Switching Circuits and Logic Design
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Lecture – 20
Minimization Using Tabular Method (Part – II)

In the last lecture we were talking about the Quine McCluskey method, for minimizing switching expressions, if you recall the basic idea was that we were considering the binary representation for the minterms, then using a set of systematic rules, we were combining adjacent minterms which were differing in index value of 1; that means, number of 1's.

And we were ultimately creating smaller and smaller terms and finally, whatever we are left were with those were the so, called prime implicants. So, what we have seen so far is how we can get all the prime implicants of a given function. So, let us continue from that point onwards. So, this will be the part two of our lecture minimization using tabular method.

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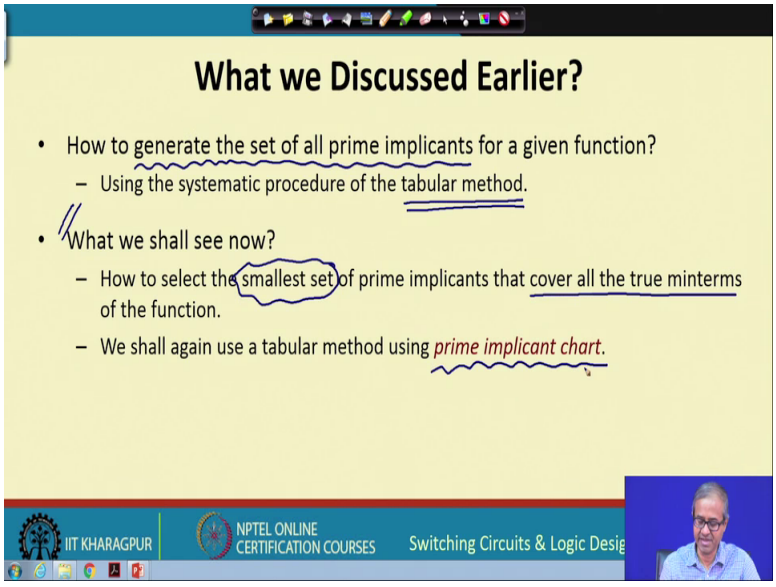

So, as I said. So, what you have seen in the last lecture is we have looked at a method to generate the set of all prime implicants for a given function right. So, we have used a tabular method a systematic method to do that, but what after that once you get the prime

implicant what is our next step. So, our next step will be to select the smallest set of prime implicants that will cover all the true minterms of the function.

This is very similar to what we were doing using the Karnaugh map method also, they are starting with the prime implicants which were the so called cubes that, we are forming on the k map we were selecting a minimum set of cubes that will be covering all the true minterms or the ones in the map. Exactly the same thing we are trying to do here.

Now, that we have got the list of all prime implicants, let us try to see, what is the best set of prime implicants, we can select out of that which will be covering all the true minterms of the function ok. Now, for doing this again we shall be using a tabular method and, the table that we will be using now is called a prime implicant chart. Let us see what are prime implicant chart looks like.

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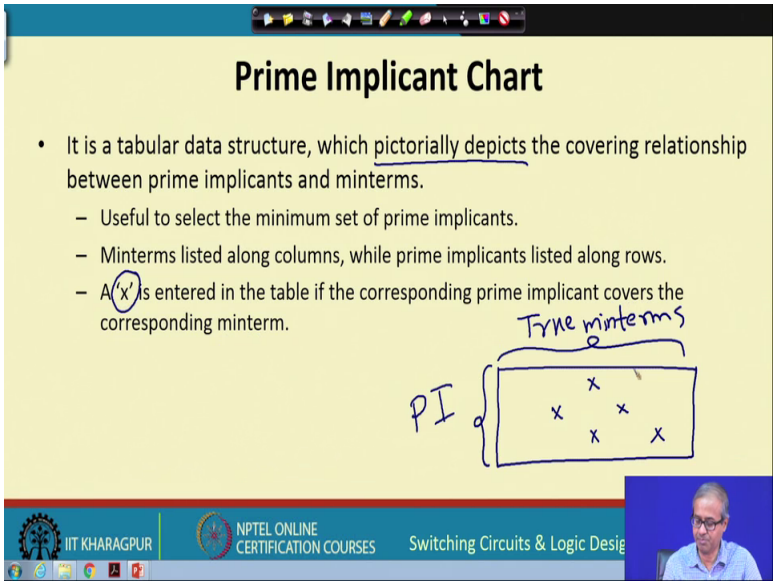

So, a prime implicant chart essentially is a tabular kind of a structure. So, basically it is a table kind of a thing. So, along the rows we list the prime implicants. So, all the prime implicants are listed along the rows and along the columns, we list all the true minterms.

So, which prime implicant is covering which minterm that we indicate by putting a cross mark this is how the table looks like.

So, let us see what is mentioned here this pictorially depicts the covering relationship; that means, which prime implicant covers which true minterms ok. So, as I said minterms are listed along the columns and, the prime implicants are listed along the rows, we shall see some examples. And will be entering some check marks some X kind of a checkmark in the table whenever a particular prime implicant is covering some minterm. So, we will put a X in the corresponding location.

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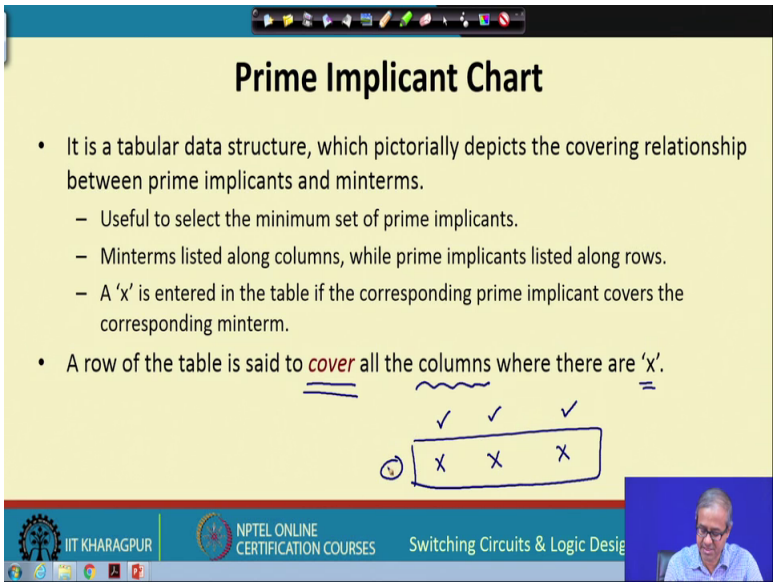

So, what we say is that a row of the table; that means, a minterm covers all the prime implicants; that means, the columns I means I means all the minterms I mean, all the minterms the columns where there are X say for example, for some prime implicant we see in the table that there is an X here, there is a X here, there is a x here. So, the corresponding true minterms, these 3 minterms are covered by this prime implicant right. This is the idea we use this term cover ok. .

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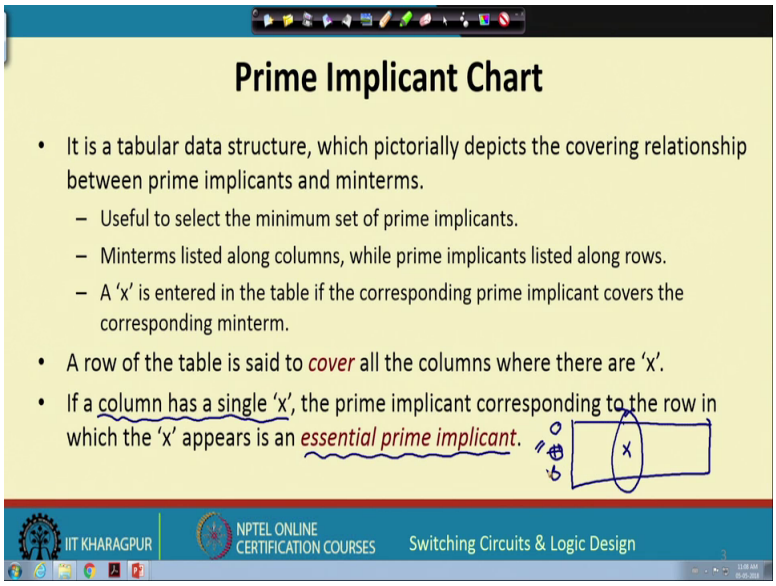

And the other point to notice that if we see that in the table let us say we we have again the table, there is some prime implicant, where we see or there are several prime implicants ok, several prime implicants are there. And you see that in some column there is a single X for example, in some column let us say this column there is a single X may be in the middle one.

What does this mean? This means this particular minterm corresponding to this column is covered by only this prime implicant not the others. So, I must select this prime implicant, otherwise this column will not get selected not get covered. So, such prime implicant which corresponds to a X, which is the only X in a column is called essential prime implicant, that is what I must include in any minimal set of prime implicants that we generate.

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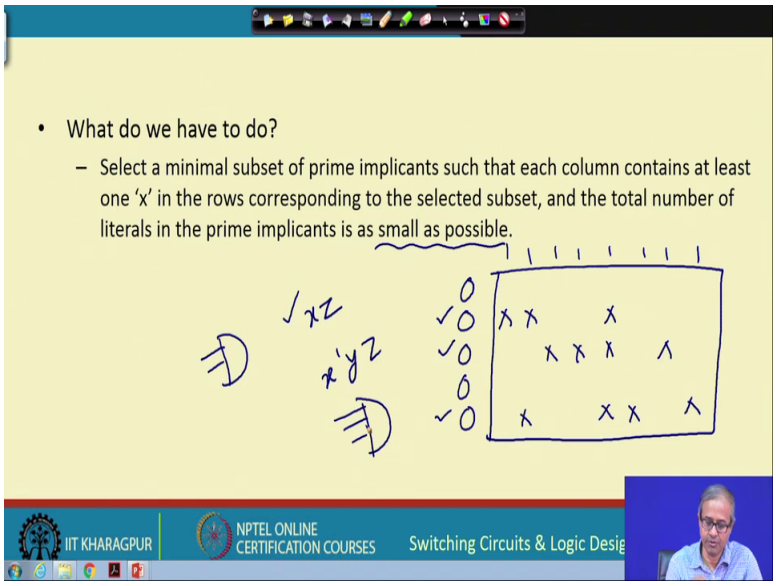

So, what we have to do finally, we will have to select a minimal subset of their prime implicant that is what I mentioned, but what is the requirement, requirement is that every column which means the true minterms, will contain at least one checkmark corresponding to the selected subset which means what I mean to say is that, suppose there is a table there are many prime implicants. Suppose I finally, select this one this one and this one. So, what I say is that the check mark should be such that, these 3 prime implicants will be covering all the columns; that means, all the true minterms.

And we have to select the set of prime implicants in such a way that the total number of literals in this prime implicant is as small as possible, as small as possible means the gate realization will be as simple as possible. Let us say one prime implicant may corresponds to x z, some other may correspond to x bar y z. So, if given a choice we will select this because, this contains smaller number of literals. So, this will be requiring a 2 input gate, but this will be requiring a 3 input gate fine. Let us take an example.

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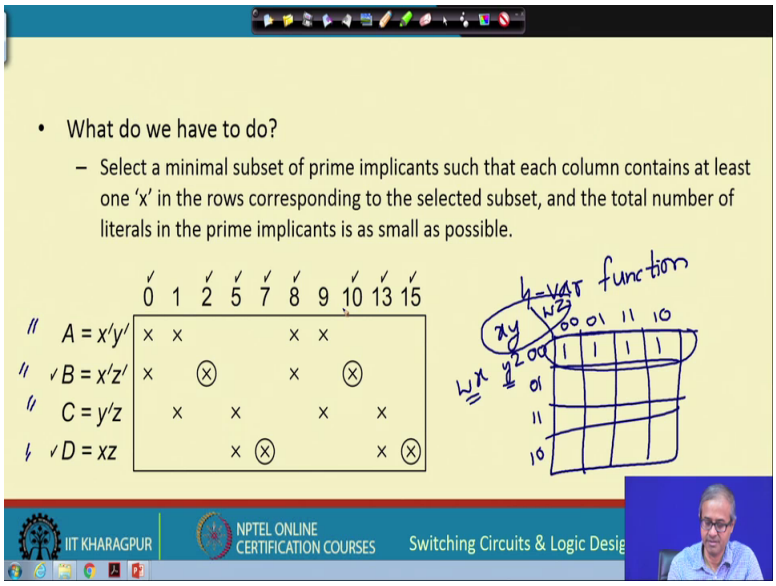

Now, here we are showing a prime implicant chart, just an example of an prime implicants chart, where I am showing 4 prime implicants, which I have called A B C and D just ignore this check marks for the time being.

And there are 10 true minterms 0 1 2 5 7 8 9 10 13 15. So, this is actually a 4 variable function, this is a 4 variable function. And these are the check marks which says that this x bar y bar this prime implicant covers, 0 1 8 and 9 this can also be reflected from the Karnaugh map if you for example, draw the Karnaugh map. Let us say if I have a Karnaugh map like this, where I have x y along this direction. And let us say w z along this direction. So, you will be having 0 0 0 1 1 1 and 1 0 here again 0 0 0 1 1 1 and 1 0. So, this x bar y bar will be corresponding to this cube, this whole row right.

So, if you see so, what this corresponds to, this corresponds to this 4 0 1 8 and 9. So, depending on in which order these are of course, I have put it in a different order, it should actually be w x and y z, if you put it w x and y z, then it will be 0 1 8 and 9, this you can check fine. So, in the same way for all the different 4 prime implicants, we list which are the true minterms which are covered.

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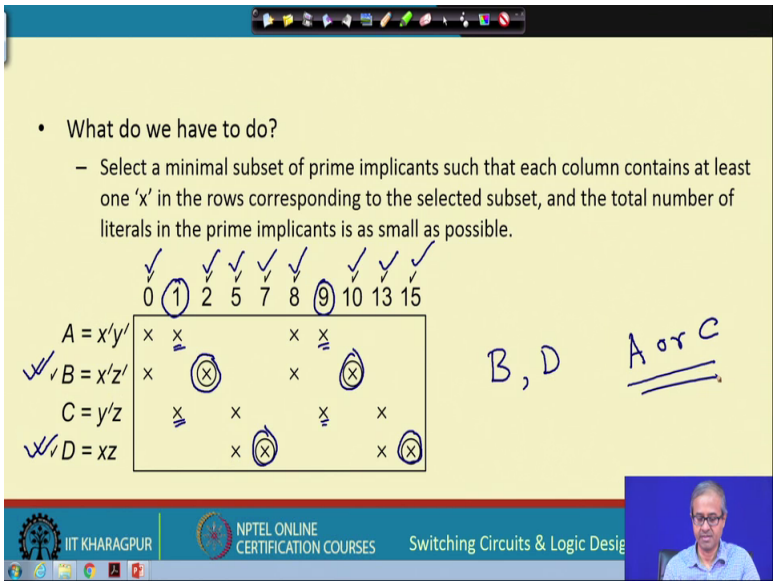

Then we look at which of the columns have a single check mark. So, I can see column number 2 here, there is a single check mark column number 7 also, this 10 also and 15 also. So, these are the only check marks in the column, which means that the corresponding prime implicants are essential. So, I must select B and D of course, there is a choice A and C that is not essential because there are multiple cross marks.

Now, see once we have selected B and D so, all the check marks corresponding to B and D will get covered B has a cross mark here, a cross mark here, a cross mark here, a cross mark here, D has a cross mark here and here. So, what are the columns which are still remaining 1 is remaining, 9 is remaining these 2 right. Now, in order to cover 1 and 9 I have a choice, I can either use A, because A covers 1 as well as 9, or I can use C, C also covers 1 as well as 9. So, I will have to choose B, I will have to choose D, then I will be choosing A or C this is a choice right. .

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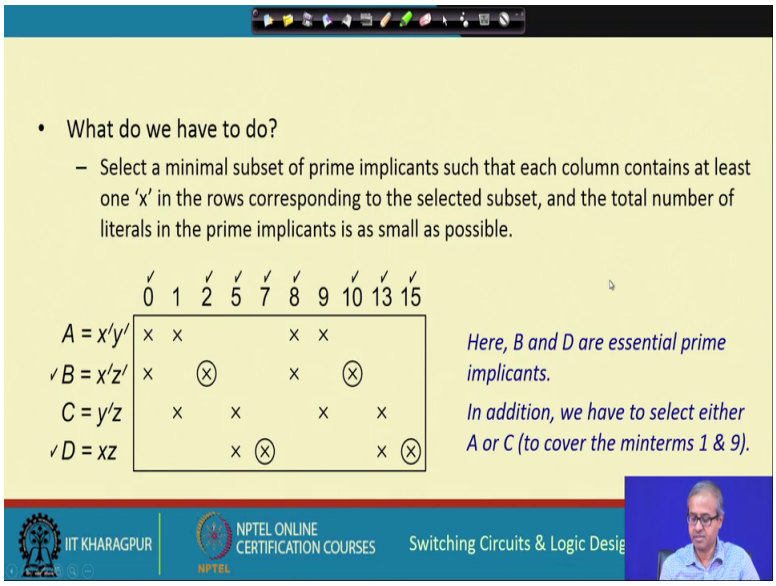

This is a choice that we have, this is exactly what we mentioned B and D are the essential prime implicants, these we must choose and addition we can choose either A or C because, this 1 and 9 these 2 minterms are left out both A and C are covering them ok, this is how we can get the set of prime implicants which can be covered.

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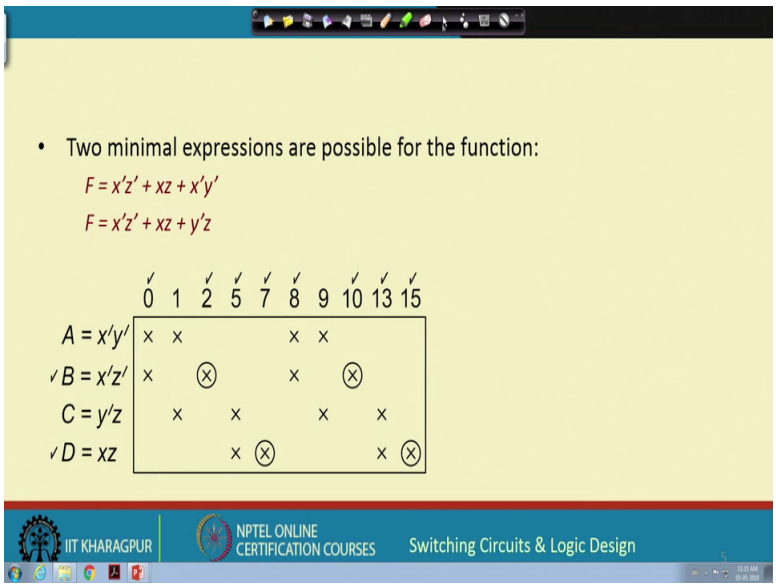

So, in this example as I said there are 2 minimal expressions possible, B and D are mandatory. So, this corresponds to B, this corresponds to D so, here also I have B here

also I have D and, I can include either A or I can include C. So, there are two possible minimum expressions for this particular function ok.

This is how we proceed with the tabular method, step 1 we create all the prime implicants of the function, step 2 we create the prime implicant chart and, try to select a subset of prime implicants which covers all the minterm true minterms. Now, there are some tricks here, which can help us in minimizing or in selecting the set of prime implicants, we shall be looking at some of these rules now ok.

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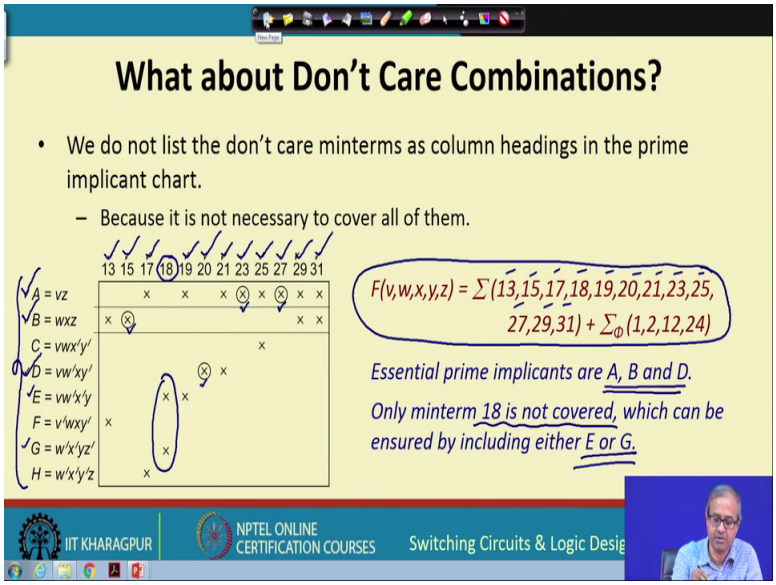

First let us see that we can have don't care combinations in a function. Now, we have seen earlier, when you have do not care combinations, we include the do not care combinations in the first step, when we were generating the prime implicants.

Because prime implicants can also include the don't cares ok, but when we create the prime implicant chart, we need not include the don't care in the columns, why because it is not necessary to cover all the don't care minterms by the set of prime implicants that we chose that is why we do not show they don't care minterms in the columns, because they need not be covered. We only show the essential minterms the ones and the function in the columns, which must be covered by the set of prime implicants ok. Let us take an example here, while here I am just showing prime implicant chart, where you see there are 8 prime implicants which have already been generated let us say in the first step.

Now, these horizontal lines indicate the size of the minterms, the first one is smallest 2 literals; second one is 3 literals remaining all 4 literals. So, the corresponding function here is this. So, we have used this don't care literals also to generate all the prime implicants, but when you list in the columns we only list these, but not the don't care ones right.

Now, let us see which of the prime implicants are essential look at the columns, where you have a single one, we have a single one here, we have a single one here, we have a single one here and also here this 4 circled ones. So, correspondingly A is essential, B is essential and, also D is essential. Now, once we have selected the essential ones you see a covers 17 19 21 23 25 27 29 31 many of them.

B covers 13 15 29 31 is already covered and D covers 20 and 21 21 is covered 20. So, what we are left out here is only 18. So, only minterm 18 is not covered. So, after we select A B D in order to cover minterm 18, we have a choice there are 2 cross marks either, we can use E or you can use G. Since both of them are having the same number of literals there cost is the same. So, here we will be choosing A B and D, essentially and either E or G right. This will be our final selection in this example ok.

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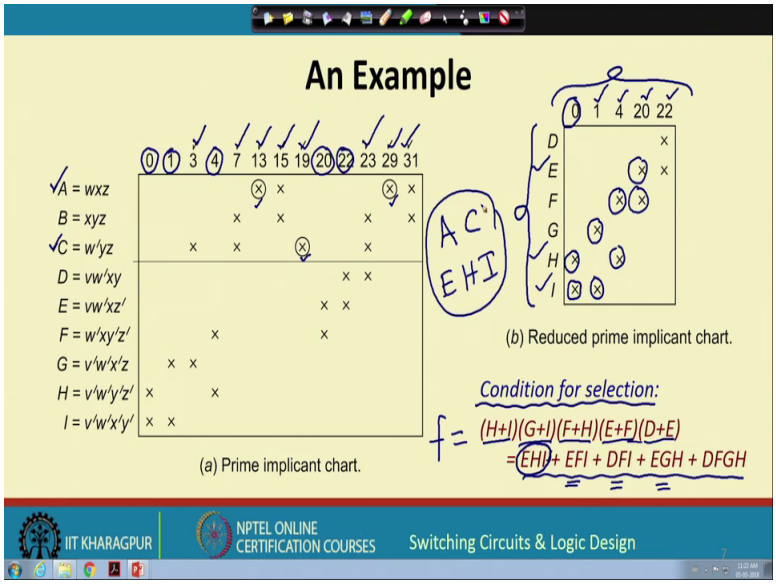

Let us take another example so, here is the here you show a similar table here the essential cross mark; that means, single cross marks are here, here and here, which means A is essential, C is essential these two are essential, because A is essential, we

have a tick mark here here here and here. And because C is essential we have a tick mark here here 19 and 23. So, what we are left out with we are left out with 0 1 4 20 and 22 and also yeah these 5.

So, the earlier example we had only one. So, just by looking at we could take a decision that which one to select, but in general for larger table, there can be many such true minterms which are still left out. So, now, we have to find out. So, mean out of the remaining minterms, which are not out of the I mean remaining prime implicants, which are not essential, what is the minimum set I have to select to cover 0 1 4 20 and 22.

So, the procedure is we first create a reduced prime implicant chart. So, what is this? So, the columns which have been covered are deleted, only this is 0 1 4 20 and 22 are left. So, you only list these you only list these 5 and, A and C have already been taken up. So, the remaining prime implicants are list in the columns.

So, now you have a choice here you see these remaining minterms can be covered by D E F G H I in this way this is check marks. Now, we can very easily find out what is the minimum set of these prime implicants that you cover all of them, how do you do it, we try to determine, a condition for selection and the condition for selection is determined in the following way. Let us look at this minterm 0, 0 is covered by either H or by I, I write H or I 1 is covered by either G or I, I write G or I 4 is covered by F or H I write F, or H 20 is covered by E or F, I write E or F and 22 is covered by D or E I write D or E.

So, if I expand using multiplication rule of switching algebra I get an expression like this. So, I can use any one of these product terms to realize my function because, I have to make this true, if this denote the function f, I have to make f equal to 1 to make f equal to 1 I can make any one of these product terms equal to 1, last one will be expensive there are 4 terms, let us take EHI for example.

So, we will be selecting E will be selecting H, we will be selecting, I there are other alternatives you can select E F I D F I E G H. So, our choice will be A and C and let us say E H I. So, we will be selecting these 5 right, this is how we do it. .

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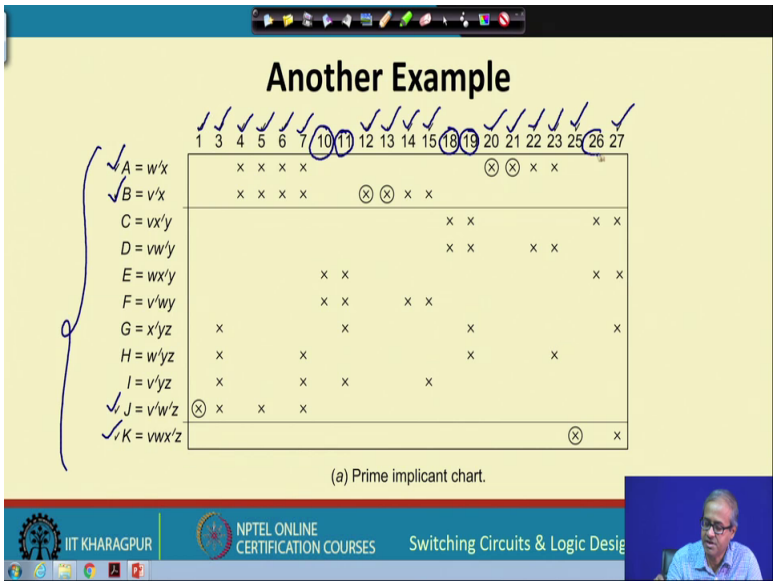

Let us take another example this is a slightly bigger example, where you see there are 4 5 6 7 8 9 10 11 prime implicants, and this circled ones are the essential columns. So, the corresponding essential prime implicants are A B and J and finally, here K. So, it covers A coveres 4 5 6 7 20 21 22 23, B covers 4 5 6 7 12 13 14 15 that is it. J covers 1 3 5 is covered 7 is covered that is it. And K covers 25 and 27. So, what you are left out with we are left out with 10 11 18 19 and 26 these five right.

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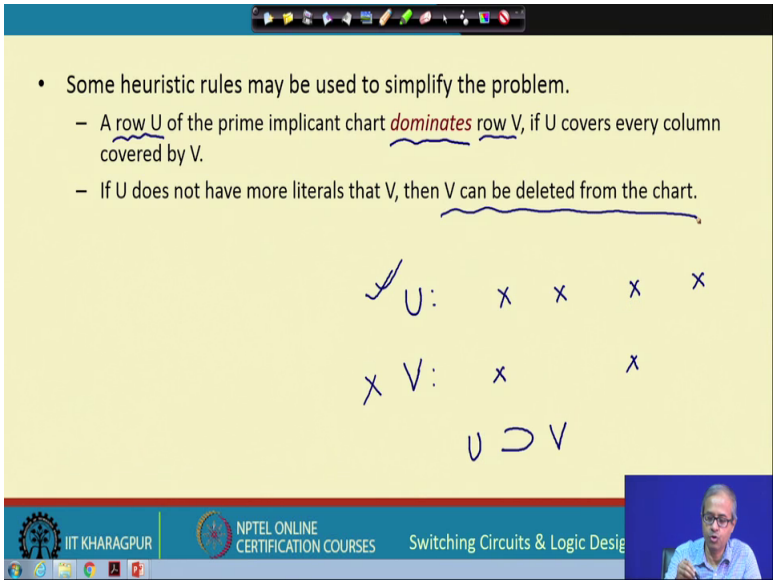

Now, we present some simple rules, using which you can further reduce the size of the table, there is a concept of domination, row can dominate another row, or column can dominate another column. So, what is the basic concept of row domination, it says a row U of the table, will dominate a row V, if U covers every column covered by V. For example, in row U I have a checkmark here, I have a checkmark here, I have a check mark here, I have a checkmark here.

And, in V let us suppose there is a checkmark here and there is a checkmark here. So, we say U dominates V because, wherever V has a checkmark, U also has a checkmark. So, in such case what you do you can delete V from the chart, because the idea is if we select U these 2 columns automatically will get selected. So, we do not need to keep V so, V can be deleted from the chart.

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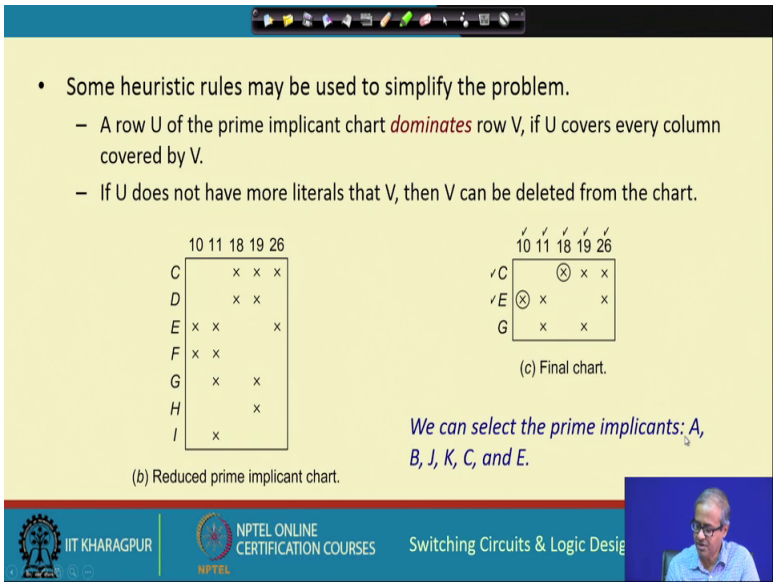

This is row domination rule so, that same example that you saw earlier 10 11 18 19 16, were remaining and these were the remaining prime implicants 7 of them. Now, if you use row domination you see D is getting dominated by C because, C has check mark here D has here so, D you can delete.

Similarly F is dominated by E checkmark E also has so, F also can be deleted H is dominated by C and also G H has a single checkmark H can be deleted. Similarly I is dominated by E as well as G, I can also be deleted. So, you are left with only C E and G. So, idea is that you make your tables smaller and, then you do the next step. So, instead

of using a large table, you minimize the size of the table. So, it becomes easier for you, you see in the minimized table you can immediately see.

Now, there is a single checkmark in column here and here so, C will be essential E will be essential. So, if you take these two all the columns are getting covered so, C and E right. So, in the earlier previous slide we had selected a sorry were selected A B J and K and, here we are selecting C and E in addition, this is how we get our final solution. .

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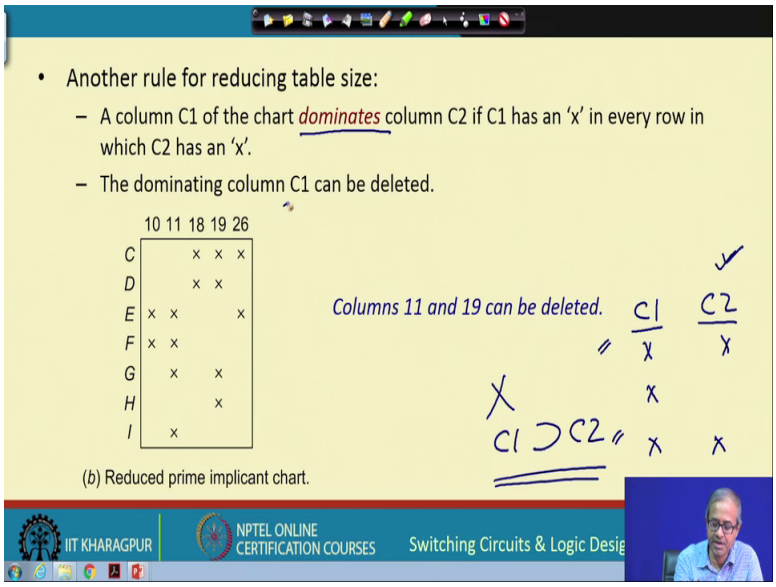

Now, there is a there is a similar rule for dominating columns, dominating column I will showing with this example, it says a column will dominate another column like, suppose I have a column C 1, I have a column C 2, if C 1 will dominate C 2 if C 1 has a cross in every row, where C 2 has a cross; that means, if I have let us say cross here here and here and C 2 has a cross here and here, then I say C 1 dominates C 2.

But for column domination the rule is different, the column which dominates that can be deleted, because you see the idea is that, if we can cover column C 2 say either by this prime implicant, or this prime implicant. So, automatically C 1 you will also get covered. So, if I cover C 2 that will imply that C 1 also gets covered that is why the dominating column C 1 can be deleted here right. So, columns so, here in this example you take this example here, column 11 can be deleted, this is column 11 you see column 1 dominates column 10 ok.

That is why column 11 can be deleted. Similarly column 19 dominates column 18. So, column 19 can also be deleted. So, you can also reduce the size of the table in terms of column. So, the idea is that when you have this prime implicant chart, you first select the essential prime implicants take them out your size of the table reduces, then you use row dominants and column dominants rule, the further reduce this size of the table, then try to find out what will be the set of minterms set of prime implicants, that will cover all the minterms, this is the basic idea behind the Quine McCluskey method.

So, with this we come to the end of this lecture. So, we have seen over the last few lectures the different ways to minimize a switching function, earlier we looked at the algebraic methods, here we talked about the graphical approach the Karnaugh map method and, also we talked about a more systematic approach a Quine McCluskey method which can be used even by hand, you can use a function of 7 or 8 variables to create the list of prime implicants that minimize and I also said. This is much easier to automate, you can write a program for the Quine McCluskey method in a much easier way as compared to the Karnaugh map method.

Thank you.