Discrete Structures
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Lecture – 01
Introduction to Propositional Logic

We start our Discrete Structure class with the fundamentals of logic. Today, we will reach the Propositional Logic.

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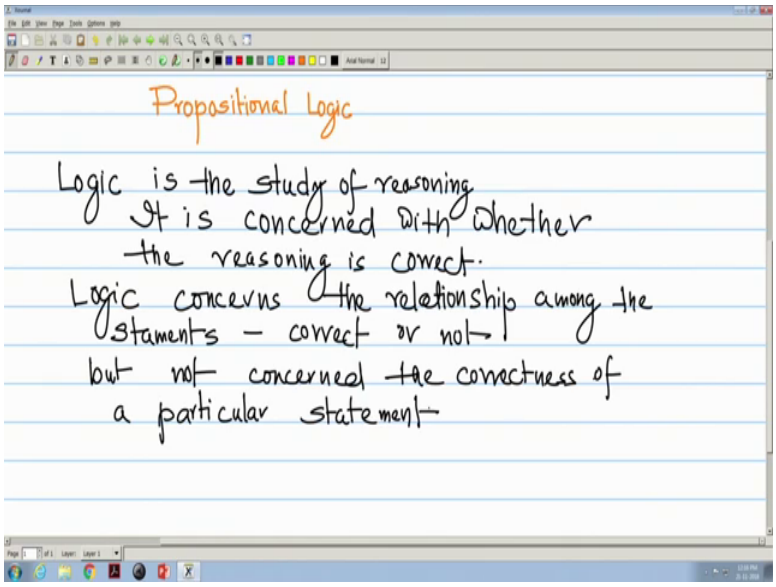

So, first we give that, what do you mean by propositional logic. What is logic? So, logic is the study of; logic is the study of reasoning; we write logic is the study of reasoning.

It is specifically concerned about the reasoning of two statements; so or whether the reasoning is correct or not. It is concerned with whether the reasoning is correct. Mainly logic concerns about the relationship between two statements, but logic does not concern about the correctness of the statement. Logic concerns the relationship among the statements particularly whether it is correct or not correct or not, but not concerned the correctness of a particular statement. We take one example first.

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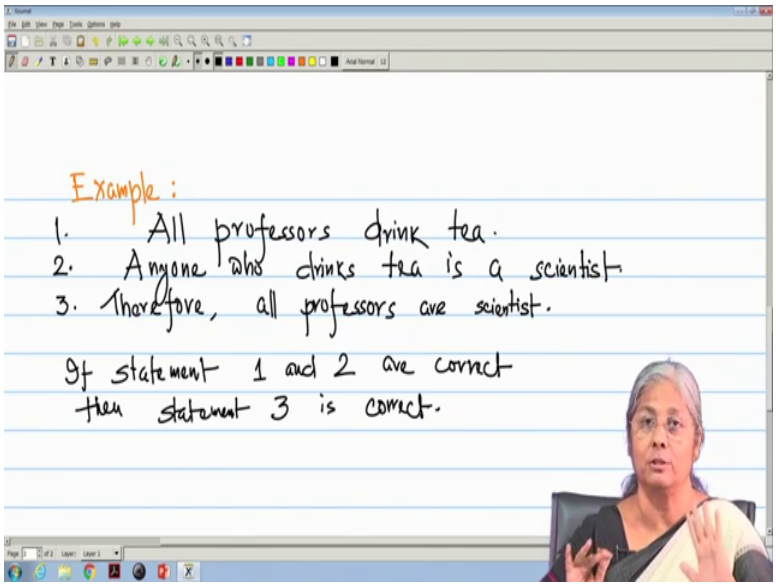

One simple example we see. See I write all professors drink tea. Second statement I write that anyone who drinks tea is a scientist. So, the third statement gives a relationship between 1 and 2 and we can tell if we think that 1 and 2 are correct, then I can write that therefore, all professors are scientist.

So, we explained this simple example. See statement – 1, tells all professors drink tea statement – 2, tells anyone who drinks tea is a scientist. So, third statement tells therefore, all professors are scientist. See the statement 1 and 2 we do not check or we are not concerned whether the statement is correct or not. But, if 1 and 2 are correct, if statement 1 and 2 are correct, then 3 is correct then statement 3 is correct. So, this means the logic. So, this is logic. So, what is the advantage of logic or why it is so important or why we will read logic. So, first we see that thing.

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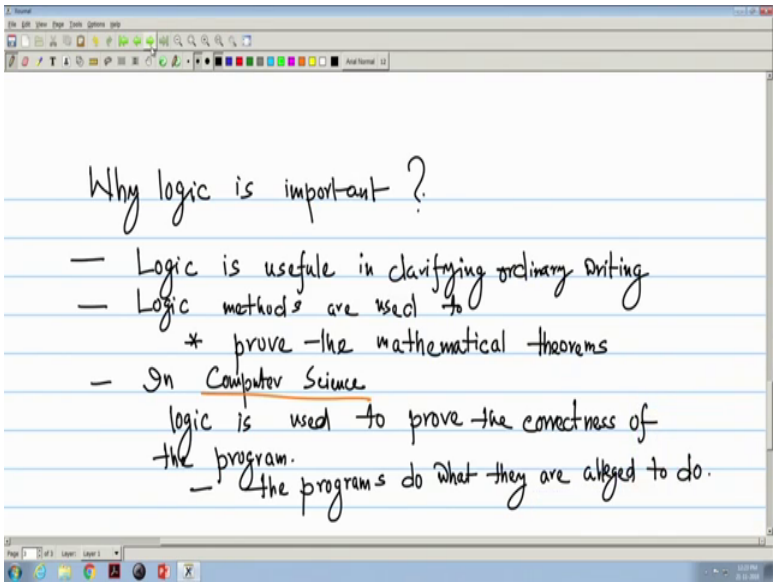

So, why logic is important? Why, we will study. Very simple way I can tell that logic is useful in clarifying ordinary writing; whatever I am writing that can be literature or that can be some science topic or our academic content. So, that must be logical and there must be some relation between the sentences that I am writing or the statements I am writing.

If I considered our technical things why logic is important, then logic methods are used are used to prove the mathematical theorem. In mathematics; we can tell that in mathematics mainly logic is used are to prove the mathematical theorems. In computer science it is used to write the correct program or to prove the correctness of the program. In computer science logic is used to prove the correctness of the program.

What do you mean by the correctness of the program? See we have to show or we have to prove that the programming is doing what it is supposed to do or what is that is to do, the programs do. So, with this small introduction now we start the propositional logic.

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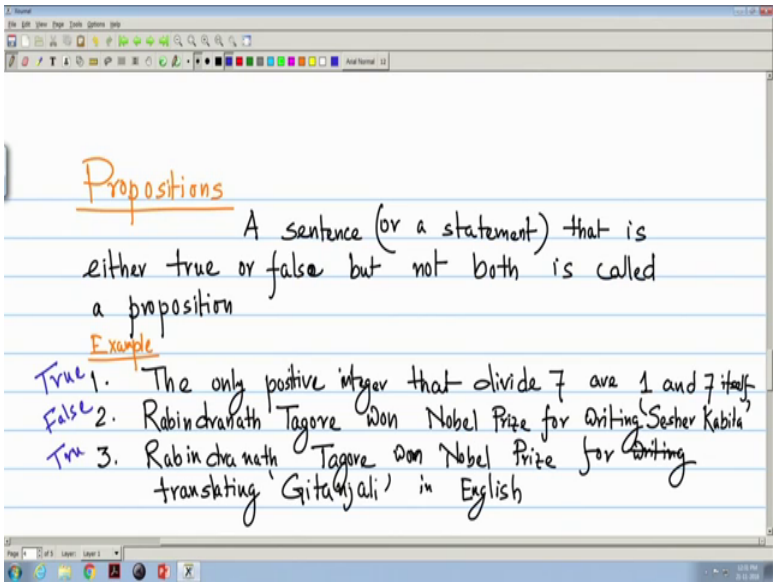

First we define the propositions. What do you mean by propositions? Simply this is a sentence or statement, but either it is true or it is false. Since just now what we read that logic will see the relationship between the statements or the correctness, but not the correctness of a particular statement. So, we define that a sentence or we can tell a statement that is either true or false, but not both is called a proposition.

We see some very simple examples so that our concept about the proposition is clear. I give some simple statements or I write those statements. First I give the only positive integer that divides 7 are 1 and 7 itself. Second statement I give that Rabindranath Tagore won Nobel Prize for writing Sesher Kabita. Third statement I write, Rabindranath Tagore won Nobel Prize for writing or better I tell for translating Gitanjali in English. We see some more examples.

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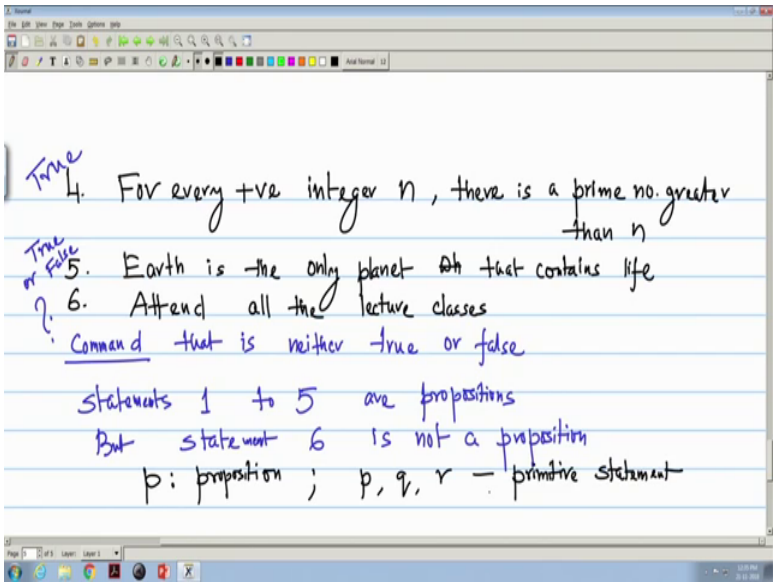

First statements I give for every positive integer n, there is a prime number greater than n. Fifth sentence I write, earth is the only planet where there exist life or that contains life. I gave you a different type of sentence, attend all the lecture classes.

Now with these six sentence I try to explain that what do we mean by proposition. We see that first one I told the only positive integer that divided 7 are 1 and 7 itself. So, it is true because 7 is a prime number. Then second statement Rabindranath Tagore won Nobel Prize for writing Sesher Kabita which is not correct, which is false. So now, we start writing that our first statement is true, second statement is false, third statement Rabindranath Tagore won Nobel Prize for translating Gitanjali in English that is true which is we know that is a true statement.

Now, next sentence that for every positive integer n, there is a prime number greater than n which tells us that the prime numbers or number of prime numbers are infinite, which is true. So, this is also true. Fifth sentence or step fifth statement tells earth is the only planet that contains life. See it may be true or false, but at a time either it is true or false not both. So, I can write the fifth sentence that may be true or false in that we know that it is true, but it may be false. So, it is true or false, but not at the same time it is not it cannot be true or false.

Sixth sentence I told attend all the lecture classes. See I cannot tell anything about the correctness, true or false. This is actually a command, ok. So, these I cannot tell about

the, this is actually a command type of command type of sentence command type of statements, that is neither true nor false. So, what I got that previous three I got one true statement – 1 true, statement – 3 true, say statement – 2 false, statement – 4 true, statement – 5 true or false and statement – 6 I cannot tell true or false it is actually a different type of thing it is a.

So, I can now conclude or I can write from these examples what I can tell that statements 1, 2, 5 are propositions. But, statement 6 is not a proposition not a proposition. So, statement or a declarative statement which is either true or false, but not both at a time is a proposition, but the command or advice this type of sentence or statements cannot be a proposition. So, we define proposition with this examples.

Normally we denote propositions by this; say p, I can tell that this is a p is proposition I can write p, q, r and that can be that denotes one particular statement that which is either true or false that can be a proposition. So, normally we call these are also sometimes we call propositions or primitive statements. What is primitive statements? That primitive statements cannot be broken into some simpler sentence, ok. So, this is primitive statement or proposition.

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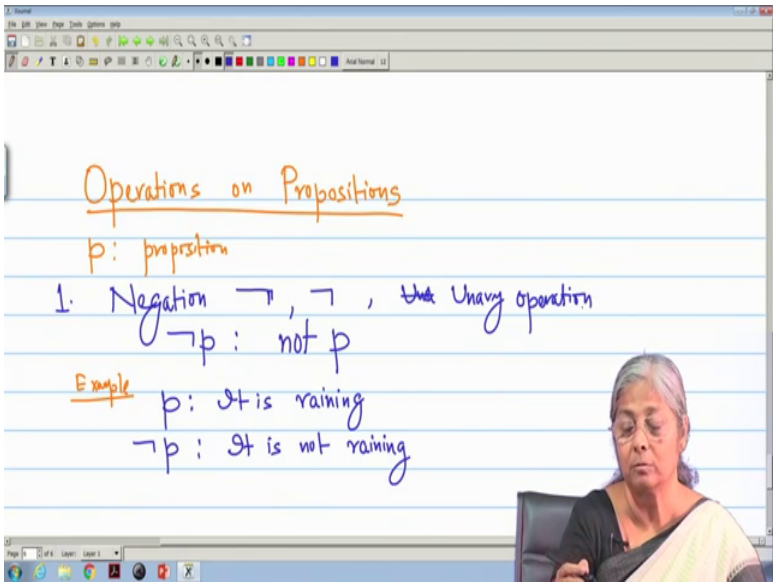

Now, we define some operations on this proposition. See these operations can be either unary or binary. What is unary; that means, this operation only operate only one single proposition. Say we consider p is a proposition, p is a proposition. Then we operation

first operation we define is a negation, normally we denote as by this or a write with this symbol we write negation; that means, if I write p is a proposition, if I write negation p then it is written that not p.

Example; if I give an example say I give an example, simple say p is a proposition: it is raining. First thing it is a declarative statement either true or false. So, what will be negation p? Negation p is it is not raining. This is a unary operator this is an unary operator because unary operation. Since it needs only one proposition or one variable I can take that one propositional variable p it works on that.

Now, some complex proposition can be obtained from the primitive statement or from that primitive proposition. We define that primitive statement cannot be broken into simpler statements or simpler propositions there is a basic thing. So, some complex proposition how we can get from there, we have some two operation for this.

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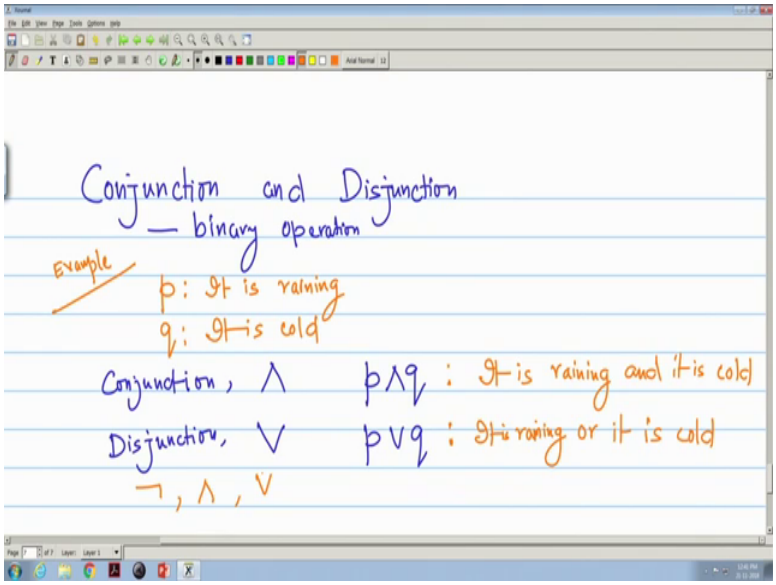

We that, operations are now these are binary operations. It is conjunctions and disjunctions; conjunction and disjunction. So, these are two binary operations. So, for this operation I need two proposition. I take the example and take quickly the example; p is one proposition: it is raining, q is: it is cold. Now, conjunction we defined like that, normally we denote this thing as this symbol we write p and q and it tells us that it is raining and it is cold.

What is disjunction? We denote by this symbol called p normally called or and this is it is raining or it is cold. So, these are three operations that negation, AND and OR these are the basic three operations on the proposition will be ready. So, we can what earlier we told that statements and these are also propositions, these are some compound propositions obtained from there.

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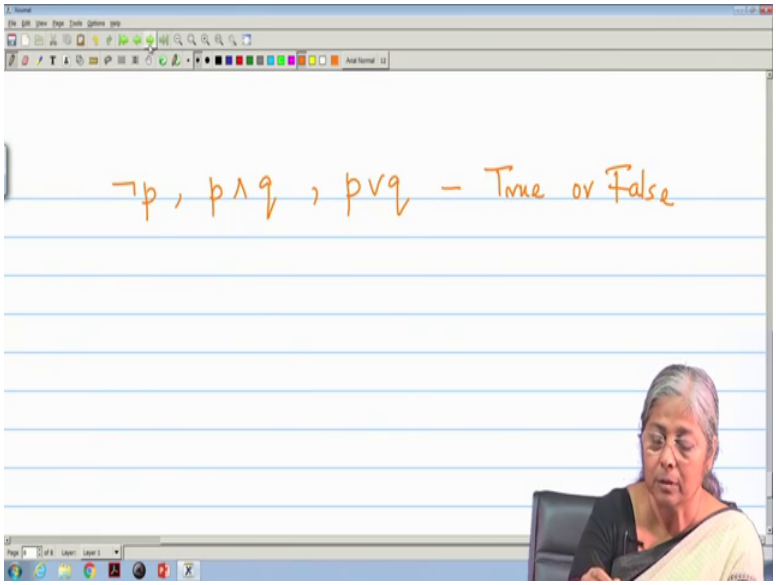

So, they are either true or false since they are propositions. So, negation p or p and q or p or q they are either, they are also either true or false, since they are also proposition.

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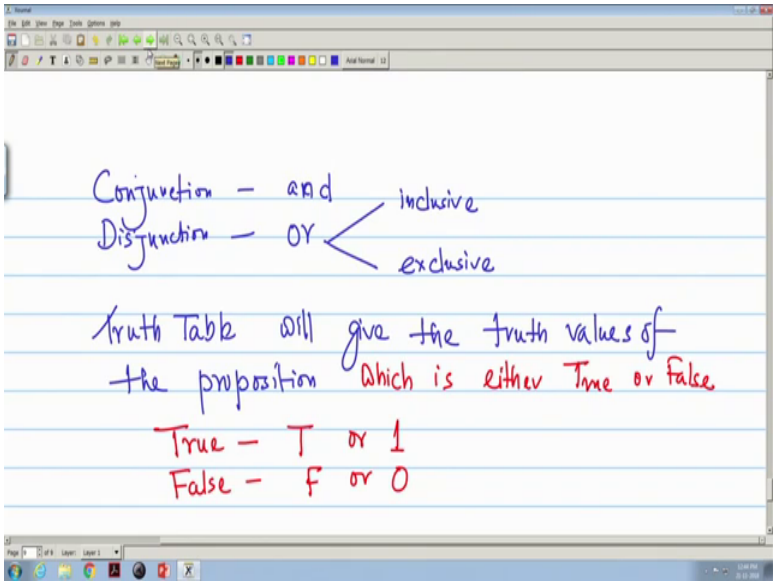

Now, another we can tell that it is normally this or these conjunction is and conjunction is and conjunction we can tell this is and. Disjunction is of two type normally we call or it is of either inclusive or what is our normal or another is called the exclusive or when the disjunction is operated on two at least two proposition, then since it is binary true proposition then it will be either inclusive on or it will be exclusive.

Now, we will define some truth table of these is op with respect to this operation. So, what is the truth table? Truth table will give the truth values of the propositions which is either true, which is either true or false we denote this true by t or 1 false by f or 0.

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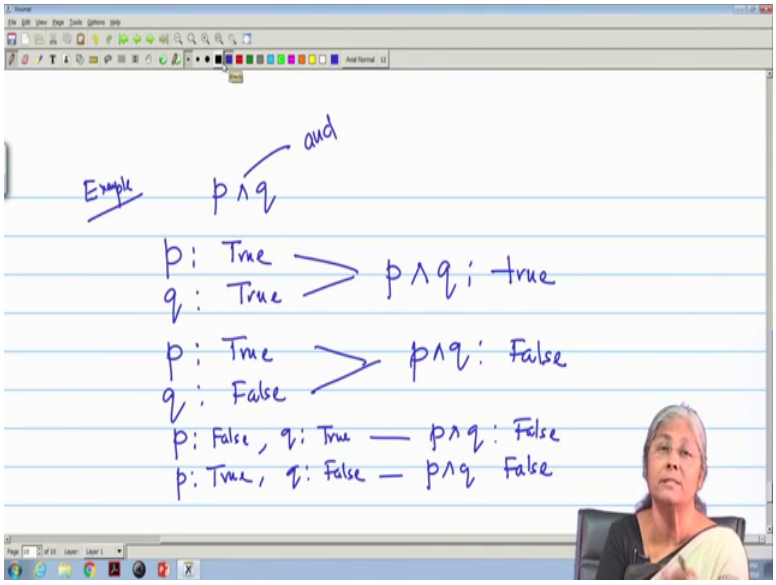

So, that means, that whenever we are say with the example that we have given that example p and q; so, if p is true, p is true q is also true then what will be the truth value of p and q. So, that gives you the one truth value of it one particular truth value of that truth table.

So, then if p q true then p and q is true we can write in this way if p is true, q is false. So, they can give p and q false. Since it is and since p and q this gives you actually and. So, now, I can quickly write that if p false q true then it will give you that p and q false, p true q false p and q false. So, in a table if we give the truth values for all the assignments of the propositions that will give us the truth table. So, now we see what are the truth table of this three basic operation.

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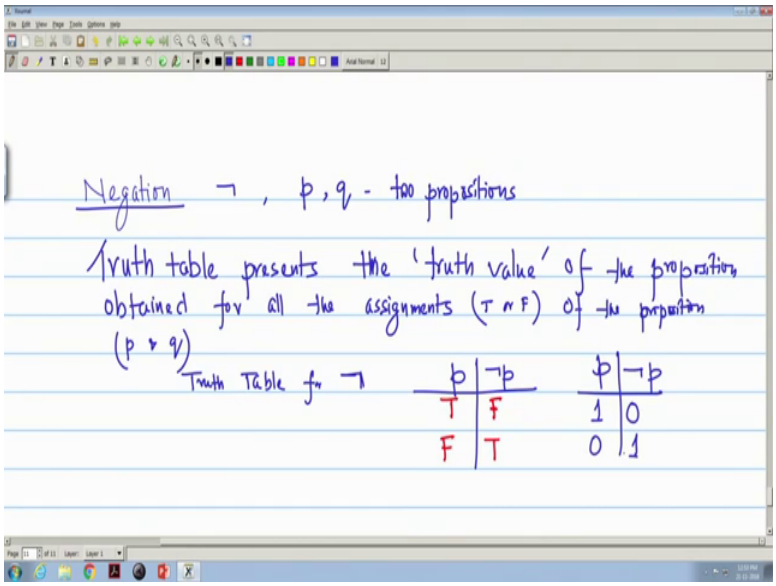

First what we have seen the negation. Negation is denoted as this sign if p and q are two propositions. So, truth table gives the; truth table presents the truth value of the proposition obtained for all the assignments; here assignments means either true or false, of the proposition or primitive proposition in this case p and q. So, truth table for negation we see truth table for negation I can write p negation p; see p can take only two values, either true or false. So, all the assignments so, I write true then negation p is false. Then if p is false negation p is true or I can also write in this way 0 or 1. So, p and negation p I can write if it is 1 it is 0, if it is 0 it is 1. So, this is the truth.

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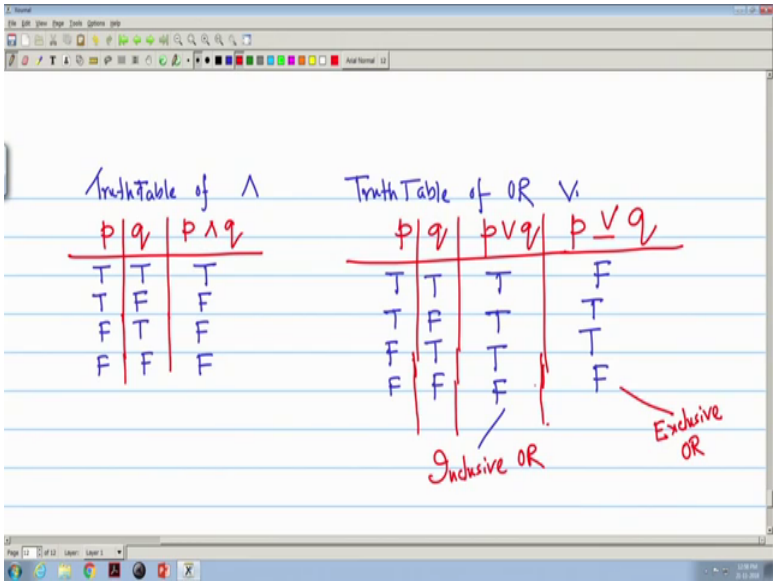

Now, we see the conjunction and disjunction; that means, if p and q then I can truth table for of conjunction. I can write p, q and p and q. So, now p, q we can take four values it can be both can be true then we and q true because and so p true q true. So, p and q true this is true and false since it is. So, the composition or the conjunction of p and q are false. Then if it is false just if it is false and true then this is false if it is both false then it is false.

Now I can write truth table of OR I can write truth table of or which is inclusive OR. So, I cannot draw the truth table p, q p OR q both if it is true, if it is true then it is true. If it is true or false since it is OR so, it can be true if it is false or true if anyone is true then it is true only if both are false then only it will be false. So, this is inclusive OR ok. Now, there is something called exclusive OR. This is we normally denote by this and we can write if it is both are true then it is false, if both are false then it is false if anyone is true then only it is true.

So, I can write that this is my inclusive OR this is my exclusive OR. So, truth table is very important for propositional logic because it gives the truth values of a complex proposition, for all the truth values of the basic proposition or the primitive proposition.

So, what we have read we did the three operation; one is unary operation, the negation the truth table of that, then we read the conjunction and disjunction to binary operations and the truth table of the p and q, the conjunction and the truth table of disjunction. Now, there are two type of disjunction, one is inclusive OR one is exclusive OR and we can write the truth table of this thing, ok.

So, now, we finish this class with these truth values and the truth tables and next class we will do we will see the other operations.