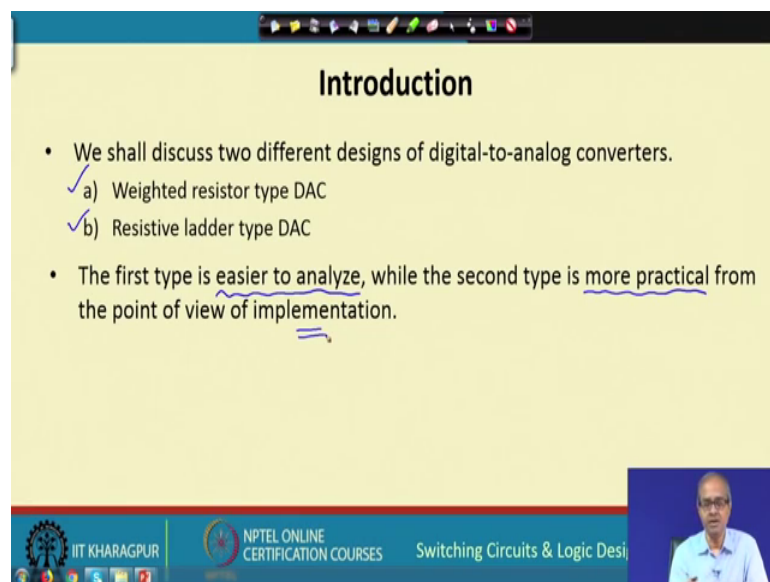


Switching Circuits and Logic Design
Prof. Indranil Sengupta
Department of Computer Science and Engineering
Indian Institute of Technology, Kharagpur

Lecture - 48
Digital-to-Analog Converter (Part II)

So, we continue with our discussion on the design of digital to analog converter. In the last lecture, if you recall, we talked about the basic characteristic of a DA converter, and the different kind of errors that may happen in such a converter. Today in this lecture we shall be talking about some of the typical designs of a DA converter, how we can build a DA converter ok? This is the second part of our lecture.

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Introduction

- We shall discuss two different designs of digital-to-analog converters.
 - ✓ a) Weighted resistor type DAC
 - ✓ b) Resistive ladder type DAC
- The first type is easier to analyze, while the second type is more practical from the point of view of implementation.

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So, here we shall be talking about two different types of DA converter as you can see. The first one is called weighted register type, second one is called resistive ladder type. Now, it will be clear when we discuss the details of this design, the first one is simpler to understand, it is easier to analyze that how it works. But, the second one it may be a little more difficult to analyze, but it is more practical, because it is easier to implement, why? The reason will become clear as we move on to the design details. Let us see how we can implement a digital to analog converter.

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Weighted Resistor Type DAC

- For an n -bit DAC, it consists of n different resistance values of magnitudes $R, 2R, 4R, \dots, 2^{n-1}R$ respectively.
 - The resistances help in generating currents inversely proportional to their magnitudes.
 - The total current is added up by an operational amplifier, and is converted to the voltage output V_{OUT} .

Handwritten notes on the slide:

- For $n=4$, the resistances are $R, 2R, 4R, 8R$.
- For $n=5$, the resistances are $R, 2R, 4R, 8R, 16R$.

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The first kind of a digital to analog converter that we talk about is called weighted resistor type DA converter. Let us, talk in general about an n -bit digital to analog converter, where we use n different resistance values and the resistance values are of magnitude $R, 2R, 4R$ up to 2 to the power n minus 1 R . Let us, say if n equal to 4 , there will be n different resistance values and the resistance values will be $R, 2R, 4R$ and $8R$. Similarly, if n equal to 5 , then will be having R up to $16R$. if n equal to 6 , we will have up to $32R$.

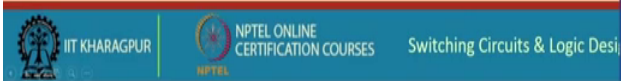
So, the idea is that you see we require so many different values of resistances. Let us say we design a 16 bit DA converter. See, for a 4 bit DA converter, 8 is how much 2 to the power 4 minus 1 that is 8 . So, for a 16 bit DA converter, we need 2 to the power 16 minus 1 or 2 to the power 15 2 to the power 15 is $32,768$ approximately. Now, the trouble is that when you require so many different values of resistances, it becomes difficult to manufacture.

If you say I require only 1 value or 2 values or 3 values of resistances, it is much easier to manufacture them accurately, but if you say I require 10 different values, it becomes difficult. So, this is one of the major drawback of this weighted register type digital to analog converter right.

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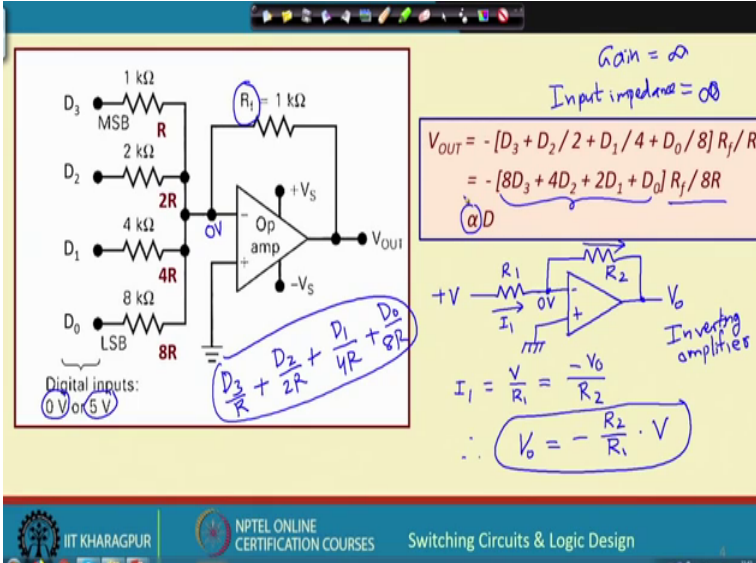
Weighted Resistor Type DAC

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 - The resistances help in generating currents inversely proportional to their magnitudes.
 - The total current is added up by an operational amplifier, and is converted to the voltage output V_{OUT} .
- Main drawback:
 - A different-valued precision resistor must be used for each bit position of the digital input.



Let us, see how this kind of a DA converter looks like this main drawback as I said different value resistances are required ok.

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Gain = ∞
Input impedance = ∞

$$V_{OUT} = -[D_3 + D_2/2 + D_1/4 + D_0/8] R_f/R$$

$$= -[8D_3 + 4D_2 + 2D_1 + D_0] R_f/8R$$

$\propto D$


Digital inputs: 0V or 5V

$\frac{D_3}{R} + \frac{D_2}{2R} + \frac{D_1}{4R} + \frac{D_0}{8R}$

$I_1 = \frac{V}{R_1} = \frac{-V_0}{R_2}$

$\therefore V_0 = -\frac{R_2}{R_1} \cdot V$

Inverting amplifiers



First you see this is the schematic diagram before explaining, how it works I will talk a little bit about this building block op amp ok. Let me talk about this then I will again come back to this diagram and explain. Now, op amp is the short form of Operational Amplifier symbolically we write it like this. There are two input terminals there is one output terminal. Now, op amp is supposed to be an ideal amplifier, when ideal amplifier

means its gain should be infinity gain is ideally infinity, but in practice it is not infinity very large. And input impedance this is another characteristic of an amplifier good amplifier, input impedance should be as low as possible. Now, for an operation amplifier we assume that the input impedance is 0.

Let us look at a typical diagram; this is a typical connection of an operation amplifier. Suppose I apply a voltage V here and this is my V output, let us say V_o , these two resistance values are R_1 and R_2 . Now, because the gain is infinity and the output voltage is finite, gain means the difference of the 2 inputs multiplied by the gain, because the output value is finite, the difference between these 2 voltage should tend to 0, because plus I am connected to ground. So, this point also should be very close to 0 volt this is the characteristic.

Now, if that is so then if I talk about the current flowing let us call it I_1 , so what will be the value of I_1 ? Say I_1 will be V minus 0 by R_1 , so V by R_1 . And this current, because input impedance is 0 or so there will be no current flowing, this current will be flowing through this R_2 . The current will be flowing in this direction this same current will be sorry I am a little this input impedance is not 0 it is infinite this is also very high sorry this input infinite should be very high not very low, output impedance should be low alright.

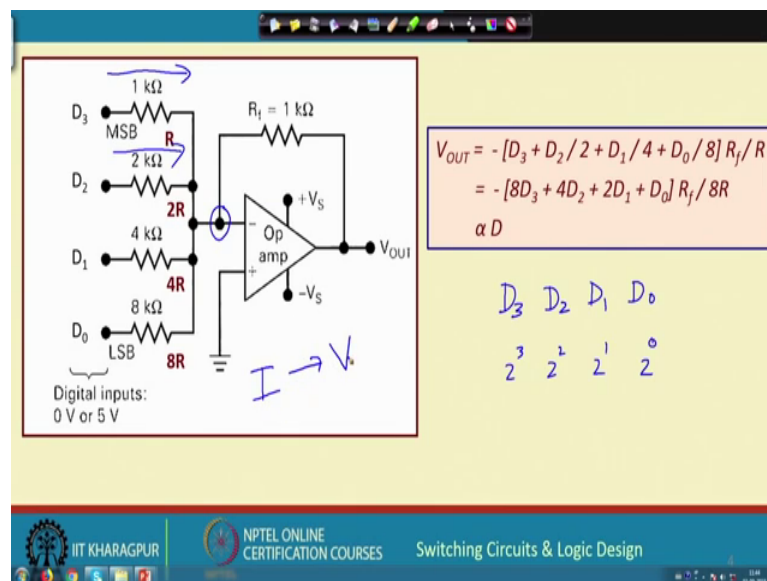
So, because input impedance is high no current is flowing inside the current will be flowing through R_2 . So, this current I_1 will be flowing through R_2 so, how much is the value 0 minus V_o by R_2 . So, it will be minus V_o by R_2 . So, if you solve, your V_o value becomes minus R_2 by R_1 into V , this is the expression for the output for. And this configuration is called inverting amplifier configuration. Now, let us now come back to this diagram here this, op amp has been used in the inverter amplifier configuration. Here exactly similar diagram the plus input is connected to ground.

And there is a resistance R_f that you have connected in the feedback from this point to the output. And in the input instead of a single voltage we have connected several resistances and $D_3, D_2, D_1, D_0, 4$ voltages and because, these are digital voltages that can be either 0 or 1. Let us say 0 means 0 volts 1 means 5 volts let us say so, because this point is at 0 volt just like I have said.

So, while calculating the value of I_1 what will be I_1 here? Let us say this is $R_2, R_4 R_8$ and example is given 1 kilo ohm, 2 kilo ohm, 4 kilo ohm, 8 ohm. So, the total current here will be D_3 divided by R plus D_2 divide by $2R$ the total current have to calculate plus D_1 by $4R$ plus D_0 by $8R$, so here the expression is written. This will be the total current flowing into here this I_1 and this will be equal to $\text{minus } V_0 \text{ by } R_f$, so V_0 or V_{out} here will be if we simplify if we take this R outside, R_f by $R D_3$ plus D_2 by $2 D_1$ by $4 D_0$ by 8 you see the same expression.

And if you just rearrange it a little bit take 8 also outside, then this becomes just the decimal equivalent of the binary number 8. You see for this 4 bit number binary number what are the weights 8 4 2 1, 2 to the power 0 2 to the power 1 2 to the power 2, 2 to the power 3. So, it is exactly the you see D_0 plus $2 D_1$, $4 D_2$, $8 D_3$, this is nothing but the decimal equivalent of the number. So, V_{out} is becoming proportional to D where the constant of proportionality is $\text{minus } R_f \text{ by } 8R$. So, by controlling the value of $R_1 R_f$ the exact voltage can be fixed up, but it will always be proportional right.

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So, this is a very simple way to build a digital to analog converter by using different values of resistances. Now, as I said I am just repeating for a 4 bit number you need different weights for the inputs. So, in order to generate these weights, we are using resistances which are also multiples of 2, $R_2 R_4 R_8 R$, so that the current flowing through this D_3 resistance is maximum, current flowing through the D_2 resistance will

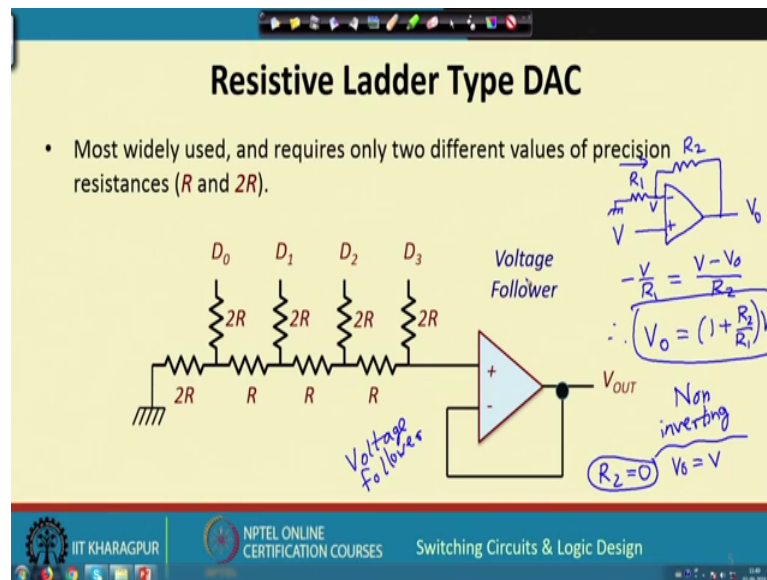
be half of that D 1 will be again half of that and D 0 will be again half of that. So, these resistances automatically generate these weights. And here the weighted sum is carried out and this op amp is actually basically it is converting this current into a voltage this is working as a current to voltage converter right.

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Let us move on to the second kind of resistive ladder DA converter, which I have said is more practical. So, here also I take the example of a 4 bit DA converter. Let us see here this D 3 and up to D 0 these are the 4 bits of the input. And as you can see only two values of resistance are required 2 R and R. There are total of 8 resistances so if it is an n-bit converter, the first thing is that I require 2 n resistance values. So, I need n plus 1 resistance values of magnitude 2 R and n minus 1 resistance values of magnitude R right.

Now, there is another point to note in this kind of a resistive ladder circuit. Here, we are not generating a current like in the weighted register type we saw earlier. In the weighted register type we had generated a current, which is proportional to the input digital word, and the op amp was converting that current into a voltage. But, here whatever we are getting here, this is a voltage and this voltage is proportional to this digital word ok. Before going into that how this voltage is proportional? Let us talk about another kind of an op amp in connection let me work it out a little bit.

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Let us look at another kind of an op amp connection. Here what I am saying is that the input voltage I am applying to the plus input let us say I am applying a voltage V here. And the resistances are connected as previously and this point I am connecting to ground. Let us, say we have R_1 here we have R_2 here and this is V_0 . Now, again we have an op amp, because the gain is infinity virtually infinite and the output voltage should be finite.

Therefore, these two input points must be approximately at the same voltage, because in the plus input I am applying V , so here also it should be V right. Now, if I again try to compute the current flowing here in this direction it is ground minus V by R_1 , so it will be minus V by R_1 . This current should be the same as the current flowing through R_2 V_0 minus V_0 by R_2

Now, if you solve this I give it an exercise for you I am jumping some steps, V_0 will be given by $1 + \frac{R_2}{R_1}$ into V and there is no minus sign here ok. So, this is called a non inverting amplifier non inverting. And another property you see in this expression if I set R_2 equal to 0, then what will happen, if R_2 equal to 0, then V_0 becomes equal to V you will see in this configuration we have done exactly that. In the feedback part there is no resistance that means R_2 is 0.

And this kind of a configuration where the output voltage is same as the input voltage this is called a voltage follower. So, in this circuit we have used a voltage follower using

op amp as you can see voltage follower fine. Now, let us see how the voltage that is being generated in this plus input becomes proportional to the input digital word step-by-step let us see. Now, as I said earlier that for this case it is a little more difficult to analyze.

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Calculation

- Let us calculate the voltages at the opamp input when exactly one of the D_i inputs is at 1 (say, $+V$ volts).
- Case 1: Input is 1000.

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So, what we do, we start by assuming that exactly one of the inputs are at 1 and the remaining inputs are at 0. Let us say to start with the first case D_3 is 1 that means most significant bit is 1 others are 0 right. Let us see what will happen in this case? So your equivalent circuit will be like this D_3 is 1 which means it is at plus V 1 means plus V, let us say 0 means ground and 0, 0, 0, 0, 0 and 0.

Now, you know that 2 resistances from any point if you have 2 resistances connected to ground, let us say 2 R and 2 R 2 resistances in parallel. This is equivalent to a single resistance of value R. Here same thing will happen you see this 2 R and this 2 R they are in parallel and connected to ground. So, at this point so equivalently you have a single resistance to ground of value R, again this R and this R are in series. So, it will be equivalent to 2 R again this 2 R 2 R will be in parallel, there will be R, again this R and R will be in series. So, in this way it will go on I urge you to work this out and it will go on.

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Calculation

- Let us calculate the voltages at the opamp input when exactly one of the D_i inputs is at 1 (say, $+V$ volts).
- Case 1: Input is 1000.**

$V_x = V/2$

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And finally, what you will get you will get an equivalent circuit like this. You will reach the last point plus V or $2R$ is here. And on the left side the this entire network, which is here this entire network becomes equivalent to a single $2R$ resistance. Again there is a voltage divider $V \ 2R \ 2R$ this is equivalent to V by 2 ok, because the total current will be V by $4R$ and the voltage V_x will be V by $4R$ multiplied by $2R$, so V by 2 . So, you see if the MSB is 1 most significant bit is 1, then the output will be V by 2 , where V is equivalent to that 1 voltage right.

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- Case 2: Input is 0100.**

$V_x = V/4$

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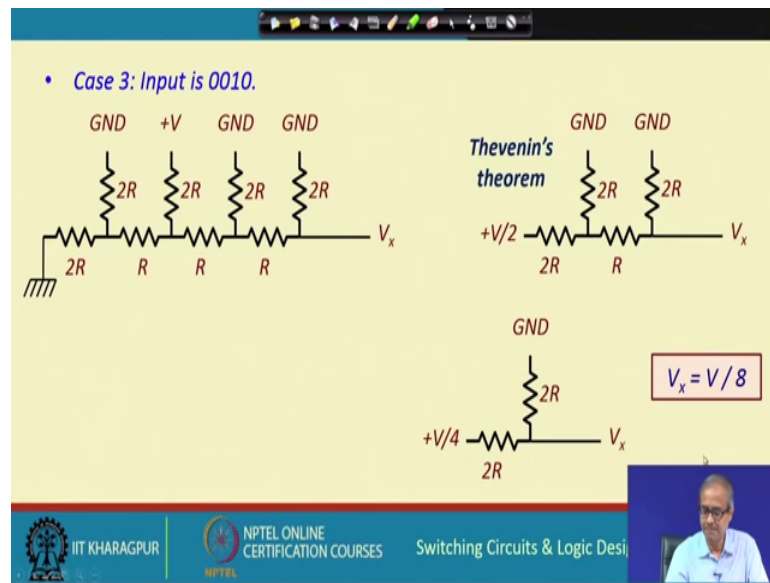
Next, let us see what will happen if the next bit is 1, next MSB is 1, 0 1 0 0. So, the equivalent circuit is this. Here the next MSB here MSB is a is a right hand side, least significant bit is a left hand side. So, this is the circuit so, in the same way, these two resistances are in parallel, there will be R R R in series again $2 R$ again this R and $2 R$ and $2 R$ will be in parallel R , R and R will be in series again $2 R$.

So, you will be getting an equivalent circuit like this when you reach this plus V point, $2 R$ on the left this entire circuit is equivalent to $2 R$. But, now you directly cannot use this parallel calculation, because if a voltage here not ground at one point it is the ground, other point voltage.

Here what you do you apply something called Thevenin's theorem, this you must have studied in your school, if you recall. Thevenin's so let me tell you what this Thevenin's theorem says. So, I am applying a Thevenin's theorem at this point so, what it says that if we have a circuit like in a V ground and $2 R$ $2 R$, this entire thing can be replaced by a resistance and a voltage source by R some resistance and a voltage source. What the resistance value can be calculated by setting all the voltages to ground like here we set it to ground $2 R$ $2 R$ in parallel becomes R so it is R .

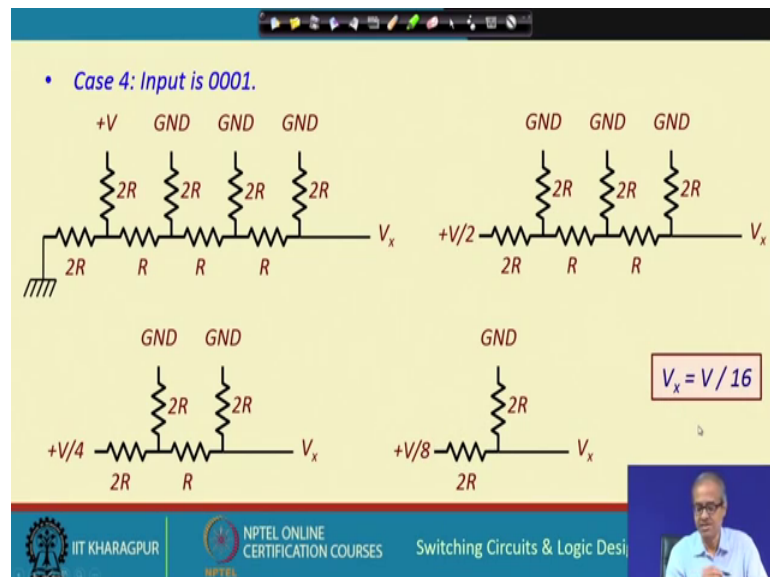
And this voltage value will be equal to what is the voltage here? If this V is applied here, so you now again forget this ground plus V $2 R$ $2 R$ ground, so at this point it will be V by 2 , so V by 2 in series with R this is what Thevenin's theorem is. So, once you do it again you have $2 R$ this $2 R$ again V by 2 $2 R$ and $2 R$ to ground. So, if the output voltage again a resistance divided $2 R$ $2 R$ to ground middle point, so it will be V by 4 V x will be V by 4 . So, you will see earlier we said when the MSB is 1 output was V by 2 , but when the next MSB 1 it is V by 4 right.

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So, if now the next MSB is 1 0 0 1 0 in a similar way you combine these two then apply Thevenin's theorem at this point yes, it will become like this. Then again apply Thevenin's theorem at this point, it will become like this you have V by $4 \cdot 2R \cdot 2R$ it becomes V by 8 you see it was V by 2 V by 4 V by 8 .

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Lastly, when the least significant bit is one, circuit will like this again similarly you start by applying Thevenin's theorem from the beginning like this and continue apply Thevenin's theorem once here, once here again once here first time here V by 4 next time

here V by 8, now v by eight two r two r ground, so here it will be half of V by V by 16. So, what we have seen is that for a 4 bit DA converter for the 4 bit positions if individually one of the bit is 1, the output voltage is coming as V by 2 V by 4 V by 8 and V by 16.

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• When all the four inputs $D = D_3D_2D_1D_0$ are applied (where $D_i = \text{GND}$ or $+V$ volts), we can apply the principle of superposition to compute the final output voltage V_A .

$$\frac{D_3}{2} + \frac{D_2}{4} + \frac{D_1}{8} + \frac{D_0}{16}$$

$$= \frac{1}{16} \cdot D$$

$V_{OUT} \propto D$

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Now, what now when all the four inputs are applied simultaneously, then we can apply another theorem or principle is this called principle of superposition. Principle of superposition applies only for linear circuits, I am not going into the detail definition, but just remember any circuit designed using resistances that is a linear circuit. Principle of superposition says that the total output voltage will be the same as the sum of the output voltages with respect to one of the inputs applied individually 1 at a time.

Like, what I am saying is that when all 4 inputs are applied, you calculate it separately. First D_3 is applied all others are grounded so what is the voltage? Then D_2 is applied all are grounded then what is the voltage? Then D_1 is applied then applied you add all of them up that will be the net V_A right.

So, this already you have seen V_A is what D_3 by 2, because, V we assumed D_2 we said it becomes by 4 D_1 by 8 D_0 by 16. So, if you take 1 by 16 common, it becomes the decimal equivalent of the number. So, V_A is proportional to D and because this is a voltage follower V_{out} it will also be proportional to D right.

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$$V_A = [D_3.(V/2) + D_2.(V/4) + D_1.(V/8) + D_0.(V/16)]$$
$$= [8D_3 + 4D_2 + 2D_1 + D_0].(V/16)$$
$$= D.(V/16)$$

Thus, $V_A \propto D$.

For an N -bit DAC, the contribution of the k -th input D_k on the output voltage will be $V / 2^{N-k}$

This is how this works; this is the calculation as I said. So, this becomes finally 3 by 16, therefore V_A is proportional to D . And just one thing to note for an n -bit DA converter now, in general if you look at the k th input D_k the corresponding contribution will be V divide by 2 to the power N minus k , because for a 4 bit convert it was V by 2 V by 4 V by 8 it comes just like this.

So, with this we come to the end of this lecture. Now, in this lecture, we talked about two different designs of a DA converter digital to analog converter. First one in the weighted resistor type, we had so many different values of resistances, because of which with respect to implementation and accuracy it becomes a problem. But for resistive ladder type DA converter although the number of resistances required a larger, but only two different values of resistances are required which can be fabricated very accurately. So, from the implementation point of view, this weighted resistor type DA converter is most commonly used. So, the next lecture we shall be starting our discussion on the reverse kind of conversion namely analog to digital converter.

Thank you.