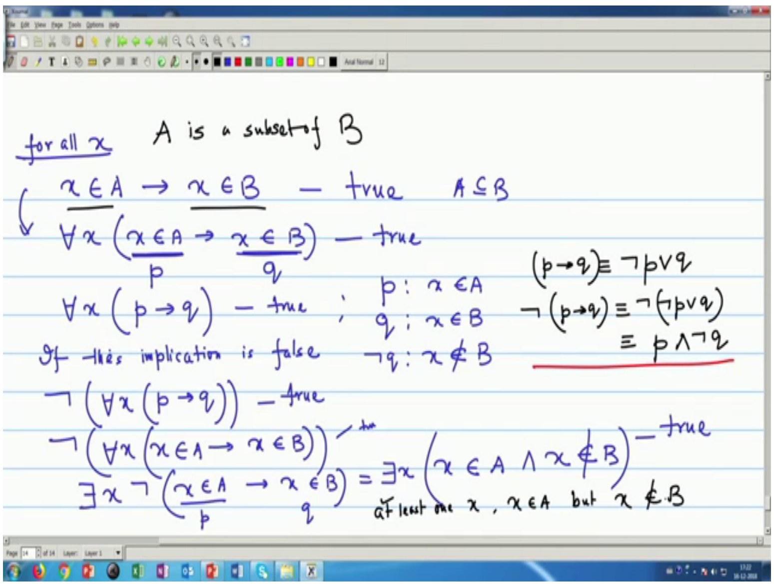

Now, x belongs to if that using the propositional logic if we write that x belongs to A implies x belongs to B. Now, if for all x it is true if it is true for all x, then we tell that this is a belongs to A is the subset of, A is a subset of B. So, for all x whether this is true. Now, if I write using our logic for all x, x belongs to A implies x belongs to B and this must be true. Now, say I this is one, say one proposition or basic statement I think that p and x belongs to B another statement q. So, if it is for all x p implies q if I think that that is true where p, I have assumed that x belongs to A and q I have assumed that x belongs to B ok.

Now, for all x I have to show that this is true for all x. Now, if it is to be false, if my statement this conditional statement because this is a conditional statement. So, if these implication or conditional statement is false; that means, negation of this is true, that is negation for all x p implies q. This is true; that means, negation if this is false, then this should be true. If this implication is false, so; that means, we remember that negation if p implies q. If we remember the formula that p implies q is equivalent to negation p or q. So, if I take negation of p implies q which is equivalent to, we apply De Morgan's law q is equivalent to p and negation q. So, if we apply here this thing; that means, my this is negation for all x my p is x belongs to A and implies sorry x belongs to A implies x belongs to B.

So, negation of for all x means there exists x, there exists x negation of x belongs to A implies x belongs to B. So, this is simply p q. So, negation p q what we have seen negation p implies q is p and negation q. So, this becomes there exists x x belongs to A p and so, and negation q; that means, I can write negation q is does not belongs to B; that means, here negation q is x does not belongs to B. So, what we get and that must be that must be true it is true, this is true. So, what we get? So, if it is a subset A is a subset of B then we tell that, A is a subset of B; that means, x belongs to A implies x belongs to B.

Now, if it is false then the negation is true and we get there exists x; that means, there exists at least 1 x, at least 1 x for which x belongs to A, but x does not belongs to B.

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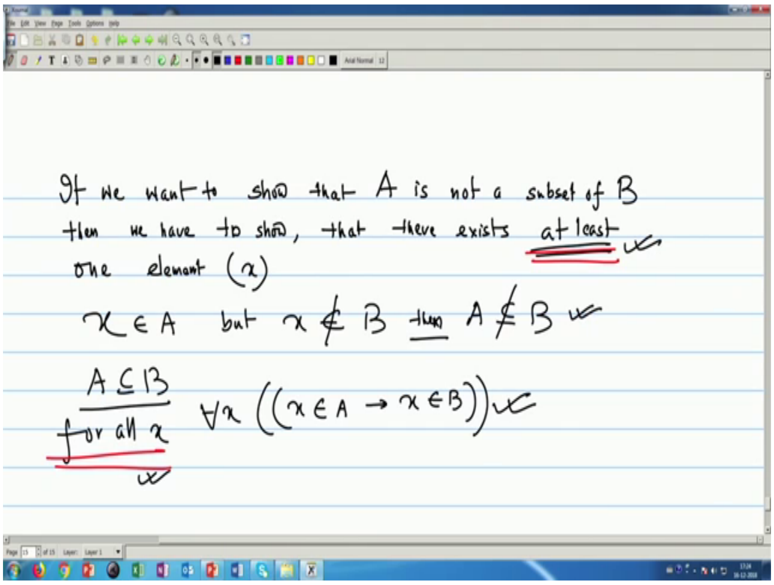

So, if we have to show that a is not A subset of B; that means, what we see that if we want to show that x is or A is not a subset of B for A B and two sets then I have to then at least one element, then we have to show that there exists at least one element say x that which belongs to A, but does not belongs to B, then we it is not, then A is not a subset of B, then A is not a subset of B, A is not a subset of B.

And, but if I have to show that A is a subset of B, then for all x we have to show for all x this is for all x, this is true for all x and here this is at least that x this implies x belongs to B. So, the summary is that if we have to show that A is a subset of B, then I have to show for all this is the thing the for all and if it is not then at least like the counter example we have to show that at least one such x exists that which belongs to A, but does not belongs to B.

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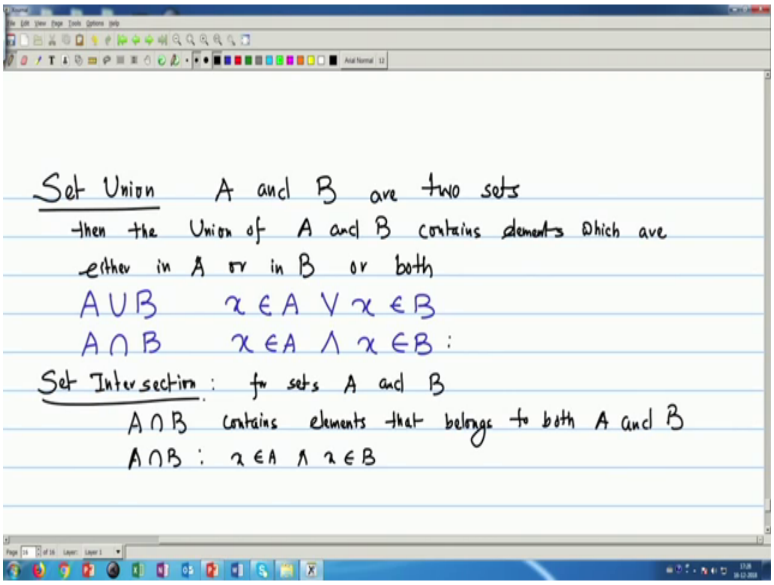

Now, with this concept, we can use the say Set Union, we can define we can define set union that A and B are two sets, then this union of A and B contains elements which are either A, either in A; that means, elements belongs to A or B or in B or both. We normally define that or denote the notation that A union B that is, but we denote these are the notation that A union B; that means, we tell that x belongs to A x belongs to A or x belongs to B. Now, A intersection B that we write x belongs to A and x belongs to B. So, intersection is all the elements. So, set intersection that for sets A and B, A intersection B contains elements that belongs to both A and B and what we write that A intersection B is equivalent to we write x belongs to A and x belongs to B and set difference, if A and B are two sets then A minus B, I can write that x belongs to A and x does not belongs to B. Similarly, B minus A that x belongs to B and x does not belongs to A.

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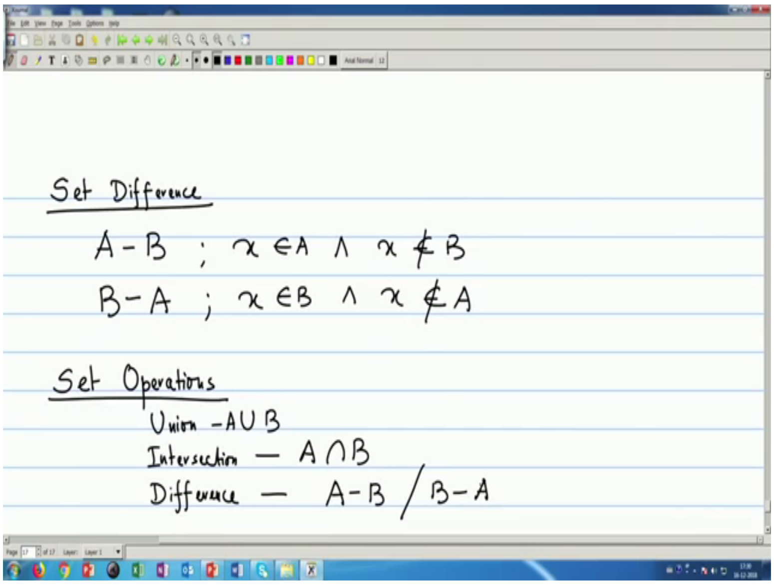

So, these are my set operations. We define union normally denote by A union B, intersection is set difference is A minus B or B minus A. Normally, these are the operations we use on the set. There are many other properties on set and that we will discuss in the next class.